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W-mass prediction in the complex NMSSM

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Abstract. *In this work, we present our recent calculation of the W boson mass in the complex Next-to-Minimal Supersymmetric Standard Model (NMSSM), which includes all state-of-the-art higher-order supersymmetric corrections to Δr. In particular, we include the full one-loop corrections and the leading and sub-leading two-loop corrections $\mathcal{O}(\alpha + \alpha_t\alpha_s + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$ from the ρ parameter. We show some numerical investigations of the dependence of $\Delta\rho$ on the renormalization schemes and of the dependence of M_W on some selected parameters. We also discuss comparisons between some public tools which compute M_W in the NMSSM. Finally we present a new version of the public Fortran code,* `NMSSMCALC`*, including a new M_W prediction.*

Keywords: W mass; ρ parameter; supersymmetry.
Classification numbers: 12.60.Jv; 12.15.Lk; 14.70.Fm.

1. Introduction

These proceedings are based on our recent work [1] which was presented at the 49th Vietnam Conference on Theoretical Physics. The W and Z bosons are the only massive gauge bosons in the Standard Model (SM). Both of them were discovered in 1983 at the super proton synchrotron (SPS). Their masses are related to the gauge couplings and the vacuum expectation value. They are important parameters of the SM and enter the SM global fits. Precise measurements of

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these masses are crucial for testing the SM. In contrast to the Z boson mass, it is more challenging to measure the W mass, since its leptonic decay products contain a neutrino which is undetectable at colliders. Theoretically, the W mass is also more sensitive to new physics. Over 40 years since its discovery, there are 10 experiments which have measured the W mass. In 2021, the combination of the measurements of the W boson mass has lead to a world average of $M_W^{\text{exp}} = 80.379 \pm 0.012\,\text{GeV}$ [2]. This value is about 2σ standard deviations higher than the SM prediction, $M_W^{\text{SM,OS}} = 80.353 \pm 0.004\,\text{GeV}$ [3] and $M_W^{\text{SM},\overline{\text{MS}}} = 80.351 \pm 0.003\,\text{GeV}$ [4] in the on-shell (OS) and $\overline{\text{MS}}$ renormalization scheme, respectively. In 2022, the CDF collaboration has reported a new result of the W boson mass [5], $M_W^{\text{CDF}} = 80.4335 \pm 0.0094\,\text{GeV}$, which is more than 7σ higher than the SM prediction. This CDF result has caused a lot of attention in the particle physics community, which one, however, should take with caution. The ATLAS collaboration recently released their precisely measured value of $M_W^{\text{ATLAS}} = 80.3665 \pm 0.0159\,\text{GeV}$ [6], while the CMS analysis quoted a value of $M_W^{\text{CMS}} = 80.3602 \pm 0.0099\,\text{GeV}$ [7] whose accuracy is compatible with the one of the CDF collaboration. Both new results are consistent with the SM prediction. Motivated by these measurements and many dedicated studies of the W mass in the SM and also beyond the SM, we present a precise calculation of the W mass in the complex Next-to Minimal Supersymmetric Standard Model (NMSSM). This is the second most studied supersymmetric extension of the SM. It was proposed to solve the μ-problem by dynamically generating it through the vacuum expectation value of a complex scalar singlet. The model has a richer phenomenology in the Higgs sector and in the neutralino sector in comparison with the minimal supersymmetric extension of the SM (MSSM), see [8] and references therein.

