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THE PROPAGATION OF THE ELECTROMAGNETIC WAVE IN MULTILAYERS STRUCTURES COMPOSED FROM CHOLESTERIC LIQUID CRYSTALS UNDER THE INFLUENCE OF THE EXTERNAL ELECTRIC FIELD

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Abstract. *The tensor solution for the electromagnetic wave being in quadrature with the surface of the cholesteric liquid crystal under the influence of the external electric field along axis of swing of crystal, has been found. Description of the reflection, transmission of the electromagnetic wave in multilayer structures composed from cholesteric liquid crystals have been shown. Futher the dependence of the reflection spectrum on the polarization of the incident beam and magnitude of the external electric field are determined.*

I. INTRODUCTION

Recently, liquid crystal in general and cholesteric liquid crystal, in particular, had attracted great interests from both theoreticians and experimenters since their physical properties are enable to be changed under the influence of some external actions, such as electromagnetic field, stresses, temperature, ... This significant property has been widely applied in various technological devices. A sery of recent works have studied the effects of external field on twist step of cholesteric liquid crystal [1-4], however all these works had not consider the external effects on propagative characterizes of electromagnetic wave on liquid crystal.

Noticed that the cholesteric liquid crystal is stratified anisotropic material, so in the general case of the oblique electromagnetic wave in the crystal plate, the exactly solution of the Maxwell's equations can not be found. Theories of the propagation of the electromagnetic wave in the cholesteric liquid crystal had been shown in the works [3-9]. These authors had used various approximate methods such as the geometrical – optics approximation, the methods of part layers (i.e separate the plate into thinner layers which are assumed as homogenous plates, then used the method of Jonnes' matrixs to survey the propagation of the electromagnetic wave in multilayers structure, without considering the repeated reflection on the space between layers). Thereby, the problem of the electromagnetic wave being in quadrature with the surface of the cholesteric liquid crystal plate was studied and found out analytical solution in [3, 7, 8]. However, these works do not mention the problem of multilayers structures composed of liquid crystal that in these cases the calculation is too complex. For example, to solve the problem of three layers structure composed from a cholesteric liquid crystal plate between two isotropic homogenous dielectric layers, we have to solve 32 equations for 32 unknown [8].

In works on crystal optics [10, 11], F.I. Fedorov, L.M. Barkovsky, and G.N. Borzdov had proposed a method called operation method which allows to describe anisotropy of material promoted general tensors form without using concrete coordinate axies. Therefore, in the problems of propagation of the electromagnetic wave in multilayer structures composed from anisotropic materials, using operation method is very compactly and effectively so that the field of reflection and refraction wave have been expressed through the field of incident wave only by tensor transformations and interesting repeated reflection on the space between layers.

In the present paper, by using the above mentioned operation method, we have derived the exactly solution for the electromagnetic wave that is in quadrature with the surface of the cholesteric liquid crystal under the influence of the external electric field along the axis of swing of crystal. Consequently, the problems of reflection, transmission of the electromagnetic wave in multilayer structures composed from cholesteric liquid crystals has been solved. Furthermore, the dependence of the reflection spectrum on the polarization of the incident beam and magnitude of the external electric field is investigated.

II. THE ELECTROMAGNETIC WAVE IN CHOLESTERIC LIQUID CRYSTAL PLATE

Let us consider a monochromatic plane wave transmitting from the isotropic material into the cholesteric liquid crystal perpendicularly. Assume that the swing of crystal is perpendicular to interface. Chose the swing of crystal is Oz axis, $z = 0$ at interface. The external electric field vector has intensity E, along the axis of swing of crystal. Optical properties of cholesteric liquid crystal is characterized by permittivity tensor $\varepsilon\left(z\right)$

$$\varepsilon\left(z\right) = S(z)\varepsilon\tilde{S}(z), \tag{1}$$

where $\varepsilon = \varepsilon(0)$ is permittivity tensor at $z = 0$, $S(z)$ is matrix rotating around Oz axis, $\tilde{S}_{\mathrm{ij}} = S_{\mathrm{ji}}$, $S(z)$ is described by

