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PHOTON–AXION CONVERSION CROSS–SECTIONS IN A RESONANT CAVITY WITH TM_{m10} MODE

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Abstract. *Photon-axion conversions in the resonant cavity with the mode (TM_{m10}) are considered in detail. The differential cross-sections and numerical evaluations are presented. It is shown that there is a resonant conversion for the considered process at low energies, in which the conversion cross-sections are much larger than those of the wave guide in the same conditions. Some estimates for experimental conditions are given from our results.*

I. INTRODUCTION

The most attractive candidate for the solution of the strong CP problem is Peccei and Quinn (PQ) mechanism [1], where the CP-violating phase θ ($\theta \leqslant 10^{-9}$) is explained by the existence of a new pseudo-scalar field, called the axion [2, 3]. At present, the axion mass is constrained by laboratory [4, 5] as well as by astrophysical and cosmological considerations [6, 7], in between 10^{-6} eV and 10^{-3} eV. If the axion has a mass near the low limit of order 10^{-5} eV, it is a good candidate for the dark matter of the universe. Besides that, an axino (the fermionic partner of the axion) naturally appears in SUSY models [8, 9] which acquire a mass from three-loop Feynman diagrams in a typical range from a few eV up to a maximum of 1 keV [10, 11]. The candidates for dark matter can appear in different model, in the 3-3-1 models [12–14] or in the supersymmetric and superstring theories [15].

The light particles with a two photon interaction can transform into photons in external electric or magnetic fields, an effect first discussed by Primakoff [16]. This effect is the basis of Sikivie's methods for the detection of axions in a resonant cavity [17]. He suggested that this method can be used to detect the hypothetical galactic axion flux that would exist if axions were the dark matter of the Universe. Various terrestrial experiments to detect invisible axions by making use of their coupling to photons have been proposed [18–20] and results of such experiments appeared recently [21–23]. The experiment CAST [24, 25] at CERN searches for axions from the sun or other sources in the universe. The experiment uses a large magnet from LHC and searches for exotic particles decaying into axions in the periodic EM field of the resonant cavity.

Consider the conversion of the photon γ with momentum q into the axion a with momentum p in an external electromagnetic field. Using the Feynman rules we get the following expression for the matrix element [26].

$$\langle p \mid M \mid q\rangle = -\frac{g_{a\gamma}}{2(2\pi)^2\sqrt{q_0 p_0}}\epsilon_\mu(q,\sigma)\epsilon^{\mu\nu\alpha\beta}q_\nu \int_V e^{ikr}F^{class}_{\alpha\beta}dr, \qquad (1)$$

where $k \equiv q - p$ is the momentum transfer to the EM field, $g_{a\gamma} \equiv g_\gamma \frac{\alpha}{\pi f_a} = g_\gamma \alpha m_a (m_u + m_d)[\pi f_\pi m_\pi \sqrt{m_u m_d}]^{-1}$ and $\epsilon^\mu(q,\sigma)$ represents the polarization vector of the photon.

Expression (1) is valid for an arbitrary external EM field. In the following we shall use it for the case, namely conversions in the periodic EM field of the resonant cavity with TM_{m10} mode. Here we use the following notations: $q \equiv \mid q \mid, p \equiv \mid p \mid = (p_0^2 - m_a^2)^{1/2}$ and θ is the angle between p and q.

II. CONVERSION IN A RESONANT CAVITY WITH TM_{M10} MODE

For the sake of simplicity we choose the nontrivial solution of the resonant cavity, namely the TM_{m10} mode [27, 28]

$$E_z = E_0 \sin(\frac{m\pi x}{a}) \sin(\frac{\pi y}{b}),$$

$$H_x = \frac{i\epsilon\pi}{\omega b} E_0 \sin(\frac{m\pi x}{a}) \cos(\frac{\pi y}{b}), \tag{2}$$

$$H_y = -\frac{i\epsilon\pi}{\omega a} E_0 \cos(\frac{m\pi x}{a}) \sin(\frac{\pi y}{b})$$

here the propagation of the EM wave is in the z-direction. Not that there is no E-wave of types (000), (001), (010), (100), (101) and (011).