2. The Complex NMSSM

The model is an extension of the MSSM, containing two Higgs doublet superfields $\hat{H}_d$, $\hat{H}_u$ and the singlet superfield $\hat{S}$. We furthermore impose a Z_3 symmetry to eliminate the linear and bilinear terms in the superpotential. The superpotential of the complex NMSSM is given by $(i, j = 1,2)$

$$\mathscr{W}_{\text{NMSSM}} = \epsilon_{ij}[y_e \hat{H}_d^i \hat{L}^j \hat{E}^c + y_d \hat{H}_d^i \hat{Q}^j \hat{D}^c - y_u \hat{H}_u^i \hat{Q}^j \hat{U}^c] - \epsilon_{ij}\lambda \hat{S}\hat{H}_d^i \hat{H}_u^j + \frac{1}{3}\kappa \hat{S}^3 \,. \tag{1}$$

The Yukawa couplings y_u, y_d and y_e are assumed to be diagonal 3×3 matrices in flavour space. The parameters λ and κ are in general complex. After electroweak symmetry breaking, the Higgs boson fields are expanded around their vacuum expectation values (VEVs) v_u, v_d, and v_s, respectively,

$$H_d = \begin{pmatrix} \frac{v_d + h_d + i a_d}{\sqrt{2}} \\ h_d^- \end{pmatrix}, \; H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{v_u + h_u + i a_u}{\sqrt{2}} \end{pmatrix}, \; S = \frac{e^{i\varphi_s}}{\sqrt{2}}(v_s + h_s + i a_s)\,, \tag{2}$$

with the CP-violating phases $\varphi_{u,s}$. The CP-even and CP-odd Higgs interaction states $(h_{d,u,s}, a_{u,d,s})$ mix to form five CP-indefinite Higgs mass eigenstates h_i $(i = 1,...,5)$, with their masses per convention ordered as $m_{h_1} \leq \cdots \leq m_{h_5}$, and one neutral Goldstone boson G^0. The charged Higgs bosons $H^\pm$ with mass $M_{H^\pm}$ and the charged Goldstone bosons $G^\pm$ are generated from the charged Higgs interaction states $h_d^\pm, h_u^\pm$. The five neutralino mass eigenstates denoted as $\tilde{\chi}_i^0$, $(i = 1,...,5)$ result from the mixing of the fermionic superpartners of the neutral Higgs bosons and the singlet field, i.e. the neutral higgsinos $\tilde{H}_u$, $\tilde{H}_d$ and the singlino $\tilde{S}$, with the neutral gauginos $\tilde{B}$ and $\tilde{W}_3$.

3. Calculational Framework

The W boson mass can be computed from the following relation [1]

$$M_W^2 = \frac{M_Z^2}{2}\left\{1+\sqrt{1-\frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2}\left(1+\Delta^{(n)}_{\texttt{NMSSMCALC}}r\right)}\right\}, \quad n=1,2\,. \tag{3}$$

In general Δr can be decomposed as

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\rm rem}. \tag{4}$$

The first term, $\Delta\alpha$, on the right-hand side of Eq. (4) is the light fermion (u,d,s,c,b,e,μ,τ) contribution to the running of the electromagnetic coupling from its definition at vanishing momentum to its value at M_Z. We separate $\Delta\alpha$ into leptonic and hadronic contributions, $\Delta\alpha = \Delta\alpha_{\rm lepton} + \Delta\alpha^{(5)}_{\rm had}$, with $\Delta\alpha^{(5)}_{\rm had} = 0.02768$ [9] and $\Delta\alpha_{\rm lepton} = 0.03150$ [10]. The second term, $\Delta\rho$, is most sensitive to beyond SM physics, and it is defined at one- and two-loop order as

$$\Delta^{(1)}\rho = \frac{\Sigma^{(1),T}_{ZZ}}{M_Z^2} - \frac{\Sigma^{(1),T}_{WW}}{M_W^2}, \quad \Delta^{(2)}\rho = -\frac{\Sigma^{(1),T}_{ZZ}}{M_Z^2}\left(\frac{\Sigma^{(1),T}_{ZZ}}{M_Z^2} - \frac{\Sigma^{(1),T}_{WW}}{M_W^2}\right) + \left(\frac{\Sigma^{(2),T}_{ZZ}}{M_Z^2} - \frac{\Sigma^{(2),T}_{WW}}{M_W^2}\right),$$