$$S\left(z\right) = \exp\left[\varphi(\mathrm{z})\mathrm{q}^{\times}\right] = I\cos\varphi(z) + \sin\varphi(z)q^{\times} + \vec{q}\otimes\vec{q}. \tag{2}$$

with $\vec{q}$ is unit real vector, normal to the interface (Oz), $I = 1 - \vec{q}\otimes\vec{q}$, $(\vec{q}\otimes\vec{q})_{\mathrm{ij}} = q_i q_j$, $\vec{q}^{\times}$ is an antisymmetric tensor of second rank, dual to the vector $\vec{q}$, $\varphi\left(z\right) = \dfrac{\pi}{L}z$, L is step of spiral, L depends on external electric field intensity [3]:

$$L = L_0\left[1 - \left(\frac{E}{E_c}\right)^3\right], \tag{3}$$

L_0 is step of spiral when external electric field is zero, E_c is the critical value of electric field intensity. When external electric field increases to the critical value E_c, spiral structure of cholesteric liquid crystal will be destroy completely. Maxwell's equations takes the form:

$$\frac{d}{dz}\begin{pmatrix} \vec{H}_t \\ \vec{q}^{\times}\vec{E} \end{pmatrix} = ikM(z)\begin{pmatrix} \vec{H}_t \\ \vec{q}^{\times}\vec{E} \end{pmatrix}, \tag{4}$$

where $M(z) = \begin{pmatrix} 0 & B(z) \\ I & 0 \end{pmatrix}$ and $B(z) = \dfrac{I\bar{\varepsilon}(z)I}{\varepsilon_q(z)}$,
k is wave number, $\vec{H}_t,\ \vec{E}_t$ are tangential components of electric field intensity vector $\vec{E}$ and magnetic field intensity vectors $\vec{H}$, $\vec{H}_t = I\,\vec{H}$, $\vec{E}_t = I\,\vec{E}$, $\varepsilon_q = \vec{q}\varepsilon\vec{q}$, $\bar{\varepsilon}$ is the mutual tensor of ε. The solution of the equation (4) has been found in following form [12]:

$$\begin{pmatrix} \vec{H}_t(z) \\ \vec{q}^{\times}\vec{E}(z) \end{pmatrix} = P\begin{pmatrix} \vec{H}_t(0) \\ \vec{q}^{\times}\vec{E}(0) \end{pmatrix}, \tag{5}$$

where $P = \int\limits_0^z (E + ikM(z)dz)$.

P is matrizant (multiplicative integral, integral exponent) [12]. The specific nature of the multiplicative integral has a connection with the noncommutation of the operators $M(z)$. If operators $M(z')$ and $M(z'')$ are permutable at two arbitrary points z' and z'': $M(z')M(z'') = M(z'')M(z')$; $z', z'' \in [z_0, z]$, then the multiplicative integral reduces to the operator $P = \exp\left\{\int\limits_{z_0}^{z} iM(z)dz\right\}$. In the general case, the multiplicative integral is calculated following [12]:

$$P = \int\limits_0^z (E + ikM(z)dz) = E + \int\limits_0^z ikM(z)dz + \int\limits_0^z ikM(z)dz\int\limits_0^z ikM(z_1)dz_1 + ... \tag{6}$$

When the electromagnetic wave being in quadrature with the surface of the cholesteric liquid crystal, having thickness l, the analytical expression of multiplicative integral will be determined.

Using formula of multiplicative derivative [12]

$$D_zX = \frac{dX}{dz}X^{-1} \tag{7}$$

we can rewritten the multiplicative integral (5) by following form:

$$P = \int\limits_0^l \left\{ I + \left[\begin{pmatrix} -\frac{\pi}{L}\vec{q}^{\times} & ikSB(0)\tilde{S} \\ ikI & -\frac{\pi}{L}\vec{q}^{\times} \end{pmatrix} + D_z \begin{pmatrix} \exp\left(\frac{\pi}{L}l\vec{q}^{\times}\right) & 0 \\ 0 & \exp\left(\frac{\pi}{L}l\vec{q}^{\times}\right) \end{pmatrix} \right] dz \right\} \tag{8}$$

Using integrate of parts formula for multiplicative integral

$$\int\limits_{t_0}^{t} \left[E + (Q + D_tX)\,dt\right] = X(t)\int\limits_{t_0}^{t} \left[E + X^{-1}QX dt\right]X(t_0) \tag{9}$$

