

TRANSMITTANCE FUNCTION OF TWO-PORTS NONLINEAR FIBER MACH-ZEHNDER INTERFEROMETER

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Abstract. *In this article a two-ports nonlinear fiber Mach-Zehnder interferometer is proposed. The expression for output-input intensity relation and the transmittance function are derived based on principle operation of optical coupler. The expression of coupling coefficient of coupler is presented. The bell shape of the transmittance function is simulated. The results lead to investigate bistability of two-ports-nonlinear fiber Mach-Zehnder interferometer and to confirm bistable character of it in the future.*

I. INTRODUCTION

Nowaday, the optical bistable device (OBD) is important in “photonics”. They are used as an optical signal shaper, A/D converter, memory element, hard limiter, logic gate, etc [1, 2]. Its pre-eminence quality belongs to bistability of output-input intensity relation. There are many kinds of OBD, whose operation is based on another principles, but most of them operate basing on principle of nonlinear interferometer [2, 6, 7]. The previous OBD are used active mediums as gas, liquid in cuvet, so their dimension is too big. With advantage in nano-semiconductor technology and optical fiber, the dimension of them is more and more reduced to nano-scale [5, 9, 12]. Up-to-date, the most useful OBD is nonlinear fiber Mach-Zehnder interferometer (NFM-ZI), which consist of a pair of fibers, one of them has nonlinear refractive index, the second has linear one. Two fibers couple one with other by two couplers (see Fig.1a). As shown in some works [3, 8], the transfer percentage of coupler depends on principle parameters as length of coupler, radius of fiber, and separate space between them. Moreover, the bistability of OBD depends also on the transfer percentage of coupler, input intensity, and the length of nonlinear fiber [5, 6, 7, 9]. Thus, all above parameters influence on output intensity of OBD, i.e. on output-input intensity relation. This influence has still not been completely investigated. From Fig.1a, it is shown that the classical NFM-ZI has four ports: input, output, “feedback”, and free.

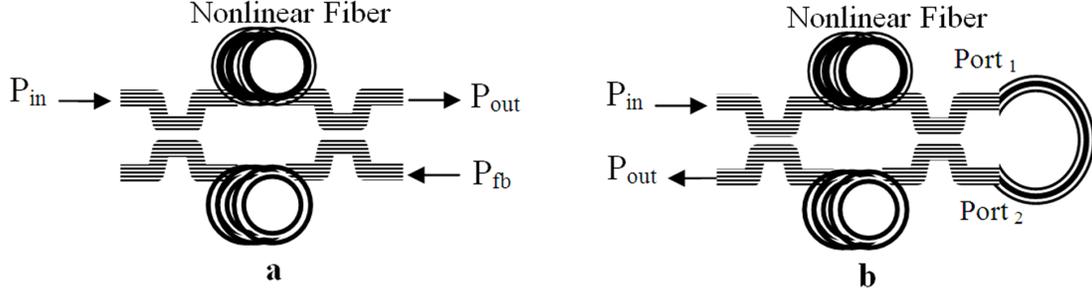


Fig. 1. (a) Four- ports NFM-ZI. (b) two-ports NFM-ZI.

With this geometrical structure, it must be used two input light sources at the same time, strong one plays role of pump, and other weak plays role of “feedback” [1].

In this paper we propose a two-ports NFM-ZI (see Fig. 1b). In Sec. II, we derive equation describing the output-input intensity relation. In Sec. III we present expression of transfer coefficient of coupler. In Sec. IV, we derive the transmittance function and simulate it.

II. OUTPUT-INPUT INTENSITY RELATION

Consider a two-ports NFM-ZI illustrated in Fig.1b. It consists of two couplers with transfer efficiencies, η_1 and η_2 [3, 8], respectively, one nonabsorbing nonlinear fiber with nonlinear refractive index coefficient, n_2 and length, l_{nl} , one nonabsorbing linear fiber with length, l_l . Two ports of second coupler are connected by a linear fiber with length, l_{con} .

Denote A_{in} be the amplitude of input light. The amplitude of light propagating through nonlinear fiber is given by [10]:

$$A_1 = \sqrt{\eta_1} A_{in} e^{i\phi_{nl}}, \quad (1)$$

where ϕ_{nl} is the phase change after fiber distance of l_{nl} . The amplitude of light propagating through linear fiber is given by:

$$A_2 = \sqrt{1 - \eta_1} A_{in} e^{i\phi_l}, \quad (2)$$

where ϕ_l is the phase change after fiber distance of l_l . The amplitude of light propagating through second coupler at **Port**₁ is given by:

$$A_{tol,1} = \sqrt{1 - \eta_1} \sqrt{1 - \eta_2} A_{in} e^{i\phi_l} + \sqrt{\eta_1} \sqrt{\eta_2} A_{in} e^{i\phi_{nl}}, \quad (3)$$

and at **Port**₂ is given by:

$$A_{tol,2} = \sqrt{1 - \eta_1} \sqrt{\eta_2} A_{in} e^{i\phi_l} + \sqrt{\eta_1} \sqrt{1 - \eta_2} A_{in} e^{i\phi_{nl}}. \quad (4)$$

