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ON YUKAWA COUPLINGS IN THE ZEE-BABU MODEL

HOANG NGOC LONG
Institute of Physics, Vietnam Academy of Science and Technology,
10 Dao Tan, Ba Dinh, Hanoi, Vietnam
VO VAN VIEN
Department of Physics, Tay Nguyen University,
587 Le Duan, Buon Ma Thuot, Vietnam

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Abstract. *The exact solution for neutrino mass matrix of the Zee-Babu model is derived. The tribimaximal mixing imposes the conditions on the Yukawa couplings, from which the normal mass hierarchy is preferred. The derived conditions give a possibility of maximal CP violation in the neutrino sector.*

I. INTRODUCTION

Nowadays, particle physicists are attracted by two exciting subjects: Higgs and neutrino physics. The neutrino mass and mixing are the first evidence of the beyond Standard Model. Many experiments show that neutrinos have tiny masses and their mixing is still mysterious [1]. Recent data are a clear sign of rather large value θ_{13} [2].

The discovery of the long-waiting Higgs boson at around 125 GeV [3] opened a new chapter in particle physics. It is essential for us to determine which model the discovered Higgs boson belongs to? For this aim, the diphoton decay of the Higgs boson plays very important role. It is expected that new physics might enter here to modify the standard model (SM) Higgs property.

Within the above mentioned reasons, the search of extended model coinciding with the current data on neutrino and Higgs physics is one of top priorities. In our opinion, the model with simplest particle content is preferred. In the SM, neutrinos are strictly massless. For neutrino mass, it was first pointed out by Zee in Ref. [4] in which new scalars are added in the Higgs sector with neutrino masses induced at the one-loop level. After that a two-loop scenario called Zee-Babu model [5] was proposed. The Zee-Babu model [4–6] with just two additional charged Higgs bosons (h^-, k^{--}) carrying lepton number 2, is very attractive. In this model, neutrinos get mass from two-loop radiative corrections, which can fit current neutrino data. Moreover, the singly and doubly charged scalars that are new in the model can explain the large annihilation cross section of a dark matter pair into two photons as hinted by the recent analysis of the Fermi γ-ray space telescope data [7], if new charged scalars are relatively light and have large couplings to a pair of dark matter particles. These new scalars can also enhance the $B(H \to \gamma\gamma)$, as the recent LHC results may suggest.

The Zee-Babu model contains the Yukawa couplings which are specific for lepton number violating processes. There are a lot of works on constraints the parameter space of the model [8, 9].

In this paper, starting from the neutrino mass matrix, we get the exact solution, i.e., the eigenstates and the eigenvalues. As a consequence, the neutrino mixing matrix is followed. With this exact solution, we can fit with current data and get constraints on the couplings. We do hope that the experiments in the near future will approve or rule out the model.

II. NEUTRINO MASS MATRIX IN THE ZEE-BABU MODEL

The Zee-Babu model [5] includes two $\mathrm{SU}(2)_L$ singlet Higgs fields, a singly charged field h^- and a doubly charged field k^{--}. Moreover, right-handed neutrinos are not introduced. The addition of these singlets gives rise to the Yukawa couplings:

$$L_Y = f_{ab}\overline{(\psi_{aL})^C}\psi_{bL}h^+ + h'_{ab}\overline{(l_{aR})^C}l_{bR}k^{++} + H.c., \tag{1}$$

where ψ_L stands for the left-handed lepton doublet, l_R for the right-handed charged lepton singlet and $(a, b = e, \mu, \tau)$ being the generation indices, a superscript ^{C} indicates charge conjugation. Here $\psi^C = C\overline{\psi}^T$ with C is the charge-conjugation matrix. The coupling constant f_{ab} is antisymmetric $(f_{ab} = -f_{ba})$, whereas h_{ab} is symmetric $(h_{ab} = h_{ba})$. Gauge invariance precludes the singlet Higgs fields from coupling to the quarks. In terms of the component fields, the interaction Lagrangian is given