(8) becomes

$$P=\begin{pmatrix}\exp\left(\frac{\pi}{L}l\vec{q}^{\times}\right) & 0\\ 0 & \exp\left(\frac{\pi}{L}l\vec{q}^{\times}\right)\end{pmatrix}\exp\begin{pmatrix}-\frac{\pi}{L}l\vec{q}^{\times} & iklB\left(0\right)\\ iklI & -\frac{\pi}{L}l\vec{q}^{\times}\end{pmatrix} \quad (10)$$

P is a characteristic matrix of the crystal plate. With matrix P, we can derive the relation among the field vectors on the first and the last boundaries of the plate. Besides, the problems of reflection, transmission of the electromagnetic wave in multilayer structures composed from cholesteric liquid crystals can been solved.

III. THE REFLECTION SPECTRUM OF THE ELECTROMAGNETIC WAVE IN MULTILAYERS STRUCTURES COMPOSED FROM CHOLESTERIC LIQUID CRYSTALS

Consider a multilayer structure that consists of sequentiality alternate two components, one from cholesteric liquid crystal layers, other from isotropic dielectric layers under an external electric magnetic field. The interested system is in a isotropic medium with refractive index n_0. The liquid crystal layers have thickness l_1, characterized by permittivity tensor $\varepsilon\left(z\right)$ (1). The isotropic dielectric layers have thickness l_2 and refractive index n. In the case the electromagnetic wave being in quadrature with the surface of the interested system, we obtain the following expression of reflection and transmission tensors [11]:

$$R=\left[\left(-n_0I,I\right)P\begin{pmatrix}I\\ -n_0I\end{pmatrix}\right]^{-}\left[\left(n_0I,-I\right)P\begin{pmatrix}I\\ n_0I\end{pmatrix}\right],$$

$$D=2n_0I\left[\left(n_0I,I\right)P^{-1}\begin{pmatrix}I\\ n_0I\end{pmatrix}\right]^{-}. \quad (11)$$

where $P=\left(P_1P_2\right)^N$, N is period of the system, P_1 is characteristic matrix of the crystal plate, given by the formula (11), P_2 is characteristic matrix of the isotropic dielectric plate, given by the following formula:

$$P_2=\exp\left(ikl_2M_2\right),\quad M_2=\begin{pmatrix}0 & n^2I\\ I & 0\end{pmatrix} \quad (12)$$

In practice, we are interested in intensity and polarization of the reflection and transmission waves through the system, concretely we will determine intensity of the transmission wave through the system consisting of polarizer, optical system and analyzer. The polarizer is characterized by dyad (12) $\prod=\vec{P}\otimes\vec{P}^{*},\quad\left|\vec{P}\right|^2=1$, the analyzer is characterized by dyad $A=\vec{A}\otimes\vec{A}^{*},\quad\left|A\right|^2=1$. Assume that intensity of the incident wave equal to unit then intensity of the reflection wave J^R and the transmission wave J^D are in form [13]:

$$J^R=\left|\vec{P}^{*}R\vec{P}\right|,\qquad J^D=\left|\vec{A}^{*}D\vec{P}\right| \quad (13)$$

With $\vec{P}^{*}$ is conjugate complex of $\vec{P}$. The formula (13) is using for elliptic polarizer and analyzer arbitrarily. For linear polarizer, P and A are real vectors. φ_1 is the angle between vectors $\vec{P}$ and $\vec{b}_0$, where $\vec{b}_0$ is the unit vector in incident plane and in quadrature with $\vec{q}$, then $\vec{P} = \cos\varphi_1 \vec{b}_0 + \sin\varphi_1 \vec{b}_0^{\times} \vec{q}$. φ_2 is the angle between vectors $\vec{P}$ and $\vec{A}$, $\vec{A} = \cos\varphi_2 \vec{P} + \sin\varphi_2 \vec{P}^{\times} \vec{q}$. For circular polarizer, P and A are complex vectors $\vec{P}^2 = 0$, $\vec{A}^2 = 0$, we have $\vec{A} = \vec{P} = \frac{\sqrt{2}}{2}\left(\vec{b}_0 \pm i\vec{a}_0\right)$,with + is correspond with waves polarizing right – circular and - is correspond with waves polarizing left – circular.

