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THE NONLINEAR ACOUSTOELECTRIC EFFECT IN A SUPERLATTICE

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Abstract. *The acoustoelectric effect in a superlattice (SL) is investigated for an acoustic wave whose wavelength $\lambda = 2\pi/q$ is smaller than the mean free path l of the electrons and hypersound in the region $ql \gg 1$. (where q is the acoustic wave number) . A nonlinear dependence of the acoustoelectric current j^{ac} on the constant electric field E is obtained by using the classical Boltzmann kenetic equation. The analytical expression for the acoustoelectric current j^{ac} is calculated for constant of momentum relaxation time. Numerical calculations is done, and the result is discussed for a typical GaAs/AlGaAs SL. It is noted that when the electric field is negative the current j^{ac} decreases, reaches a minimum and rises. On the other hand, when the electric field is positive the current increases, reaches a maximum and then falls off. A similar observation has been noted for an acoustoelectric interaction in a multilayered structure resulting from the analysis of Si/SiO_2 structure. The dominant mechanism for such a behavior is attributed to the periodicity of the energy spectrum of electron along the SL axis.*

I. INTRODUCTION

When an acoustic wave is absorbed by a conductor, the transfer of the momentum from the acoustic wave to the conduction electron may give rise to a current usually called the acoustoelectric current j^{ac} or a constant electric field E^{ac} in the case of an open circuit. The study of this effect is crucial because of the complementary role it may play in the understanding the properties of the SL, which, we believe, should find an important place in the acoustoelectronic devices. The study of acoustoelectric effect in bulk materials have received a lot of attention [1-5]. Recently, there have been a growing interest in observing this effect in mesoscopic structures [6-8]. The interaction between surface acoustic wave (SAW) and mobile charges in semiconductor layered structures and quantum wells is an important method to study the dynamic properties of low-dimensional systems. The SAW method was applied to study the quantum Hall effects [9-11], the fractional quantum Hall effect [12], and the electron transport through a quantum point contact [13, 14]. It has also been noted that the transverse acoustoelectric voltage (TAV) is sensitive to the mobility and to the carrier concentration in the semiconductor, thus it has been used to provide a characterization of electric properties of semiconductors [15]. Interface state density [16], junction depth [17], and carrier mobility [18] have been measured with this method. Especially, in recent time the acoustoelectric effect was studied in both a one-dimensional channel [19] and in a finite-length ballistic quantum channel [20–22]. In addition, the acoustoelectric effect was measured by an experiment in a submicron-separated quantum wire [23], in a carbon nanotube [24], in an InGaAs quantum well [25]. The SAW method was also applied to the study acoustoelectric effect and acoustomagnetoelectric effect [26–28].

In this paper, we examine this effect in a superlattic for the case when electron relaxation time is not dependent on the energy and we will show that the presence of minibands in the SL will result in a nonlinear dependence of the j^{ac} on the wavenumber q. Also, in the presence of an applied constant electric field E a threshold value E_0 is obtained where the acoustoelectric current changes direction.

This paper is organized as followed. In Section 2, we outline the theory and conditions necessary to solve the problem, in Section 3 we discuss the results, and in Section 4 we come to a conclusion.

II. ACOUSTOELECTRIC CURRENT

By using the classical Boltzmann kenetic equation method in [26,27,28], we calculated the acoustoelectric current in SL. The acoustic wave is considered a hypersould in the region $ql \gg 1$ (l is the electron mean free path, $\vec{q}$ is the acoustic wave). Under such circumstances, the acoustic wave can be interpreted as monochromatic phonons having the 3D phonon distribution function $N(\vec{k})$, which can be presented in the form [28]

$$N(\vec{k}) = \frac{(2\pi)^3}{\hbar\omega_{\vec{q}} v_s}\Phi\delta(\vec{k}-\vec{q}), \tag{1}$$

where $\hbar = 1$, $\vec{k}$ is the current phonon wave vector, Φ is the sound flux density, $\omega_{\vec{q}}$ and v_s are the frequency, and the group velocity of sound wave with the wave vector $\vec{q}$, respectively.