$$\begin{aligned}\mathcal{L}_Y =& 2\left[f_{e\mu}(\bar{\nu_e^c}\mu_L - \bar{\nu_\mu^c}e_L) + f_{e\tau}(\bar{\nu_e^c}\tau_L - \bar{\nu_\tau^c}e_L) + f_{\mu\tau}(\bar{\nu_\mu^c}\tau_L - \bar{\nu_\tau^c}\mu_L)\right]h^+ \\ &+ \left[h_{ee}\bar{e^c}e_R + h_{\mu\mu}\bar{\mu^c}\mu_R + h_{\tau\tau}\bar{\tau^c}\tau_R + h_{e\mu}\bar{e^c}\mu_R + h_{e\tau}\bar{e^c}\tau_R + h_{\mu\tau}\bar{\mu^c}\tau_R\right]k^{++} + H.c. \end{aligned} \tag{2}$$

where we have used $h_{aa} = h'_{aa}, h_{ab} = 2h'_{ab}$ for $a \neq b$. Eq. (1) conserves lepton number, therefore, itself cannot be responsible for neutrino mass generation.

The Higgs potential contains the terms:

$$V(\phi, h^+, k^{++}) = \mu(h^-h^-k^{++} + h^+h^+k^{--}) + \cdots, \tag{3}$$

which violate lepton number by two units. They are expected to cause Majorana neutrino masses.

In the literature, Majorana neutrino masses are generated at the two-loop level via the diagram shown in [6] and again depicted in Fig.1. The corresponding mass matrix for Majorana neutrinos is as follows

$$M_{ab} \;\;=\;\; 8\mu f_{ac}h^*_{cd}m_c m_d I_{cd}(f^+)_{db}, \tag{4}$$

The integral I_{cd} is given by [10]

$$\begin{aligned} I_{cd} =& \int\frac{d^4k}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\frac{1}{k^2-m_c^2}\frac{1}{k^2-M_h^2}\frac{1}{q^2-m_d^2} \\ &\times\frac{1}{q^2-M_h^2}\frac{1}{(k-q)^2-M_k^2} \end{aligned} \tag{5}$$

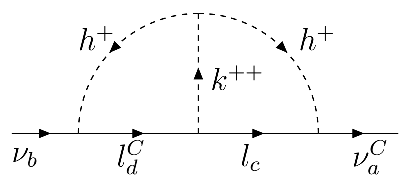

Fig. 1. The two-loop diagram in the Zee-Babu model.

Note that Eq. (5) is simplified by neglecting the charged lepton masses [12].

To evaluate the above integral, ones have neglected the charged lepton masses in the denominator, since these masses are much smaller than the charged scalar masses M_h and M_k. Then

$$I_{cd} \simeq I = \frac{1}{(16\pi^2)^2}\frac{1}{M^2}\frac{\pi^2}{3}\tilde{I}(r), \;\; M \equiv \max(M_k, M_h) \tag{6}$$

which does not depend on lepton masses. Here $\tilde{I}(r)$ is a function of the ratio of the masses of the charged Higgses $r \equiv M_k^2/M_h^2$,

$$\tilde{I}(r) = \begin{cases} 1 + \frac{3}{\pi^2}(\log^2 r - 1) & \text{for } r \gg 1 \\ 1 & \text{for } r \to 0, \end{cases} \tag{7}$$

which is close to 1 for a wide range of scalar masses.

The neutrino mass matrix arisen from (4) is given by

$$\mathcal{M}_\nu = -I\mu f_{\mu\tau}^2 \times$$
$$\begin{pmatrix} \epsilon^2\omega_{\tau\tau} + 2\epsilon\epsilon'\omega_{\mu\tau} + \epsilon'^2\omega_{\mu\mu} & \epsilon\omega_{\tau\tau} + \epsilon'(\omega_{\mu\tau} - \epsilon\omega_{e\tau} - \epsilon'\omega_{e\mu}) & -\epsilon'\omega_{\mu\mu} - \epsilon(\omega_{\mu\tau} + \epsilon\omega_{e\tau} + \epsilon'\omega_{e\mu}) \\ \star & \omega_{\tau\tau} + \epsilon'^2\omega_{ee} - 2\epsilon'\omega_{e\tau} & \epsilon\epsilon'\omega_{ee} - \omega_{\mu\tau} - \epsilon\omega_{e\tau} + \epsilon'\omega_{e\mu} \\ \star & \star & \omega_{\mu\mu} + 2\epsilon\omega_{e\mu} + \epsilon^2\omega_{ee} \end{pmatrix} \tag{8}$$

where we have redefined parameters:

$$\epsilon \equiv \frac{f_{e\tau}}{f_{\mu\tau}}, \qquad \epsilon' \equiv \frac{f_{e\mu}}{f_{\mu\tau}} \qquad \omega_{ab} \equiv m_a h_{ab}^* m_b \tag{9}$$