where $\Sigma^{(n),T}_{VV}$, $V=W,Z$ are the transverse parts of the gauge boson self-energies evaluated at $n=1,2$-loop order and at zero external momentum. The gauge boson self-energies in the NMSSM, however, include the SM-like corrections as a subset. When investigating the size of new physics effects, it is convenient to disentangle the SM-like corrections and the SUSY corrections as

$$\Delta^{(\alpha_i^2)}_{\rm NMSSM}\rho = \Delta^{(\alpha_i^2)}_{\rm SUSY}\rho + \Delta^{(\alpha_i^2)}_{\rm SM}\rho\,. \tag{5}$$

The third term, $\Delta r_{\rm rem}$, denotes all remaining contributions. In `NMSSMCALC`, we include the following contributions to Δr,

$$\Delta^{(n)}_{\texttt{NMSSMCALC}}r = \Delta^{\rm lit.}_{\rm SM}r + \Delta^{(n)}_{\rm SUSY}r\,, \tag{6}$$

and the SM corrections from the literature consist of the following terms,

$$\Delta^{\rm lit.}_{\rm SM}r = \Delta^{(1)}r + \Delta^{(\alpha\alpha_s)}r + \Delta^{(\alpha\alpha_s^2)}r + \Delta^{(\alpha^2)}r + \Delta^{(G_\mu^2 m_t^4\alpha_s)}r + \Delta^{(G_\mu^2 m_t^6)}r + \Delta^{(G_\mu^2 m_t^2\alpha_s^3)}r\,.$$

For a complete list of references we refer the reader to [11].

For $n=1$, we compute ourselves the full one-loop correction to Δr in SM and NMSSM by calculating one-loop corrections to the $\mu\to e\bar{\nu}_e\nu_\mu$ decay. It can be written as

$$\Delta^{(1)}r = \frac{\Sigma^{WW}_T(0) - \delta M_W^2}{M_W^2} + 2\delta Z_e - 2\frac{\delta s_W}{s_W} + \frac{1}{2}\left(\delta Z^\mu + \delta Z^e + \delta Z^{\nu_\mu} + \delta Z^{\nu_e}\right) + \Delta r_\triangle + \Delta r_\Box\,.$$

Here $\delta M_W^2, \delta Z_e, \delta s_W$ are the counterterms of the W mass, the electric coupling e, and the sine of the weak mixing angle being derived from the OS relation $s_W^2 = 1 - M_W^2/M_Z^2$. The wave function counterterms of the external leptons are denoted by δZ^l ($l=\mu,e,\nu_\mu,\nu_e$), while the triangle and box contributions are $\Delta r_\triangle$ and $\Delta r_\Box$, respectively. These are evaluated for vanishing lepton masses. For $n=2$, we include only the two-loop corrections of orders $O(\alpha_t\alpha_s)$ and $\mathcal{O}((\alpha_t+\alpha_\lambda+\alpha_\kappa)^2)$ to $\Delta\rho$. These corrections were adapted from our previous calculations of the two-loop $\mathcal{O}(\alpha_t\alpha_s)$ [12], $\mathcal{O}(\alpha_t^2)$ [13], $\mathcal{O}((\alpha_t+\alpha_\lambda+\alpha_\kappa)^2)$ [14] corrections to the loop-corrected Higgs boson masses.

4. Numerical results

This section highlights some main and interesting results of our study. The complete numerical investigation can be found in [1]. For illustrative purposes, the results are shown for a set of two parameter points, the first one `P1` (see [1] for the set of parameters) obtained from our simple scan and the second one `BP3` taken from [15]. The first point, `P1`, features rather light electroweakinos. The second, `BP3`, is characterized by large one-loop corrections to the W boson mass due to very light sleptons with masses of $\mathcal{O}(100\,\mathrm{GeV})$. Furthermore, singlet-like CP-even and -odd Higgs bosons have masses lighter than 50 GeV in `BP3`.