Substitution of (2) into (1) gives us the following expression for the matrix element

$$\langle p \mid M \mid q \rangle = -\frac{g_{a\gamma}}{(2\pi)^2 \sqrt{q_0 p_0}} [(\epsilon_1(\mathbf{q},\tau) q_2 - \epsilon_2(\mathbf{q},\tau) q_1) F_z + \epsilon_1(\mathbf{q},\tau) q_0 F_x + \epsilon_2(\mathbf{q},\tau) q_0 F_y] \tag{3}$$

where $p_0 \equiv p_0 + \omega$, $\vec{k} = \vec{q} - \vec{p}$ and

$$F_z = -\frac{8E_0}{ab}\Big(a(q_x - p_x)\cos\frac{a(q_x-p_x)}{2}\sin\frac{m\pi}{2} - m\pi\cos\frac{m\pi}{2}\sin\frac{a(q_x-p_x)}{2}\Big)\times \\ b(q_y-p_y)\cos\frac{b(q_y-p_y)}{2}\sin\frac{d(q_z-p_z)}{2}\times \\ \Big[\Big(\frac{m^2\pi^2}{a^2} - (q_x-p_x)^2\Big)\Big(\frac{\pi^2}{b^2} - (q_y-p_y)^2\Big)(q_z-p_z)\Big]^{-1}$$

$$F_x = -\frac{8\pi\epsilon E_0}{\omega a b^2}\Big(a(q_x - p_x)\cos\frac{a(q_x-p_x)}{2}\sin\frac{m\pi}{2} - m\pi\cos\frac{m\pi}{2}\sin\frac{a(q_x-p_x)}{2}\Big)\times \\ \pi\cos\frac{b(q_y-p_y)}{2}\sin\frac{d(q_z-p_z)}{2}\times \\ \Big[\Big(\frac{m^2\pi^2}{a^2} - (q_x-p_x)^2\Big)\Big(\frac{\pi^2}{b^2} - (q_y-p_y)^2\Big)(q_z-p_z)\Big]^{-1} \tag{4}$$

$$F_y = \frac{8\pi\epsilon E_0}{\omega a^2 b}\Big(m\pi\cos\frac{a(q_x-p_x)}{2}\sin\frac{m\pi}{2} - a(q_x-p_x)\cos\frac{m\pi}{2}\sin\frac{a(q_x-p_x)}{2}\Big)\times$$
$$b(q_y-p_y)\cos\frac{b(q_y-p_y)}{2}\sin\frac{d(q_z-p_z)}{2}\times$$
$$\Big[\Big(\frac{m^2\pi^2}{a^2}-(q_x-p_x)^2\Big)\Big(\frac{\pi^2}{b^2}-(q_y-p_y)^2\Big)(q_z-p_z)\Big]^{-1}$$

where a, b and d are three dimensions of the cavity, ω is the frequency of the EM field and m = 0,1,2,3... Substituting Eq. (4) into Eq. (3) we finally obtain the DCS for conversions

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\frac{g_{a\gamma}^2}{(2\pi)^2}\frac{p_0}{q_0}[(q_x^2+q_y^2)F_z^2 + (1-\frac{q_x^2}{q^2})q_0^2F_x^2 + (1-\frac{q_y^2}{q^2})q_0^2F_y^2$$
$$-2q_xq_yF_xF_y - 2q_0q_xF_yF_z + 2q_0q_yF_xF_z]. \quad (5)$$

In the first case, we assume that the momentum of the photon is parallel to the z-axis, we have $q = q_z$ and $q_x = q_y = 0$ (the direction of the EM field). From the Eq. (5) we obtain the DCS

$$\frac{d\sigma}{d\Omega'} = \frac{1}{2}\frac{g_{a\gamma}^2}{(2\pi)^2}p_0q_0[F_x^2+F_y^2] = \frac{1}{2}\frac{g_{a\gamma}^2}{(2\pi)^2}q^2(1+\frac{\omega}{q})[F_x^2+F_y^2]. \quad (6)$$

where

$$F_x = -\frac{8\pi\epsilon E_0}{\omega a b^2}\Big(-ap_x\cos\frac{ap_x}{2}\sin\frac{m\pi}{2} + m\pi\cos\frac{m\pi}{2}\sin\frac{ap_x}{2}\Big)\times$$
$$\pi\cos\frac{bp_y}{2}\sin\frac{d(q-p_z)}{2}\times$$
$$\Big[\Big(\frac{m^2\pi^2}{a^2}-p_x^2\Big)\Big(\frac{\pi^2}{b^2}-p_y^2\Big)(q-p_z)\Big]^{-1}, \quad (7)$$

and

$$F_y = \frac{8\pi\epsilon E_0}{\omega a^2 b}\Big(ap_x\cos\frac{m\pi}{2}\sin\frac{ap_x}{2} - m\pi\cos\frac{ap_x}{2}\sin\frac{m\pi}{2}\Big)\times$$
$$\Big(bp_y\cos\frac{bp_y}{2}\sin\frac{\pi}{2}\sin\frac{d(q-p_z)}{2}\times$$
$$\Big[\Big(\frac{m^2\pi^2}{a^2}-p_x^2\Big)\Big(\frac{\pi^2}{b^2}-p_y^2\Big)(q-p_z)\Big]^{-1}. \quad (8)$$