Let us denote

$$\begin{aligned} \omega'_{\tau\tau} &\equiv \omega_{\tau\tau} + \epsilon'^2\omega_{ee} - 2\epsilon'\omega_{e\tau}, \\ \omega'_{\mu\tau} &\equiv \omega_{\mu\tau} + \epsilon\omega_{e\tau} - \epsilon'\omega_{e\mu} - \epsilon\epsilon'\omega_{ee}, \\ \omega'_{\mu\mu} &\equiv \omega_{\mu\mu} + 2\epsilon\omega_{e\mu} + \epsilon^2\omega_{ee}. \end{aligned} \tag{10}$$

then the neutrino mass matrix can be rewritten in the compact form

$$\mathcal{M}_\nu = -I\mu f_{\mu\tau}^2 \begin{pmatrix} \epsilon^2\omega'_{\tau\tau} + 2\epsilon\epsilon'\omega'_{\mu\tau} + \epsilon'^2\omega'_{\mu\mu} & \epsilon\omega'_{\tau\tau} + \epsilon'\omega'_{\mu\tau} & -\epsilon\omega'_{\mu\tau} - \epsilon'\omega'_{\mu\mu} \\ \star & \omega'_{\tau\tau} & -\omega'_{\mu\tau} \\ \star & \star & \omega'_{\mu\mu} \end{pmatrix}. \tag{11}$$

The above matrix has three exact eigenvalues given by

$$\begin{aligned}
\lambda_1 =& 0, \\
\lambda_2 =& \frac{\mu I f_{\mu\tau}^2}{2}\left\{\omega'_{\mu\mu}(1+\epsilon'^2) + 2\epsilon\epsilon'\omega'_{\mu\tau} + \omega'_{\tau\tau}(1+\epsilon^2)\right. \\
& \left. - \sqrt{\left[\omega'_{\mu\mu}(1+\epsilon'^2) + 2\epsilon\epsilon'\omega'_{\mu\tau} + \omega'_{\tau\tau}(1+\epsilon^2)\right]^2 + 4(1+\epsilon^2+\epsilon'^2)(\omega'^2_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})}\right\} \\
\equiv& \frac{\mu I f_{\mu\tau}^2}{2}\left(F_{\mu\tau} - \sqrt{F_{\mu\tau}^2 + 4(1+\epsilon^2+\epsilon'^2)(\omega'^2_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})}\right), \\
\lambda_3 =& \frac{\mu I f_{\mu\tau}^2}{2}\left\{\omega'_{\mu\mu}(1+\epsilon'^2) + 2\epsilon\epsilon'\omega'_{\mu\tau} + \omega'_{\tau\tau}(1+\epsilon^2)\right. \\
& \left. + \sqrt{\left[\omega'_{\mu\mu}(1+\epsilon'^2) + 2\epsilon\epsilon'\omega'_{\mu\tau} + \omega'_{\tau\tau}(1+\epsilon^2)\right]^2 + 4(1+\epsilon^2+\epsilon'^2)(\omega'^2_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})}\right\} \\
\equiv& \frac{\mu I f_{\mu\tau}^2}{2}\left(F_{\mu\tau} + \sqrt{F_{\mu\tau}^2 + 4(1+\epsilon^2+\epsilon'^2)(\omega'^2_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})}\right),
\end{aligned} \tag{12}$$

where we have denoted

$$F_{\mu\tau} \equiv \omega'_{\mu\mu}(1+\epsilon'^2) + 2\epsilon\epsilon'\omega'_{\mu\tau} + \omega'_{\tau\tau}(1+\epsilon^2). \tag{13}$$