4.1. $\Delta\rho$ *and* Δr

In the upper left panel of Fig. 1, we present the prediction for the ρ parameter starting from the parameter point `P1` as a function $\sqrt{\lambda^2+\kappa^2}$ where λ and κ are the NMSSM-specific superpotential parameters. Note that we have introduced the following notation in the figure,

$$\alpha_{\rm new}^2 \equiv \alpha_t\alpha_s + (\alpha_\lambda + \alpha_\kappa + \alpha_t)^2 \ . \tag{7}$$

Since the ρ parameter strongly depends on the top quark mass, we show results using both the OS (full lines) and the $\overline{\rm DR}$[1] (dashed lines) renormalization schemes in the top/stop sector. We observe an increase in $\Delta\rho$ with increasing $\sqrt{\lambda^2+\kappa^2}$ which first is rather weak and becomes stronger for very large values of $\sqrt{\lambda^2+\kappa^2}$. This behaviour is due to an increase of the $SU(2)$ mass splittings between the neutral and charged Higgs bosons on the one side and the neutral and charged electroweakinos on the other side. Numerical instabilities can occur in the $\mathcal{O}((\alpha_t+\alpha_\lambda+\alpha_\kappa)^2)$ corrections when λ and κ are small and/or negative squared tree-level masses appear at the given loop-order. In these cases, the program automatically falls back to the $\mathcal{O}(\alpha_t\alpha_s)$ predictions for both the W-mass and $\Delta\rho$. In the upper left panel of Fig. 1 we observe good convergence for $\sqrt{\lambda^2+\kappa^2} > 0.1$ for M_W at $\mathcal{O}(\alpha_{\rm new}^2)$. For $\sqrt{\lambda^2+\kappa^2} < 0.1$ the M_W prediction does not converge and therefore the ρ parameter prediction is used at $\mathcal{O}(\alpha_t\alpha_s)$ which explains the jump of the red line onto the blue line.

The lower panel of Fig. 1 (left) plots $\Delta_{\rm ren}^{\rho}$,

$$\Delta_{\rm ren}^{\rho} = \frac{\Delta\rho^{\rm OS} - \Delta\rho^{\overline{\rm DR}}}{\Delta\rho^{\overline{\rm DR}}}, \tag{8}$$

obtained at the three considered loop orders as functions of $\sqrt{\lambda^2+\kappa^2}$. We observe a renormalization scheme dependence of up to $50\%, 22\%$ and 16% for the one-loop, $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}(\alpha_{\rm new}^2)$ results, respectively. Therefore, including the two-loop QCD and EW corrections can significantly reduce the theory uncertainty of the ρ parameter.

In the right panel of Fig. 1 we plot individual contributions to Δr obtained after the M_W iteration has converged. The blue solid line is the overall result of Δr produced using `NMSSMCALC` , including all available corrections. The green dotted line represents one-loop SM contributions, including the most substantial $\Delta\alpha$ contributions. The green dashed line represents the contribution

[1] The $\overline{\rm DR}$ renormalization scheme uses dimensional reduction in order to regularize UV-diverent loop integrals. Similar to the minimal-substraction scheme, its counterterms consist of the divergent part $\Delta_{\rm UV} = \frac{1}{\epsilon} - \gamma_E + \log(4\pi)$ plus a SUSY-restoring finite part.