In the sphere coordinate, we have $p_x = p\sin\theta\cos\varphi'$; $p_y = p\sin\theta\sin\varphi'$ and $p_z = p\cos\theta$, where φ' is the angle between the x-axis and the projection of $\mathbf{p}$ on the xy-plane and $d\Omega' = d\varphi' dcos\theta$. From Eq. (6), Eq. (7) and Eq. (8) it is easy to show that there are no conversions when $\theta = 0$ and $\varphi' = \frac{\pi}{2}$. When $\theta = \frac{\pi}{2}$ and $\varphi' = 0$ and in the limit $q \to \frac{\pi}{d}$, we get the DCS for conversions

$$\frac{d\sigma}{d\Omega'} = \frac{8g_{a\gamma}^2E_0^2}{a^2\pi^2\omega^2}\frac{1}{(\frac{m^2\pi^2}{a^2}-p^2)^2}(1+\frac{\omega d}{\pi})\Big\{-ap\cos\frac{ap}{2}\sin\frac{m\pi}{2} + m\pi\cos\frac{m\pi}{2}\sin\frac{ap}{2}\Big\}^2 \quad (9)$$

Next, if the momentum of the photon is perpendicular to the direction of the EM field, i.e. in the y-axis, we have $q = q_y$ and $q_x = q_z = 0$. From the Eq. (6) we obtain the DCS

$$\frac{d\sigma}{d\Omega''} = \frac{1}{2}\frac{g^2_{a\gamma}}{(2\pi)^2}p_0 q_0[F_x^2 + F_z^2] = \frac{1}{2}\frac{g^2_{a\gamma}}{(2\pi)^2}q^2(1+\frac{\omega}{q})[F_x^2 + F_z^2]. \quad (10)$$

where

$$F_z = -\frac{8E_0}{ab}\Big(-ap_x\cos\frac{ap_x}{2}\sin\frac{m\pi}{2} + m\pi\cos\frac{m\pi}{2}\sin\frac{ap_x}{2}\Big) \times b(q-p_y)\cos\frac{b(q-p_y)}{2}\sin\frac{dp_z}{2}\Big[\Big(\frac{m^2\pi^2}{a^2} - p_x^2\Big)\Big(\frac{\pi^2}{b^2} - (q-p_y)^2\Big)p_z\Big]^{-1} \quad (11)$$

and

$$F_x = -\frac{8\pi\epsilon E_0}{\omega ab^2}\Big(-ap_x cos\frac{ap_x}{2}\sin\frac{m\pi}{2} + m\pi\cos\frac{m\pi}{2}\sin\frac{ap_x}{2}\Big)\pi\cos\frac{b(q-p_y)}{2}\sin\frac{dp_z}{2} \times \Big[\Big(\frac{m^2\pi^2}{a^2} - p_x^2\Big)\Big(\frac{\pi^2}{b^2} - (q-p_y)^2\Big)p_z\Big]^{-1} \quad (12)$$

In the sphere coordinate, we have $p_x = p\sin\theta\cos\varphi''$; $p_y = p\cos\theta$ and $p_z = p\sin\theta\sin\varphi''$, where φ'' is the angle between the z-axis and the projection of $\mathbf{p}$ on the xz-plane and $d\Omega'' = d\varphi'' d\cos\theta$. From Eq. (10), Eq. (11) and Eq. (12) it is easy to show that there are no conversions when $\theta = \varphi'' = 0$. When $\theta = \varphi'' = \frac{\pi}{2}$ and in the limit $p \to \frac{m\pi}{a}$, we get the DCS for conversions

$$\frac{d\sigma}{d\Omega''} = \frac{1}{2}\frac{g^2_{a\gamma}E_0^2 d^2 a^2}{\pi^2\omega^2 m^2 (2\pi)^2(\frac{\pi^2}{b^2} - q^2)^2}q^2(1+\frac{\omega}{q})\{m\pi - sin m\pi\}^2\Big[\omega q + \frac{\pi^2}{b^2}\Big]^2 cos^2\frac{bq}{2}. \quad (13)$$

From Eq. (13), we see that the DCS depends quadratically on the amplitude E_0, photon momentum q and two dimensions of the resonant cavity. and it does not depend on each value of m.