The massless eigenstate is given by

$$\nu_1 = \frac{1}{\sqrt{f_{e\mu}^2 + f_{e\tau}^2 + f_{\mu\tau}^2}}(f_{\mu\tau}\nu_e - f_{e\tau}\nu_\mu + f_{e\mu}\nu_\tau). \tag{14}$$

The mass matrix (11) is diagonalized as

$$U^T\mathcal{M}_\nu U = \mathrm{diag}(0, \lambda_2, \lambda_3), \tag{15}$$

where

$$U = \begin{pmatrix} \frac{1}{\sqrt{1+\epsilon^2+\epsilon'^2}} & \frac{A_1}{\sqrt{1+A_1^2+B_1^2}} & -\frac{A_2}{\sqrt{1+A_2^2+B_2^2}} \\ -\frac{\epsilon}{\sqrt{1+\epsilon^2+\epsilon'^2}} & \frac{B_1}{\sqrt{1+A_1^2+B_1^2}} & -\frac{B_2}{\sqrt{1+A_2^2+B_2^2}} \\ \frac{\epsilon'}{\sqrt{1+\epsilon^2+\epsilon'^2}} & \frac{1}{\sqrt{1+A_1^2+B_1^2}} & -\frac{1}{\sqrt{1+A_2^2+B_2^2}} \end{pmatrix} \tag{16}$$

with the new notations

$$\begin{aligned} A_{1,2} &\equiv -\left\{2\epsilon'\omega'_{\mu\tau}(1+\epsilon^2)+\epsilon^3\omega'_{\tau\tau}+\epsilon[(\epsilon'^2-1)\omega'_{\mu\mu}+\omega'_{\tau\tau}]\right. \\ &\mp \left.\epsilon\sqrt{F^2_{\mu\tau}+4(1+\epsilon^2+\epsilon'^2)(\omega'^2_{\mu\tau}-\omega'_{\mu\mu}\omega'_{\tau\tau})}\right\} / \left(2\epsilon\epsilon'\omega'_{\mu\mu}+2(1+\epsilon^2)\omega'_{\mu\tau}\right), \quad (17) \\ B_{1,2} &\equiv \frac{(1+\epsilon'^2)\omega'_{\mu\mu}-(1+\epsilon^2)\omega'_{\tau\tau}\pm\epsilon\sqrt{F^2_{\mu\tau}+4(1+\epsilon^2+\epsilon'^2)(\omega'^2_{\mu\tau}-\omega'_{\mu\mu}\omega'_{\tau\tau})}}{2\left[\epsilon\epsilon'\omega'_{\mu\mu}+(1+\epsilon^2)\omega'_{\mu\tau}\right]}. \quad (18) \end{aligned}$$

The eigenstates ν_i corresponding to the eigenvalues $\lambda_i, (i=1,2,3)$ are found to be

$$\begin{aligned} \nu_1 &= \frac{1}{\sqrt{1+\epsilon^2+\epsilon'^2}}\nu_e-\frac{\epsilon}{\sqrt{1+\epsilon^2+\epsilon'^2}}\nu_\mu+\frac{\epsilon'}{\sqrt{1+\epsilon^2+\epsilon'^2}}\nu_\tau \\ &\equiv \frac{1}{\sqrt{f^2_{e\mu}+f^2_{e\tau}+f^2_{\mu\tau}}}(f_{\mu\tau}\nu_e-f_{e\tau}\nu_\mu+f_{e\mu}\nu_\tau), \\ \nu_2 &= \frac{A_1}{\sqrt{1+A_1^2+B_1^2}}\nu_e+\frac{B_1}{\sqrt{1+A_1^2+B_1^2}}\nu_\mu+\frac{1}{\sqrt{1+A_1^2+B_1^2}}\nu_\tau, \\ \nu_3 &= \frac{-A_2}{\sqrt{1+A_2^2+B_2^2}}\nu_e+\frac{-B_2}{\sqrt{1+A_2^2+B_2^2}}\nu_\mu+\frac{-1}{\sqrt{1+A_2^2+B_2^2}}\nu_\tau. \end{aligned} \quad (19)$$