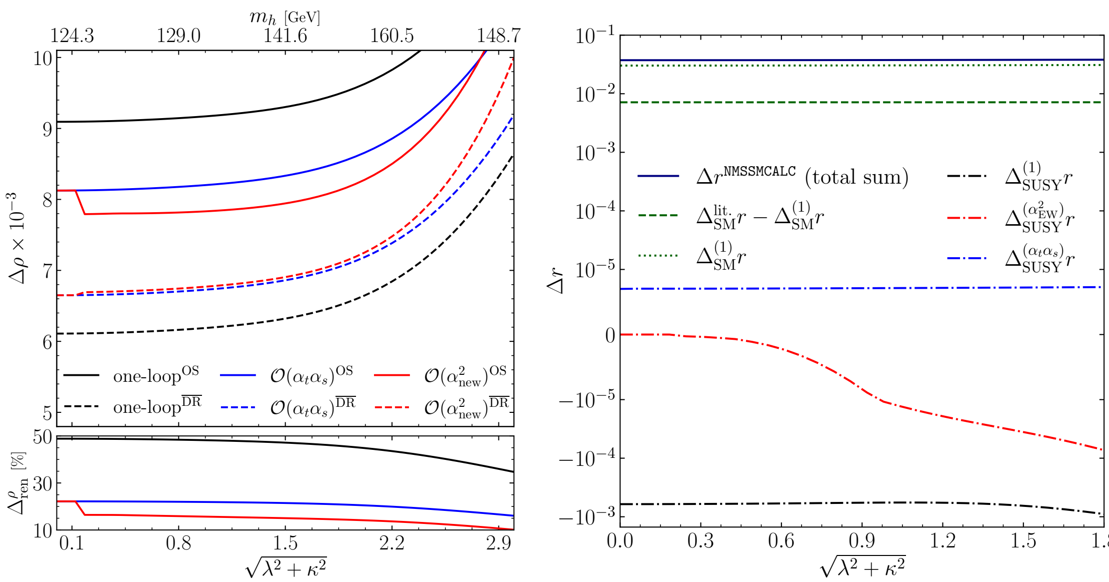

Fig. 1. Upper left: The ρ parameter at full one-loop order (black), two-loop $\mathcal{O}(\alpha_t\alpha_s)$ (blue) and two-loop $\mathcal{O}(\alpha^2_{\rm new})$ (red) as function of $\sqrt{\lambda^2+\kappa^2}$. The dashed lines are for the $\overline{\rm DR}$ scheme in the top/stop sector while the full lines are for the OS scheme. Lower left: Renormalization scheme dependence $\Delta^{\rho}_{\rm ren}$ of the ρ parameter at full one-loop order (black), at two-loop $\mathcal{O}(\alpha_t\alpha_s)$ (blue) and $\mathcal{O}(\alpha^2_{\rm new})$ (red). Right: Individual contributions to Δr predicted by `NMSSMCALC` . For $|\Delta r| > 10^{-5}$ a log-scale is chosen and a linear scale otherwise. These plots are taken from [1].

of higher-order SM results in the literature to Δr. The one-loop SUSY contribution (black dash-dotted) is the third-largest contribution and is negative, resulting in a positive shift to M_W. It is followed by the negative EW contributions (red dash-dotted). The SUSY QCD corrections (blue dash-dotted) are positive and numerically in competition with the EW corrections for $\sqrt{\lambda^2+\kappa^2} \gtrsim 0.6-0.9$. Note that we used a log-scale for $|\Delta r| > 10^{-5}$ and a linear scale otherwise to emphasize the $\sqrt{\lambda^2+\kappa^2}$-dependence of the two-loop SUSY corrections.

4.2. M_W and comparisons between tools

In the left panel of Fig. 2, we have plotted M_W as function of $\sqrt{\lambda^2+\kappa^2}$ using four public tools (`FlexibleSUSY` , `NMSSMTools` , `SARAH/SPheno` and `NMSSMCALC`) which can perform the M_W calculation in the NMSSM. The first source of differences relates to the incorporation of the SM higher-order corrections. The three codes `FlexibleSUSY` , `NMSSMTools` , `SARAH/SPheno` have used the fitting formula [4]

$$M_W^{\substack{\tt FlexibleSUSY \\ \tt SARAH/SPheno}} = \sqrt{M_W^{\rm SM\ fit.^2}(m_h, m_t, \alpha, \alpha_s)\left[1 + \frac{s_W^2}{c_W^2 - s_W^2}\Delta r^{(n)}_{\rm SUSY}\right]}, \tag{9}$$

where $M_W^{\rm SM\ fit.}$ is a numerical fit that incorporates the SM higher-order corrections as a function of the SM input parameters. This method loses the implicit dependence of $\Delta^{\rm lit.}_{\rm SM} r$ on the