It is assumed that the sound wave and the applied electric field $\vec{E}$ propagates along the z axis of the SL. The problem was solved in the quasi-classical case, i.e. $2\Delta \gg \tau^{-1}$, $2\Delta \gg eEd$ (τ is the relaxation time, d is the period of the SL, 2Δ is the width of the lowest energy miniband and e is the electron charge). The density of the acoustoelectric current can be written in the form [29]

$$j^{ac} = \frac{2e}{(2\pi)^3}\int U^{ac}\psi_i(\vec{p})d^3p. \tag{2}$$

Here, $\psi_i(\vec{p})$ is the solution of the Boltzmann kinetic equation in the absence of the magnetic field, $\vec{p}$ is the electron momentum and

$$\begin{aligned} U^{ac} = \frac{2\pi\Phi}{\omega_{\vec{q}} v_s}\Big\{ &\mid G_{\vec{p}-\vec{q},\vec{p}} \mid^2 [f(\epsilon_{n,\vec{p}-\vec{q}}) - f(\epsilon_{n,\vec{p}})]\delta(\epsilon_{n,\vec{p}-\vec{q}} - \epsilon_{n,\vec{p}} + \omega_{\vec{q}}) \\ &+ \mid G_{\vec{p}+\vec{q},\vec{p}} \mid^2 [f(\epsilon_{n,\vec{p}+\vec{q}}) - f(\epsilon_{n,\vec{p}})]\delta(\epsilon_{n,\vec{p}+\vec{q}} - \epsilon_{n,\vec{p}} - \omega_{\vec{q}})\Big\}, \end{aligned} \tag{3}$$

where $f(\epsilon_{n,\vec{p}})$ is the distribution function and $\epsilon_{n,\vec{p}}$ is the energy spectrum of the electron, n denotes quantization of the energy spectrum, and $G_{\vec{p}\pm\vec{q},\vec{p}}$ is the matrix element of the electron-phonon interaction. Introducing a new term $\vec{p}' = \vec{p} - \vec{q}$ in the first term of the integrals in Eq.(3) and taking into account the fact that

$$\mid G_{\vec{p},\vec{p}'} \mid^2 = \mid G_{\vec{p}',\vec{p}} \mid^2, \tag{4}$$

the matrix element of the electron-phonon interaction for $qd \ll 1$ is given by

$$\mid G\vec{p},\vec{q} \mid^2 = \frac{\wedge^2 \vec{q}^2}{2\sigma\omega_{\vec{q}}}, \tag{5}$$

where $\wedge$ is the deformation potential constant and σ is the density of the SL.

We can express Eq.(2) in the form

$$\begin{aligned} j_i^{ac} = -\frac{e\Phi}{2\pi^2 v_s \omega_{\vec{q}}} \int \mid G_{\vec{p}+\vec{q},\vec{p}} \mid^2 [f(\epsilon_{n,\vec{p}+\vec{q}}) - f(\epsilon_{n,\vec{p}})] \times \\ \times [\psi_i(\vec{p}+\vec{q}) - \psi_i(\vec{p})] \delta(\epsilon_{n,\vec{p}+\vec{q}} - \epsilon_{n,\vec{p}} - \omega_{\vec{q}}) d^3\vec{p}, \end{aligned} \tag{6}$$

where $\psi_i(\vec{p})$, as indicated in [29], is the mean free path $l_i(\vec{p})$.

In solving Eq. (6) we considered a situation whereby the sound was propagating along the SL axis (Oz), thus the acoustoelectric current in Eq.(6) in the direction of the SL axis becomes

$$j_z^{ac} = -\frac{e\Phi\vec{q}^2\tau\wedge^2}{4\pi^2 v_s \omega_{\vec{q}}^2 \sigma} \int [f(\epsilon_{n,\vec{p}+\vec{q}}) - f(\epsilon_{n,\vec{p}})][l_z(\vec{p}+\vec{q}) - l_z(\vec{p})] \delta(\epsilon_{n,\vec{p}+\vec{q}} - \epsilon_{n,\vec{p}} - \omega_{\vec{q}}) d^3\vec{p}. \tag{7}$$