From the explicit expressions of eigenstates, we obtain some useful relations

$$\begin{aligned} A_1A_2+B_1B_2+1 &= 0, \\ A_1-\epsilon B_1+\epsilon' &= 0, \\ A_2-\epsilon B_2+\epsilon' &= 0, \\ (A_1-A_2)/(B_1-B_2) &= \epsilon. \end{aligned} \quad (20)$$

and

$$\begin{aligned} A_1A_2 &= \frac{(\epsilon'^2-\epsilon^2)\omega'_{\mu\tau}+\epsilon\epsilon'(\omega'_{\tau\tau}-\omega'_{\mu\mu})}{\epsilon\epsilon'\omega'_{\mu\mu}+(1+\epsilon^2)\omega'_{\mu\tau}}, \\ B_1B_2 &= -\frac{(1+\epsilon'^2)\omega'_{\mu\tau}+\epsilon\epsilon'\omega'_{\tau\tau}}{\epsilon\epsilon'\omega'_{\mu\mu}+(1+\epsilon^2)\omega'_{\mu\tau}}. \end{aligned} \quad (21)$$

III. CONSTRAINTS FROM THE TRIBIMAXIMAL MIXING

The current data on neutrino mass and mixing show that the tribimaximal mixing is very realistic [14]

$$U_{HPS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (22)$$

Comparing (16) with (22) yields the following conditions

$$\epsilon = -\epsilon' = \frac{1}{2}, \quad (23)$$

$$A_2 = 0, \quad (24)$$

$$B_1 = 1, \quad (25)$$

$$A_1 = 1, \quad (26)$$

$$B_2 = -1 \quad (27)$$

Eq. (23) leads to

$$f_{e\tau} = -f_{e\mu} = \frac{1}{2} f_{\mu\tau} \quad (28)$$

Substitution of (23) into expressions of A_1, A_2, B_1 and B_2 yields

$$A_1 = -\frac{(3\omega'_{\mu\mu} + 10\omega'_{\mu\tau} - 5\omega'_{\tau\tau}) + \sqrt{(3\omega'_{\mu\mu} + 10\omega'_{\mu\tau} - 5\omega'_{\tau\tau})^2 + 16(\omega'_{\mu\mu} - 5\omega'_{\mu\tau})(\omega'_{\mu\mu} - \omega'_{\tau\tau})}}{4(\omega'_{\mu\mu} - 5\omega'_{\mu\tau})}, \quad (29)$$

$$A_2 = \frac{-(3\omega'_{\mu\mu} + 10\omega'_{\mu\tau} - 5\omega'_{\tau\tau}) + \sqrt{(3\omega'_{\mu\mu} + 10\omega'_{\mu\tau} - 5\omega'_{\tau\tau})^2 + 16(\omega'_{\mu\mu} - 5\omega'_{\mu\tau})(\omega'_{\mu\mu} - \omega'_{\tau\tau})}}{4(\omega'_{\mu\mu} - 5\omega'_{\mu\tau})}, \quad (30)$$

$$B_1 = -\frac{5(\omega'_{\mu\mu} - \omega'_{\tau\tau}) + \sqrt{(5\omega'_{\mu\mu} - 2\omega'_{\mu\tau} + 5\omega'_{\tau\tau})^2 + 96(\omega'^2_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})}}{2(\omega'_{\mu\mu} - 5\omega'_{\mu\tau})}, \quad (31)$$

$$B_2 = \frac{-5(\omega'_{\mu\mu} - \omega'_{\tau\tau}) + \sqrt{(5\omega'_{\mu\mu} - 2\omega'_{\mu\tau} + 5\omega'_{\tau\tau})^2 + 96(\omega'^2_{\mu\tau} - \omega'_{\mu\mu}\omega'_{\tau\tau})}}{2(\omega'_{\mu\mu} - 5\omega'_{\mu\tau})}, \quad (32)$$

It can be checked that with the help of (23), all remaining conditions [from (24) to (27)] are satisfied if

$$\omega'_{\mu\mu} = \omega'_{\tau\tau} \quad (33)$$

equivalently for the Yukawa couplings

$$\omega_{\mu\mu} + \omega_{e\mu} = \omega_{\tau\tau} + \omega_{e\tau} \quad (34)$$

Note that the derived limit is slightly differs from those given in [8].