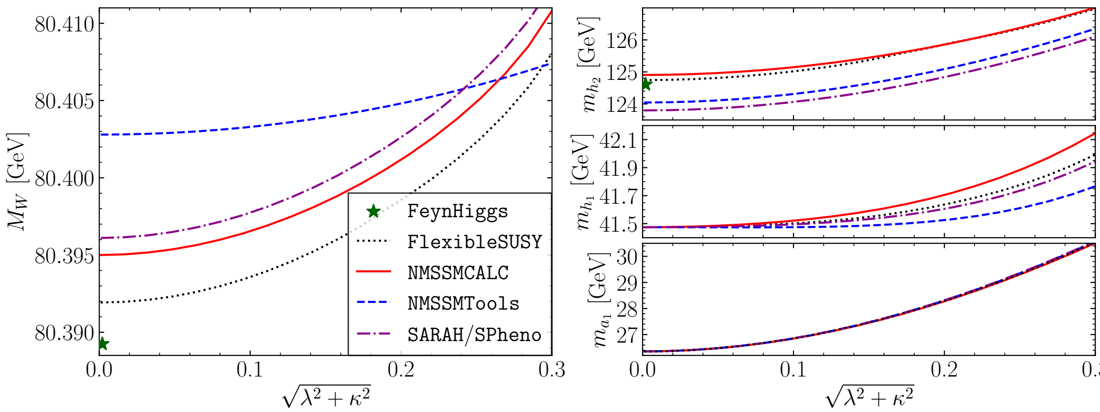

Fig. 2. Comparison between `NMSSMCALC` (red solid), `NMSSMTools` (blue dashed), `SARAH/SPheno` (violet dash-dotted) and `FlexibleSUSY` (black dotted) for the parameter point `BP3` as a function of $\sqrt{\lambda^2+\kappa^2}$. Left: W-mass prediction. Right (up to down): prediction of the SM-like Higgs boson mass m_{h_2}, the lightest CP-even Higgs mass m_{h_1} and lightest CP-odd mass m_{a_1}, respectively.

value of M_W which is correctly taken into account in `NMSSMCALC` . The second source of difference is the treatment of the SUSY input parameters. In `FlexibleSUSY` , Eq. (9) is evaluated with all running parameters at M_Z, while `SARAH/SPheno` uses parameters defined at the SUSY input scale, and `NMSSMCALC` and `NMSSMTools` compute M_W using the running SUSY input parameters given at the SUSY input scale $M^2_{\text{SUSY}} = m_{\tilde{t}_R} m_{\tilde{Q}_3}$. The $\tan\beta$ is also treated differently in different codes. `NMSSMCALC` and `NMSSMTools` interpret $\tan\beta$ at M_Z rather than M_{SUSY}, the two other codes use $\tan\beta(M_{\text{SUSY}})$. Last but not least the Higgs mass in the W mass prediction is different. A common approach is to use the loop-corrected Higgs mass to ensure that the NMSSM in the decoupling limit gives the same numerical value for M_W as the SM. This approach has been implemented in `FlexibleSUSY` , `NMSSMCALC` and `SARAH/SPheno` . `NMSSMTools`, however, uses a fixed value of $m_h = 125.2\,\text{GeV}$. In addition, the SM-like Higgs boson is not

Table 1. Comparison of the prediction for the SM-like Higgs boson mass, the W boson mass and the new physics contribution to the muon anomalous magnetic moment using `FlexibleSUSY` , `NMSSMCALC` , `NMSSMTools` , and `SARAH/SPheno` for two parameter points (P1 and PB3), see details in [1].