To calculate, we use the parameters of system:

$$\varepsilon_1 = 2,290; \quad \varepsilon_2 = 2,143; \quad L_0 = 20\mu m; \quad l_1 = 25\mu m, n_= 2,417; \quad l_2 = 100\mu m, \quad n_o = 1, \tag{14}$$

ε_1, ε_2 are main values of tensor ε of cholesteric liquid crystal plate.

$$\varepsilon = \varepsilon_1 \vec{a}_0 \otimes \vec{a}_0 + \varepsilon_2 \vec{b}_0 \otimes \vec{b}_0 + \varepsilon_2 \vec{q} \otimes \vec{q}, \tag{15}$$

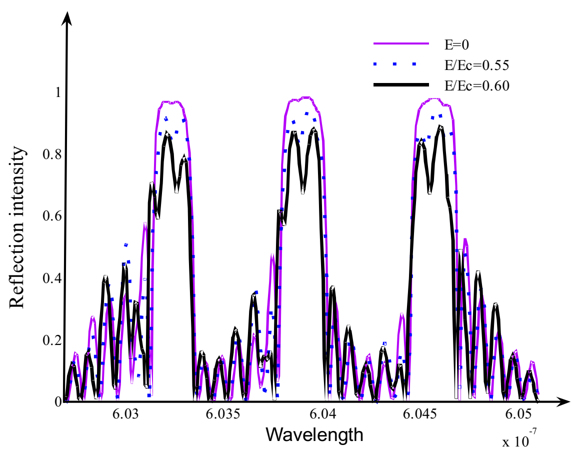

Fig. 1. The dependence of the reflection intensity on wavelength for the different values of the magnitudes of the external electric field when the incident beam is linear polarization with $\varphi_1 = 30^0$, $\varphi_2 = 15^0$, $N = 7$

Fig. 1 show the spectrum of the reflection wave's magnitude for the different values of the external electric field when the incident wave polarizing linearly is perpendicular to the multilayer structure that consists of sequentiality alternate two components, one from cholesteric liquid crystal layers, other from isotropic dielectric layers (the period of the system N = 7). Fig. 2 show the dependence of the reflection wave's magnitude $\Delta R = R - R_0$ on wavelength for the different values of the external electric field. Where alternately R_0 and R is the reflection wave's magnitude when the external electric field is zero and is not equal zero.

From figures, we have proved that the reflection intensity varies following the magnitudes of the external magnetic field. The change is clearer when the magnitude of the external electric field is up to 50 per cent of the critical value of electric field intensity (Fig. 1). When $E/E_c < 0.03$, the deference between R and R_0 is insignificant $\Delta R < 0.06$. (Fig. 2).

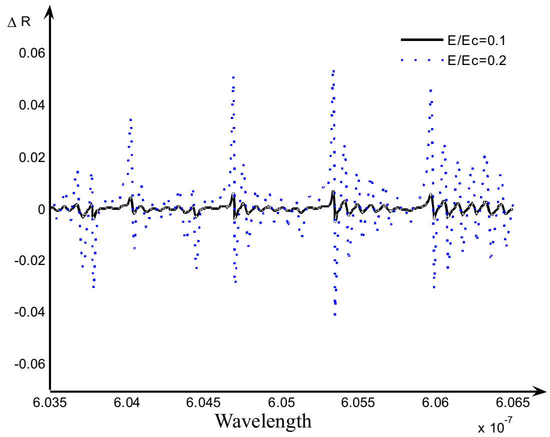

Fig. 2. The changes of the reflection intensity ΔR on wavelength for the different values of the magnitudes of the external electric field when the incident beam is linear polarization with $\varphi_1 = 30^0$, $\varphi_2 = 15^0$, $N = 7$

IV. STUDY ON THE POLARIZATION OF REFLECTION AND TRANSMISSION WAVES

The reflection and transmission intensity $\vec{H}^R$, $\vec{H}^D$ vectors are expressed by means of the reflection and transmission tensors in following form:

$$\vec{H}^R = R\vec{H}_0 \quad \vec{H}^D = D\vec{H}_0 \tag{16}$$

The polarization of the reflection and transmission waves are described in [10]. For reflection wave, we use the following form:

$$\gamma^R = \frac{\left|(\vec{H}^R)^2\right|}{\left|(\vec{H}^R)\right|^2}, \tag{17}$$