From the conditions (23) and (33) we obtain [1]

$$\begin{aligned} m_1 \equiv& \lambda_1 = 0, \\ m_2 \equiv& \lambda_2 = \mu I f^2_{\mu\tau}(\omega'_{\mu\mu} + \omega'_{\mu\tau}) \equiv \mu I f^2_{\mu\tau}(\omega'_{\tau\tau} + \omega'_{\mu\tau}), \\ m_3 \equiv& \lambda_3 = \frac{3}{2}\mu I f^2_{\mu\tau}(\omega'_{\mu\mu} - \omega'_{\mu\tau}) \equiv \frac{3}{2}\mu I f^2_{\mu\tau}(\omega'_{\tau\tau} - \omega'_{\mu\tau}). \end{aligned} \quad (35)$$

[1]The integration in Fig.1 is linear divergent and has a surface term [11], which give a similar form of mass matrix.

or

$$\omega'_{\tau\tau} \equiv \omega'_{\mu\mu} = \frac{1}{\mu I f^2_{\mu\tau}}\left(\frac{m_2}{2} + \frac{m_3}{3}\right), \ \omega'_{\mu\tau} = \frac{1}{\mu I f^2_{\mu\tau}}\left(\frac{m_2}{2} - \frac{m_3}{3}\right). \tag{36}$$

Using the experimental constraints [17]

$$\begin{aligned} |m_2^2 - m_1^2| &\equiv m_2^2 = 7.59 \times 10^{-5}\text{eV}^2, \\ |m_3^2 - m_2^2| &= 2.43 \times 10^{-3}\text{eV}^2 \end{aligned} \tag{37}$$

and assuming m_2, m_3 to be real, we obtain following four possibilities

- The first case

$$\omega'_{\mu\tau} = -\frac{0.0293855}{\mu I f^2_{\mu\tau}}, \quad \omega'_{\mu\mu} = \frac{0.0206734}{\mu I f^2_{\mu\tau}}, \tag{38}$$

$$\frac{\omega'_{\mu\mu}}{\omega'_{\mu\tau}} \equiv \frac{\omega'_{\tau\tau}}{\omega'_{\mu\tau}} = -0.703526, \tag{39}$$

$$m_2 \equiv -0.00871\text{eV}, \ m_3 = 0.0500591\text{eV}. \tag{40}$$

- The second case

$$\omega'_{\mu\tau} = \frac{0.0206735}{\mu I f^2_{\mu\tau}}, \quad \omega'_{\mu\mu} = -\frac{0.0293856}{\mu I f^2_{\mu\tau}}, \tag{41}$$

$$\frac{\omega'_{\mu\mu}}{\omega'_{\mu\tau}} \equiv \frac{\omega'_{\tau\tau}}{\omega'_{\mu\tau}} = -1.42141, \tag{42}$$

$$m_2 \equiv -0.00871\text{eV}, \ m_3 = -0.0500591\text{eV}. \tag{43}$$

- The third case

$$\omega'_{\mu\tau} = -\frac{0.0206735}{\mu I f^2_{\mu\tau}}, \quad \omega'_{\mu\mu} = \frac{0.0293856}{\mu I f^2_{\mu\tau}}, \tag{44}$$

$$\frac{\omega'_{\mu\mu}}{\omega'_{\mu\tau}} \equiv \frac{\omega'_{\tau\tau}}{\omega'_{\mu\tau}} = -1.42141, \tag{45}$$

$$m_2 \equiv 0.00871\text{eV}, \ m_3 = 0.0500591\text{eV}. \tag{46}$$

- The fourth case

$$\omega'_{\mu\tau} = \frac{0.0293855}{\mu I f^2_{\mu\tau}}, \quad \omega'_{\mu\mu} = -\frac{0.0206734}{\mu I f^2_{\mu\tau}}, \tag{47}$$

$$\frac{\omega'_{\mu\mu}}{\omega'_{\mu\tau}} \equiv \frac{\omega'_{\tau\tau}}{\omega'_{\mu\tau}} = -0.703526, \tag{48}$$

$$m_2 \equiv 0.00871\text{eV}, \ m_3 = -0.0500591\text{eV}. \tag{49}$$

From the expressions from (38) to (49), it follows

$$m_2 = \pm 0.00871 \text{ eV}, \ m_3 = \pm 0.0500591 \text{ eV}, \tag{50}$$

$$\omega'_{\mu\mu} = \omega'_{\tau\tau} = \pm 0.703526\omega'_{\mu\tau}, \ \text{or} \ \omega'_{\mu\mu} = \omega'_{\tau\tau} = \pm 1.42141\omega'_{\mu\tau}. \tag{51}$$

The expressions (50) shows that $\omega'_{\mu\mu}, \omega'_{\tau\tau}$ and $\omega'_{\mu\tau}$ are the same order, and neutrinos follow the normal mass hierarchy.