From (3) and (4) the amplitude of feedback light from second coupler to nonlinear branch is:

$$A_f = \left(\frac{\sqrt{\eta_2} [\sqrt{1-\eta_1}\sqrt{\eta_2}A_{in}e^{i\phi_l} + \sqrt{\eta_1}\sqrt{1-\eta_2}A_{in}e^{i\phi_{nl}}] +}{\sqrt{1-\eta_2} [\sqrt{1-\eta_1}\sqrt{1-\eta_2}A_{in}e^{i\phi_l} + \sqrt{\eta_1}\sqrt{\eta_2}A_{in}e^{i\phi_{nl}}]} \right) e^{i\phi_{lcon}}, \quad (5)$$

and the amplitude of light from second coupler to linear branch is:

$$A_{lin} = \left(\frac{\sqrt{1-\eta_2} [\sqrt{1-\eta_1}\sqrt{\eta_2}A_{in}e^{i\phi_l} + \sqrt{\eta_1}\sqrt{1-\eta_2}A_{in}e^{i\phi_{nl}}] +}{\sqrt{\eta_2} [\sqrt{1-\eta_1}\sqrt{1-\eta_2}A_{in}e^{i\phi_l} + \sqrt{\eta_1}\sqrt{\eta_2}A_{in}e^{i\phi_{nl}}]} \right) e^{i\phi_{lcon}}. \quad (6)$$

From (5) and (6) the total amplitude of output light (goes outside from first coupler) is given by:

$$\begin{aligned} A_{out} &= \sqrt{1-\eta_1}A_f + \sqrt{\eta_1}A_{lin} \\ &= \sqrt{1-\eta_1} \left(\frac{\sqrt{\eta_2} [\sqrt{1-\eta_1}\sqrt{\eta_2}A_{in}e^{i\phi_l} + \sqrt{\eta_1}\sqrt{1-\eta_2}A_{in}e^{i\phi_{nl}}] +}{\sqrt{1-\eta_2} [\sqrt{1-\eta_1}\sqrt{1-\eta_2}A_{in}e^{i\phi_l} + \sqrt{\eta_1}\sqrt{\eta_2}A_{in}e^{i\phi_{nl}}]} \right) e^{i\phi_{lcon}} + \\ &\quad \sqrt{\eta_1} \left(\frac{\sqrt{1-\eta_2} [\sqrt{1-\eta_1}\sqrt{\eta_2}A_{in}e^{i\phi_l} + \sqrt{\eta_1}\sqrt{1-\eta_2}A_{in}e^{i\phi_{nl}}] +}{\sqrt{\eta_2} [\sqrt{1-\eta_1}\sqrt{1-\eta_2}A_{in}e^{i\phi_l} + \sqrt{\eta_1}\sqrt{\eta_2}A_{in}e^{i\phi_{nl}}]} \right) e^{i\phi_{lcon}}, \end{aligned} \quad (7)$$

or

$$A_{out} = A_{lin}e^{i\phi_{lcon}}e^{i\phi_l} \left(\frac{\eta_2(1-\eta_1) + (1-\eta_1)(1-\eta_2) + 2\sqrt{\eta_1\eta_2(1-\eta_1)(1-\eta_2)}}{\left[\eta_1(1-\eta_2) + \eta_1\eta_2 + 2\sqrt{\eta_1\eta_2(1-\eta_1)(1-\eta_2)} \right] e^{i(\phi_{nl}-\phi_l)}} \right). \quad (8)$$

From (8) the intensity of output light is given as follows:

$$\begin{aligned} I_{out} &= \frac{1}{2}\epsilon_0c|A_{out}|^2 \\ &= I_{in} \left(\frac{\left[\eta_2(1-\eta_1) + (1-\eta_1)(1-\eta_2) + 2\sqrt{\eta_1\eta_2(1-\eta_1)(1-\eta_2)} \right]^2 +}{\left[\eta_1(1-\eta_2) + \eta_1\eta_2 + 2\sqrt{\eta_1\eta_2(1-\eta_1)(1-\eta_2)} \right]^2 +} \right. \\ &\quad \left. 2 \left[\eta_1(1-\eta_2) + \eta_1\eta_2 + 2\sqrt{\eta_1\eta_2(1-\eta_1)(1-\eta_2)} \right] \cos(\phi + \Delta\phi)} \right), \end{aligned} \quad (9)$$

where $\phi_{nl} - \phi_l = \phi + \Delta\phi$, ϕ is the phase shift of light in two branches, $\Delta\phi$ is initial phase shift of input light [11].