		FlexibleSUSY	NMSSMCALC	NMSSMTools	SARAH/SPheno
P1	m_h [GeV]	119.77	119.19	118.61	118.95
	M_W [MeV]	80366.3	80365.7	80370.8±23	80366.2
	$a_\mu \times 10^9$	0.29±0.01	0.256	0.329±0.03	0.33
BP3	m_h [GeV]	125.60	125.63	124.63	123.97
	M_W [MeV]	80396.9	80400.0	80404.2±22	80401.3
	$a_\mu \times 10^9$	2.98±0.45	2.89	3.19±0.34	3.70

necessarily the lightest scalar in the spectrum since the singlet-like states can in principle be lighter. For this reason, NMSSMCALC automatically determines the SM-like Higgs boson (based on the structure of the mixing matrix) which is to be used in the SM part of the calculation. In case of FlexibleSUSY, this information can be given via the SLHA input file by the user, while SARAH/SPheno always assumes it to be the lightest scalar state. The M_W prediction of FlexibleSUSY, NMSSMCALC and SARAH/SPheno shows almost exactly the same behaviour with increasing $\sqrt{\lambda^2+\kappa^2}$. The NMSSMCALC M_W prediction differs from FlexibleSUSY (SARAH/SPheno) by at most 1.7 MeV (3.1 MeV) which is less than the SM uncertainty. The NMSSMTools prediction, however, appears to be flatter for large values of $\sqrt{\lambda^2+\kappa^2}$. We believe this is due to NMSSMTools employing loop-corrected scalar masses throughout the M_W calculation. In the MSSM limit $\sqrt{\lambda^2+\kappa^2} \to 0$, we are able to also compare with the code FeynHiggs 2.19.0, which calculates m_h and M_W in the MSSM rather than the NMSSM. In Fig. 2, the FeynHiggs prediction is depicted with the green star mark. FeynHiggs yields the smallest M_W prediction which is still in good agreement with the other codes given the SM uncertainty alone. In particular the difference to the NMSSMCALC prediction is about 5.7 MeV.

In Table 1, we compare the predictions for the SM-like Higgs mass, the M_W mass and the muon anomalous magnetic moment a_μ obtained for two benchmark points (P1 and BP3). Higher-order corrections to the muon anomalous magnetic moment a_μ are known to have a connection to large corrections to M_W [3]. We present also the uncertainty estimates for M_W and a_μ that are computed by the programs NMSSMTools and FlexibleSUSY. The uncertainty for m_h due to missing higher-orders can be estimated to be at least about 1GeV [13, 14, 16]. Despite the fact that there are many differences in the treatment of the parameters between the four programs, the obtained results for the loop-corrected SM-like Higgs boson mass, for the anomalous magnetic moment a_μ and for M_W are overall in good agreement.

5. Conclusions

The W mass is an important parameter of the SM and it is very sensitive to physics beyond the SM. Precise measurements of it provide a test for the consistency of the SM. Using the electromagnetic coupling, the Z boson mass and the Fermi constant as input, M_W can be computed through the three-body decay of the muon using perturbation theory. We have presented a consistent inclusion of the full one-loop corrections to Δr and partial two-loop $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}(\alpha_{\text{new}}^2)$ through the ρ parameter in the complex NMSSM. These higher order terms improve the prediction of the W boson mass. The effects arising at $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}(\alpha_{\text{new}}^2)$ for M_W are of the order of a few MeV which is smaller than the parametric uncertainty of the top mass and is of similar size as the missing higher-order SM corrections. We have implemented all corrections in the new version of the Fortran code NMSSMCALC:

https://www.itp.kit.edu/~maggie/NMSSMCALC/.

We have also presented a comparison between four public tools on the prediction of M_W as well as the Higgs boson mass and the muon anomalous magnetic moment. We discussed the sources of differences between these codes and showed that their predictions are overall in agreement within their theoretical uncertainties.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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