III. DISCUSSION AND CONCLUSION

i) To compare the results with the wave guide (for details, see Ref. 20) we can introduce the DCS's ration R between the TM_{110} mode of the resonant cavity and the TE_{10} mode of the wave guide (for the best case)

$$R = \frac{(DCS)_{110}}{(DCS)_{10}} \sim \frac{q^2}{\omega^2}\Big(1 + \frac{\omega}{q}\Big)^2. \quad (14)$$

From Eq. (14), in the limit $\omega^2 = m_a^2 \ll q^2$, we have $(DCS)_{110} \gg (DCS)_{10}$. This means that *the conversion cross-sections of the resonant cavity are much larger than those of the wave guide in the same conditions.*

ii) From Eq. (13) we evaluate the DCS for conversion in C.G.S. Gauss units. Using data as in Refs. 17 and 20, $E_0 = 10^6$ cm$^{-1/2}g^{1/2}s^{-1}$ a = b = d = 100 cm and $\omega = m_a = 10^{-6}$ eV. The DCS for Eq. (13) as a function of the momentum q of photon were plotted in Fig. 1. We can see from the figure, the momentum of the photons is perpendicular to the momentum of axions, there only exists the resonant conversion at the low value of the momentum $q \simeq 4.7 \times 10^{-2}$ eV, the DCS is given by 2.2×10^{-34} cm^2 and at the high value of the momentum q, DCS's have very small values. This is important for taking the experiment to detect axion.

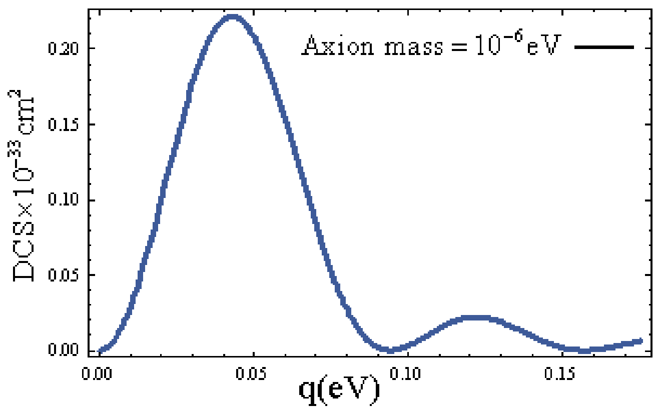

Fig. 1. DCS as a function of the moment q of photon in case $\theta \approx \pi/2$ and $\varphi'' \approx \pi/2$.

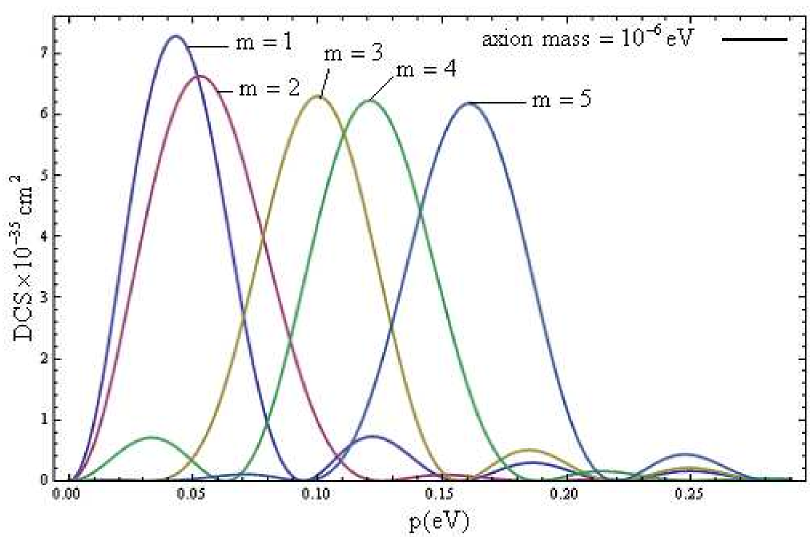

Fig. 2. DCS as a function of the moment p of axion in case $\theta \approx \pi/2$ and $\varphi' \approx 0$.

iii) When the momentum of photon is parallel to the z-axis (the direction of the EM field), the DCS for Eq. (9) with $m = 1, 2, 3, 4, 5$, is plotted in Fig. 2. It shows that the resonant conversion with the lowest mode ($m = 1$) exists at the value $p \simeq 4.7 \times 10^{-2} eV$, the DCS is given by 7.4×10^{-35} cm^2. This is the best case for conversions. Additional, the DCS for the resonant conversions reduce and move to the higher values of the momentum p of axion.

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