The distribution function in the presence of the constant applied field $\vec{E}$ is obtained by solving the Boltzmann equation in the τ approximation. This function is given

$$f(\vec{p}) = \int_0^\infty \frac{dt}{\tau} \exp(-t/\tau) f_0(\vec{p} - e\vec{E}t), \tag{8}$$

where

$$f_0(\vec{p}) = \Theta(\epsilon_F - \epsilon_{n,\vec{p}}) = \begin{cases} 0 & \epsilon_{n,\vec{p}} > \epsilon_F \\ 1 & \epsilon_{n,\vec{p}} < \epsilon_F, \end{cases} \tag{9}$$

where ϵ_F is Fermi energy. The energy spectrum $\epsilon_{n,\vec{p}}$ of the electron in the SL is given using the usual notation by

$$\epsilon_{n,\vec{p}} = \frac{\vec{p}_\perp^2}{2m} + \Delta_n(1 - \cos(p_z d)) \tag{10}$$

where, $\vec{p}_\perp$ and p_z are the transverse and longitudinal (relative to the SL axis) components of the quasi-momentum, respectively; Δ_n is the half width of the nth allowed miniband, m is the effective mass of electron and d is the SL period.

We assumed that electrons are confined to the lowest conduction miniband ($n = 1$) and omitted the miniband index. That is to say that the field does not induce transitions between the filled and empty minibands, thus the Δ_n can be written as Δ.

Substituting Eq.(8) and Eq. (10) into Eq. (7) we obtained the acoustoelectric current

$$j_z^{ac} = j_0^{ac} \int_0^\infty \frac{dt}{\tau} \exp(\frac{-t}{\tau}) \Big[A \sin\frac{qd}{2} + (1 - A^2)^{1/2} \cos\frac{qd}{2}\Big] \sin(eEt - \frac{q}{2})d, \tag{11}$$

where

$$j_0^{ac} = \frac{em\Phi\vec{q}^2\tau \wedge^2 \Delta}{\pi v_s \omega_{\vec{q}}^2 \sigma}\Big[1 - \Big(\frac{\omega_{\vec{q}}}{2\Delta\sin(qd/2)}\Big)^2\Big]^{1/2},$$

and

$$A = \frac{\omega_{\vec{q}}}{2\Delta} + \cos\frac{qd}{2}\Big[1 - \Big(\frac{\omega_{\vec{q}}}{2\Delta\sin(qd/2)}\Big)^2\Big]^{1/2}.$$

The Eq.(11) is acoustoelectric current in the case of the presence of external electric field $\vec{E}$ applied along the z axis and the degenerate electron gas.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this situation Eq.(11) was solved analytically and the result were given as

$$j_z^{ac} = -j_0^{ac}\frac{1}{(eEd\tau)^2+1}\Big[1 - eEd\tau\tan^{-1}(qd/2)\Big]\times$$
$$\times\Big[A\sin^2(qd/2) + (1-A^2)^{1/2}\sin(qd/2)\cos(qd/2)\Big], \tag{12}$$

Eq. (12) is the analytical expression of acoustoelectric current in the case of the presence of external electric field. From Eq. (12) it is observed that if

$$E > E_0 = \frac{1}{ed\tau}\tan(qd/2)$$

then the acoustoelectric current changes sign and the value E_0 can be interpreted as a threshold field. E_0 is a function of the SL parameters d, frequency $\omega_{\vec{q}}$ and the wavenumber q. For instance, at $d = 10^{-8}m, \tau = 10^{-12}s$, and $\omega_{\vec{q}} = 10^{11}s^{-1}$, the threshold field $E_0 = 31.94Vcm^{-1}$ which is small but can still be observed. The about obtained result is similar to the result in [27]. In the absence of the constant applied field $E = 0$, from Eq.(12) we obtain

$$j_z^{ac} = -j_0^{ac}\Big[A\sin^2(qd/2) + (1-A^2)^{1/2}\sin(qd/2)\cos(qd/2)\Big]. \tag{13}$$

We can see from the Eq. (13), when $qd \ll 1$ and $\omega_q \gg 2\Delta\sin(qd/2)$, j^{ac} is very small, i.e. there appears a transparency window. This is a outcome of the conversation law. Under this condition there is no absorption of acoustic waves, hence no acoustoelectric current [31]. The SL can be used as an acoustic wave filter.