For transmission wave, polarization is described as

$$\gamma^D = \frac{\left|(\vec{H}^D)^2\right|}{\left|(\vec{H}^D)\right|^2} \tag{18}$$

When the incident wave is linear polarization, then $\gamma = 1$. If the incident wave is elliptic polarization $0 < \gamma < 1$

Fig. 3 show the dependence of the polarization of the reflection wave on wavelength for the different values of the magnitudes of the external electric field when the incident beam is in quadrature with the surface of the system consist of a liquid crystal plate on an isotropic dielectric layer. The parameters of system is used in (14). From the figure, we have proved that the more magnitude of the external magnetic field is, the more obviously the polarization of reflection wave is.

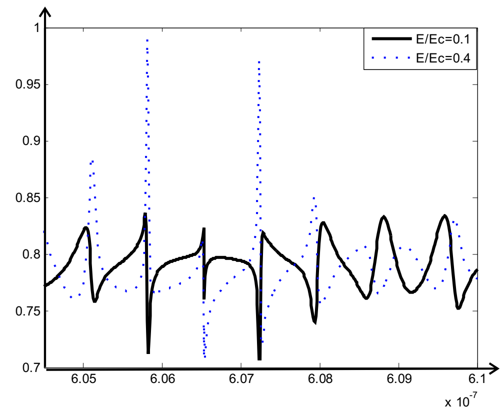

Fig. 3. The dependence of the polarization of the reflection wave on wavelength for the different values of the magnitudes of the external electric field when the incident beam is linear polarization with $\varphi_1 = 30^0$, $\varphi_2 = 15^0$

V. CONCLUSION

In a summary, using the operation method, we have found the exactly solution for the electromagnetic wave be in quadrature with the surface of the cholesteric liquid crystal under the influence of the external electric field, along axis of swing of crystal. As a result, we have solved the problems of reflection, transmission of the electromagnetic wave in multilayer structures composed from cholesteric liquid crystals in analytical-tensor form without any approximation explaining the specific of the repeated reflection on the space between layers. Our results therefore can used for arbitrary multilayer structures composed from different cholesteric liquid crystals.

REFERENCES

[1] V. A. Beliakov, V. E. Dmitrienco, *Physika Tverdovo Tela*, **17** (2) (1975)491-495.
[2] S. M. Osadchi, *Crystallographia*, **29** (5) (1984) 976-983.
[3] V. A. Beliakov, A. C. Conhin, *Optika Cholestericheskix Judkix Crystal*, Moscow: Nauka, 1982, 360 p.
[4] A. A. Gevorgan, *J. Tekhn. Phys.*, **70** (9) (2000) 75-82.
[5] I. V. Valukh, A. V. Clobodanuk, X. I. Valukh, E. Osterman, K. Scary, *Opt. J.*, **70** (7) (2003) 24-28.
[6] M. D Arutinhan., G. A. Vardanhan, A. A. Gevorgan, *Opt. J.*, **74** (4) (2007) 16-23.
[7] G. A. Vardanhan, A. A. Gevorgan, *Crystalographia*, **42** (4) (1997) 790-797.
[8] A. A. Gevorgan, K. V. Papoian, O. V. Pikichan, *Optika i Spectroskopia*, **88** (2000) 647-655.
[9] Pho Thi Nguyet Hang, L. M. Barkovsky, Proc. Hanoi Univ. Tech., Vietnam (1996)11-18.
[10] F. I. Fedorov, *Cheoria Gyrotropy*, Minsk: Nauka i Tekhnica,1976, 456 p.
[11] L. M. Barkovsky, G. N. Borzdov, *J. Phys. A. Math. Gen.*, **20** (1987) 1095-1106.
[12] F. R. Gantmacher, *Theory of Matrices*, Moscow: Nauka, 1988, 548 p.
[13] G. N. Borzdov, *Vesci Academi Nauk BSSR*, **3** (1977) 85-90.

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