Table 1. The values of γ corresponding to m_2, m_3

m_2 [eV]	m_3 [eV]	γ[rad]
-0.00871	0.0500591	$0.9872\pi/2$
0.00871	-0.0500591	$0.9872\pi/2$
0.00871	0.0500591	$1.0123\pi/2$
-0.00871	-0.0500591	$1.0123\pi/2$

Using the standard parametrization of the neutrino mixing matrix (the PMNS matrix) in terms of three angles and CP violating phases [15]

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\ e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\ e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{52}$$

where δ and γ are the Dirac and Majorana CP phases, respectively, and $s_{ij}(c_{ij}) = \sin\theta_{ij}(\cos\theta_{ij}) \geq 0$. The above Majorana mass matrix is diagonalized by the PMNS matrix

$$U^T \mathcal{M}_\nu U = \mathcal{M}_{diag} = \mathrm{diag}(m_1, m_2, m_3). \tag{53}$$

In the case of the normal mass hierarchy, the four parameters are described as [8, 12]

$$\begin{aligned}
\epsilon &= \tan\theta_{12}\frac{s_{23}}{c_{13}} + \tan\theta_{13}\, e^{i\delta}, \\
\epsilon' &= \tan\theta_{12}\frac{s_{23}}{c_{13}} - \tan\theta_{13}\, e^{i\delta}, \\
\frac{\omega'_{\mu\tau}}{\omega'_{\mu\mu}} &= -\frac{c_{13}^2 s_{23} c_{23}}{c_{13}^2 c_{23}^2 + r_{2/3}(s_{12}s_{13}c_{23}\ e^{-i\delta} + c_{12}s_{23})^2 e^{-i\gamma}} \\
&\quad -\frac{r_{2/3}(s_{12}s_{13}c_{23}\ e^{-i\delta} + c_{12}s_{23})(s_{12}s_{13}s_{23}\ e^{-i\delta} - c_{12}c_{23})e^{-i\gamma}}{c_{13}^2 c_{23}^2 + r_{2/3}(s_{12}s_{13}c_{23}\ e^{-i\delta} + c_{12}s_{23})^2 e^{-i\gamma}},
\end{aligned} \tag{54}$$

$$\frac{\omega'_{\tau\tau}}{\omega'_{\mu\mu}} = \frac{c_{13}^2 s_{23}^2 + r_{2/3}(s_{12}s_{13}s_{23}\ e^{-i\delta} - c_{12}c_{23})^2 e^{-i\gamma}}{c_{13}^2 c_{23}^2 + r_{2/3}(s_{12}s_{13}c_{23}\ e^{-i\delta} + c_{12}s_{23})^2 e^{-i\gamma}}, \tag{55}$$

with $r_{2/3} = m_2/m_3$.

We can easily see that with the help of (33), Eq. (55) is automatically satisfied. On the other hand, from (54) ones can find the values of γ corresponding to those of m_2, m_3 specified in (40), (43), (46) and (49) shown in Table 1, in which the values of γ is approximately equal to $\frac{\pi}{2}$. So the condition (33) leads to the maximal CP violation: $\sin\gamma_{CP} \simeq 1$, as mentioned in Ref. [16].

IV. SUMMARY

In this paper we have derived the exact eigenvalues and states of neutrino mass matrix in the Zee-Babu model. The tribimaximal mixing which is very realistic, imposes some conditions on the Yukawa couplings. The constraints derived in this work slightly differ from other ones given in the literature. The analysis showed that there exists possibility of maximal CP violation in the neutrino sector of the model. This requires reevaluation of the parameters in the model. This problem will be presented elsewhere.

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