The dependence of j_z^{ac} on E was investigated in Fig. 1 and Fig. 2. The dependence of j_z^{ac}/j_0^{ac} on E for given $\omega_{\vec{q}}$ is not linear. These peaks increase with increasing of $\omega_{\vec{q}}$. More interesting is the nature of the acoustoelectric current. It is observed that when the electric field is nagative the current decreases, reaches a minimum and then rises in a manner similar to that observed during a positive differential conductivity. On the other hand, when the electric field is positive the current increases, reaches a maximum then decreases. Threshold field E_0 also increases with increasing in $\omega_{\vec{q}}$. It is worthy to note that a similar nonlinear relation was obtained for a TAV experiment on $Si/Si0_2$ and this result agreed with result [30].

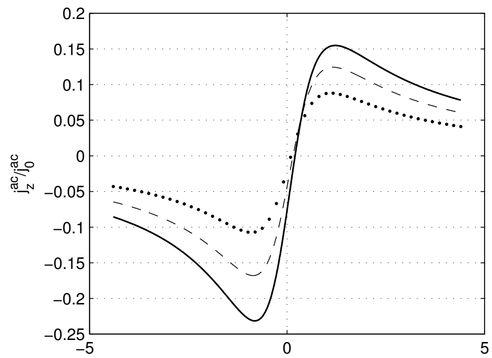

Fig. 1. The dependence of j_z^{ac}/j_0^{ac} on the $eEd\tau/\hbar$: $\omega_{\vec{q}} = 2.10^{11}s^{-1}$ (solid line); $\omega_{\vec{q}} = 1.5.10^{11}s^{-1}$ (dashed line); $\omega_{\vec{q}} = 1.10^{11}s^{-1}$ (dotted line).

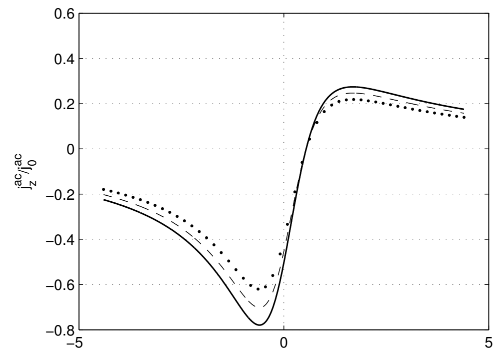

Fig. 2. The dependence of j_z^{ac}/j_0^{ac} on the $eEd\tau/\hbar$: $\Delta = 0.04$ eV (solid line); $\Delta = 0.07$ eV (dashed line); $\Delta = 0.1$ eV (dotted line).

In Fig. 2 the dependence of j_z^{ac}/j_0^{ac} on E is plotted for given Δ. It is noted that the acoustoelectric current has a peak at some values of E. These peaks decrease with increasing of Δ, which is similar to the dependence of j_z^{ac}/j_0^{ac} on E for given $\omega_{\vec{q}}$. It is obtained that when the electric field is negative the current decreases, reaches a minimum and then rises in a manner similar to that observed during a positive differential conductivity. When the electric field is positive the current increases, reaches a maximum and then decreases. This can be attributed to the Bragg reflection at the band edge.

IV. CONCLUSION

In this paper, we have obtained analytical expressions for the acoustoelectric current in a degenerate electron gas SL in the presence of constant electric field. We have shown the strong nonlinear dependence of j_z^{ac}/j_0^{ac} on the applied electric field E The dominant mechanism for such nonlinear behavior is the periodicity of the electron energy spectrum along the SL axis. We obtained that a transparency window is formed whenever $qd \ll 1$ and $\omega_q \gg 2\Delta\sin(qd/2)$ and j_z^{ac} is very small. We attributed the cause to the presence of the conservation laws and suggested the use of SL as a phonon filter.

The above results indicate that there exists some peaks which disappear in the bulk semiconductor [32].

The numerical result obtained for a GaAs/AlGaAs SL shows that there exists a threshold field E_0 for which the acoustoelectric current changes direction the threshold field increases with an increasing ω_q. Our result indicates that the acoustoelectric current exists even if the relaxation time τ of the carrier does not depend on the carrier energy. This differs from the bulk semiconductor, because in the bulk semiconductor [31] where the acoustic electric effect vanishes for a constant relaxation time.