Now, we introduce the phase shift of two branches. It is known that [4]

$$\phi = \frac{2\pi}{\lambda} (l_{nl} - l_l), \quad (10)$$

where l_{nl} , l_l are optical paths in nonlinear and linear branches, respectively. Consider the physical length of fiber in two branches are equal, L , so their optical paths are given by:

$$l_{nl} = (n_0 + n_2I_{ctr})L, \quad l_l = n_0L. \quad (11)$$

Substituting (11) into (10) we have:

$$\phi = \frac{2\pi}{\lambda} n_2I_{ctr}L, \quad (12)$$

where I_{ctr} is the intensity of control light in nonlinear branch. As shown in work of Soljacic [12], the control intensity is a sum of average intensities of contributing lights, so from (1), (6) and (7) we have:

$$\begin{aligned}
I_{ctr} &= \frac{I_{out}}{1 - \eta_1} + I_1 - \frac{\eta_1 I_{lin}}{1 - \eta_1} \\
&= \frac{I_{out}}{1 - \eta_1} + I_{in} \eta_1 + \\
&\quad I_{in} \left[\eta_1 + 4(1 - \eta_1)(1 - \eta_2)\eta_2 + 4\sqrt{(1 - \eta_1)(1 - \eta_2)\eta_1\eta_2} \cos(\phi + \Delta\varphi) \right] \\
&= \frac{I_{out}}{1 - \eta_1} + I_{in} [2\eta_1 + 4(1 - \eta_1)(1 - \eta_2)\eta_2].
\end{aligned} \tag{13}$$

Substituting (13) and (12) into (9), the output-input intensity relation is given as follows:

$$I_{out} = I_{in} \left(\begin{aligned} &\left[(1 - \eta_1) + 2\sqrt{\eta_1\eta_2(1 - \eta_1)(1 - \eta_2)} \right]^2 + \\ &\left[\eta_1 + 2\sqrt{\eta_1\eta_2(1 - \eta_1)(1 - \eta_2)} \right]^2 + 2 \left[\eta_1 + 2\sqrt{\eta_1\eta_2(1 - \eta_1)(1 - \eta_2)} \right] \times \\ &\cos \left\{ \frac{2\pi n_2 L}{\lambda(1 - \eta_1)} \left[I_{out} + I_{in} \left(2(1 - \eta_1)\eta_1 + 4(1 - \eta_1)^2(1 - \eta_2)\eta_2 \right) \right] + \Delta\varphi \right\} \end{aligned} \right). \tag{14}$$

III. TRANSFER COEFFICIENT OF FIBER COUPLER

Consider a fiber coupler with principle parameters as illustrated in Fig.2. Its transfer coefficient is given by:

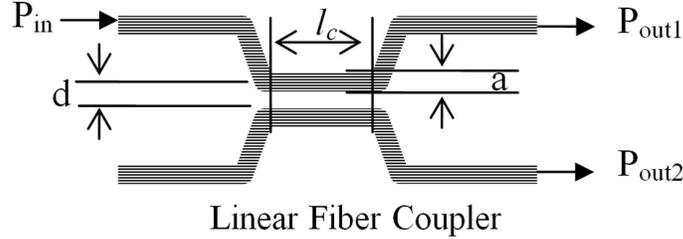


Fig. 2. Schematic of fiber coupler.

$$\eta = \frac{P_{out1}(l_c)}{P_{in}(0)} = \cos^2(Cl_c), \quad 1 - \eta = \frac{P_{out2}(l_c)}{P_{in}(0)} = \sin^2(Cl_c), \tag{15}$$

where C is the coupling parameter for two fibers with radius a , and separate space d , refractive indexes n_1 and n_2 , which can be approximatively given by [8]:

$$C = \frac{\pi \sqrt{\delta}}{2a} \exp \left\{ - \left(A + B \frac{d}{a} + D \frac{d^2}{a^2} \right) \right\}, \tag{16}$$

Where

$$\begin{cases} \delta = \frac{n_1^2 - n_2^2}{n_1^2}; A = 5.2789 - 3.6631V + 0.3841V^2; B = 0.7769 - 1.2252V + 0.3841V^2; \\ D = -0.0175 - 0.0064V - 0.0009V^2; \quad V = \frac{2\pi a \sqrt{n_1^2 - n_2^2}}{\lambda_0}. \end{cases} \quad (17)$$

From (14) to (17) we can see that the output intensity of two-ports NFI depends not only on typical parameters as input intensity, length of nonlinear fiber, nonlinear refractive index coefficient, initial phase shift, but also on principle parameters of coupler as radius, separate space, refractive index, and length.