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REFERENCES

[1] R. H. Parmenter, *Phys. Rev.,* **B89** (1953) 990.
[2] M. Rotter, A. V. Kalameitsev, A .O. Grovorov, *Phys. Rev. Lett.* **82** (1999) 2171.
[3] P. E. Lippens, M. Lannoo, J. F. Pouliquen, *J. Appl. Phys.* **66** (1989) 1209.
[4] V. V. Afonin, M. Gal'prin Yu, *Semiconductor* **27** (1993) 61.
[5] E. M. Epshtein, V. Gulyaev Yu, *Sov. Phys. Solid State.* **9** (1967) 288.
[6] J. M. Shilton, D. R. Mace, V. I. Talyanskii, *J.Phys: Condens. Matter.* **8** (1996) 377.
[7] J. M. Shilton, V. I. Talyanskii, *J.Phys: Condens. Matter.* **56** (1996) L531.
[8] F. A. Maa, Y. Galperin, *Phys. Rev.* **B56** (1997) 4028.
[9] A. Wixforth, W. Schlapp, *Phys. Rev.* **B 40** (1989) 7874.
[10] R. L. Willett, R. R. Ruel, K. W. West, *Phys. Rev. Lett.* **71** (1993) 3846.
[11] I. L. Drichko, A. M. Dyakonov, *Soviet Phys. St. Sol.* **31** (1997) 451.
[12] R. L. Willet, M. A. Paalanen, R. R. Ruel, *Phys. Rev. Lett.* **65** (1990) 112.
[13] J. M. Shilton, D. R. Mace, V. I. Talyanskii, *Phys. Rev.* **B51** (1995) 14770.
[14] V. I. Talyanskii, M. Pepper, J. M. Shilton, *Phys. Rev.* **B56** (1997) 15180.
[15] P. Das, R. T. Webster, H. Estrada-Varquez, W. C. Wang, *Surf. Sci.* **86** (1979) 848.
[16] B. Davari, P. Das, R. Bharat, *J. Appl. Phys.* **54** (1983) 415.
[17] B. Davari, P. Das, *Proc. IEEE Ultrason. Symp.* **52** (1986) 457.
[18] A. Bers, J. H. Cafarella, B. E. Burke, *Appl. Phys. Lett.* **22** (1982) 169.
[19] J. M. Shilton, D. R. Mace, V. I. Talyanskii, *J. Phys:Condens. Matter.***8** (1996) 337.
[20] O. E. Wohlman, Y. Levinson, Yu. M. Galperin, *Phys.Rev.* **B62** (2000) 7283.
[21] N. A. Zimbovskaya, G. Gumbs, *J. Phys: Condens. Matter.* **13** (2001) 409.
[22] Yu. M. Galperin, O. E. Wohlman, Y. Levinson, *Phys. Rev.* **B63** (2001) 153309.
[23] J. Cunningham, M. Pepper, V. I. Talyanskii, *Appl.Phys.Lett.* **86** (2005) 152105.
[24] B. Reulet, A.Yu. Kasumov, M. Kociak, *Appl. Phys. Lett.* **85** (2000) 2829.
[25] M. R. Astley, M. Kataoka, C. J. B. Ford, *J. Appl. Phys.* **103** (2008) 096102.
[26] S. Y. Mensah, F. K. A. Allotey *J. Phys: Condens. Matter.* **6** (1994) 6793.
[27] S. Y. Mensah, F. K. A. Allotey, *J. Phys: Condens. Matter.* **12** (2000) 5225.
[28] S. Y. Mensah, N. G. Mensah, *J. Phys: Superlattices and Micros.* **37** (2005) 87.
[29] M. I. Kaganov, Sh. T. Mevlynt, I. M. Suslev, *Sov. Phys. JETP.* **51** (1980) 189.
[30] F. Palma, *J. Apply. Phys.* **66** (1989) 292.
[31] G. Weinreich, T. M. Sanders, and H. G. White, *Phys. Rev.* **1** (1959) 114.
[32] G. Johri, H. N. Spector, *Phys. Rev.* **15** (1977) 4955.

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