IV. TRANSMITTANCE FUNCTION

Putting a normalized intensity, $X = \frac{n_2 L I}{\lambda}$, the Eq.(14) is rewritten as follows:

$$X_{out} = X_{in} \left(\begin{array}{l} \left[(1 - \eta_1) + 2\sqrt{\eta_1 \eta_2 (1 - \eta_1) (1 - \eta_2)} \right]^2 + \\ \left[\eta_1 + 2\sqrt{\eta_1 \eta_2 (1 - \eta_1) (1 - \eta_2)} \right]^2 + 2 \left[\eta_1 + 2\sqrt{\eta_1 \eta_2 (1 - \eta_1) (1 - \eta_2)} \right] \times \\ \cos \left\{ \frac{2\pi}{1 - \eta_1} \left[X_{out} + X_{in} \left(2(1 - \eta_1) \eta_1 + 4(1 - \eta_1)^2 (1 - \eta_2) \eta_2 \right) \right] + \Delta\varphi \right\} \end{array} \right). \quad (18)$$

From this equation, the basic equation for bistable device $X_{in} = \frac{X_{out}}{\mathfrak{S}(X_{out})}$ can be determined with the transmittance function is given by:

$$\mathfrak{S}(X_{out}) = \left(\begin{array}{l} \left[(1 - \eta_1) + 2\sqrt{\eta_1 \eta_2 (1 - \eta_1) (1 - \eta_2)} \right]^2 + \\ \left[\eta_1 + 2\sqrt{\eta_1 \eta_2 (1 - \eta_1) (1 - \eta_2)} \right]^2 + 2 \left[\eta_1 + 2\sqrt{\eta_1 \eta_2 (1 - \eta_1) (1 - \eta_2)} \right] \times \\ \cos \left\{ \frac{2\pi}{1 - \eta_1} \left[X_{out} + X_{in} \left(2(1 - \eta_1) \eta_1 + 4(1 - \eta_1)^2 (1 - \eta_2) \eta_2 \right) \right] + \Delta\varphi \right\} \end{array} \right). \quad (19)$$

Figure 3 presents an example simulated to show transmittance function. The bell shape, i.e. the principle shape of transmittance function gives to arise bistability of two-ports NFM-ZI, appears in suitable range of output intensity. In the range of I_{out} from 0 to $8.10^8 W/cm^2$ (Fig.3a) there are three critical points (marked by cycles), every of them is a state-jump point. In the range of I_{out} from 7.5 to $15.10^8 W/cm^2$ (Fig.3b), there are other three state-jump points.

Such, from (14), (16), (19), and Fig. 3, we can see that two-ports NFM-ZI has bistable character and will operate as a bistable device with suitable input intensity.

V. CONCLUSION

The expressions of the output-input intensity relation and the transmittance function for two-ports NFM-ZI are derived. The simulated bell shape of the transmittance function result in that the proposed two-ports NFM-ZI has bistable character and will operate as a bistable device. The bistability of it will be investigated in the next work to

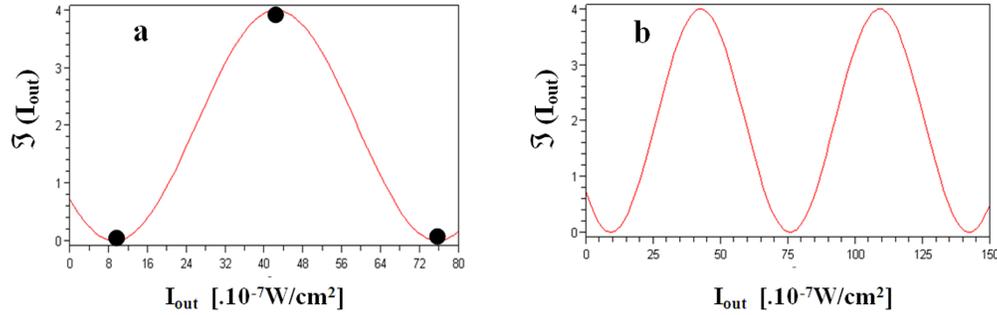


Fig. 3. Transmittance function $\mathfrak{S}(I_{out})$.
a: $I_{out} = (0 \div 8 \cdot 10^8) W/cm^2$. **b:** $I_{out} = (0 \div 15 \cdot 10^8) W/cm^2$
with $L=100\text{mm}$, $n_{nl} = 10^{-14} \text{cm}^2/W$, $I_{in} = 10^{10} W/cm^2$, $\eta_1 = \eta_2 = 0.5$, $\Delta\varphi = -0.06\pi$.

confirm the above conclusion. Moreover, the bistability of two-ports NFM-ZI depends not only on input intensity, but on coupler's transfer coefficient, which depends on principle parameters of coupler. It means that it needs to investigate in detail the influence of all principle parameters on bistability of two-ports NFM-ZI, before designing it.

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Received 12 September 2009.