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INHOMOGENEOUS COSMOLOGICAL MODELS IN BIMETRIC THEORY OF GRAVITATION

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Abstract. *Plane symmetric inhomogeneous cosmological models for perfect fluid distribution are investigated in the frame work of Rosen's bimetric theory of gravitation. A negative pressure corresponding to false vacuum state is considered. Some of the physical and kinematical properties of the models are discussed.*

I. INTRODUCTION

In recent times there have been a lot of research interests in modified theories of gravity because of their success in explaining a lot of observational data. In this context, Rosen's bimetric theory of gravitation [1] has attracted much attention [2-11].The motivation behind this proposition of this biometric theory is to avoid the problem of singularity occurring in Einstein's general relativity. The two metric tensors assumed in the bimetric theory are the Reimannian metric tensor g_{ij} which interacts with the matter and a flat background metric f_{ij}that describes the inertia forces.

Even though the large scale universe is homogeneous, some fluctuations are essential to trigger galaxy formation. At some stage small initial perturbations must have evolved into gravitationally bound systems. These fluctuations cause some local inhomogenerity. If these metric fluctuations have a scale-dependent random-phase character, then amplitude is within the range $10^{-4} - 10^{-5}$ [12]. Most of the cosmological models are based on the assumption of high degree of isotropy and homogeneity of the universe which can be easily described by the Freidman-Robertson-Walker (FRW) models. In order to take account of local fluctuations leading to inhomogeneity in the space-time which may be responsible for galaxy formation, it is wise to think of some anisotropic and inhomogeneous models to understand the evolution of the early phase of universe. There are good many works on inhomogeneous models. Bali and Tyagi [13], Pradhan *et al.* [14, 15] have investigated different aspects of inhomogeneous models using different space-time.

In this work, we have investigated the inhomogeneous plane symmetric models for perfect fluid. Considering a false vacuum state corresponding to the way inflationary cosmological models [16] assume, we have tried to get some plausible solutions to the Rosen's field equations. These solutions provide some interesting features of the universe.

The organization of the paper is as follows: In Sec. 2, basic field equations in the frame work of Rosen's Bimetric theory of gravitation are derived. In Sec. 3, some inflationary cosmological models are presented and at the end, the conclusions of the work are presented in Sec. 4.

II. BASIC EQUATIONS

At every point of space-time, there exist two line elements:

$$ds^2 = g_{ij}\, dx^i\, dx^j \tag{1}$$

and

$$d\sigma^2 = f_{ij}\, dx^i\, dx^j. \tag{2}$$

For an inhomogenous plane symmetric metric considered in the form

$$ds^2 = A(-dt^2 + dz^2) + B(dx^2 + dy^2) \tag{3}$$

the background flat metric will be

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{4}$$

where A and B are the metric potentials considered as functions of $z$and t.

The field equations of Rosen's bimetric theory of gravitation are

$$N^i_j - \frac{1}{2} N \delta^i_j = -8\pi k T^i_j \tag{5}$$

where $N^i_j = \frac{1}{2} f^{ab}(g^{hi} g_{hj|a})_{|b}$, $k = \sqrt{\frac{g}{f}}$, $g = \det(g_{ij})$ and $f = \det(f_{ij})$. The vertical bar denotes covariant differentiation. The energy momentum tensor T^i_j for a perfect fluid distribution is given by

$$T^i_j = (\rho + p) u^i u_j + p g^i_j \tag{6}$$

where ρ is the rest energy density of the system and p is the proper pressure. The four velocity vector u^isatisfies the condition $g_{ij} u^i u^j = -1$.

In commoving coordinates, the field equations (5) for the metrics (3) and (4) take the explicit forms

$$\left(\frac{A''}{A} - \left(\frac{A'}{A}\right)^2\right) - \left(\frac{\ddot{A}}{A} - \left(\frac{\dot{A}}{A}\right)^2\right) = 16\pi k p \tag{7}$$

$$\left(\frac{B''}{B} - \left(\frac{B'}{B}\right)^2\right) - \left(\frac{\ddot{B}}{B} - \left(\frac{\dot{B}}{B}\right)^2\right) = 16\pi k p \tag{8}$$

$$\left(\frac{B''}{B} - \left(\frac{B'}{B}\right)^2\right) - \left(\frac{\ddot{B}}{B} - \left(\frac{\dot{B}}{B}\right)^2\right) = -16\pi k \rho \tag{9}$$

where the overhead primes and dots denote differentiation of the metric potential with respect to $z$and $t$respectively.

Let us consider that

$$A = a(z)\,\lambda(t) \tag{10}$$

and

$$B = b(z)\,\theta(t) \tag{11}$$

so that

$$\frac{A'}{A} = \frac{a'}{a} = \alpha(z), \tag{12}$$

$$\frac{\dot{A}}{A} = \frac{\dot{\lambda}}{\lambda} = \nu(t), \tag{13}$$

$$\frac{B'}{B} = \frac{b'}{b} = \beta(z), \tag{14}$$

and

$$\frac{\dot{B}}{B} = \frac{\dot{\theta}}{\theta} = \mu(t). \tag{15}$$

With (12, 13) and (14, 15), the field equations (7)-(9) reduce to

$$\alpha' - \dot{\nu} = 16\pi k p \tag{16}$$

$$\beta' - \dot{\mu} = 16\pi k p \tag{17}$$

$$\beta' - \dot{\mu} = -16\pi k \rho \tag{18}$$

From (17) and (18) it is clear that $p + \rho = 0$, which refers to the false vacuum state. Eventhough for real energy condition, the only values p and ρ can take are $p = \rho = 0$, keeping an eye on the accelerated expansion phase of the universe (inflationary phase), we may consider $p = -\rho$ with finite non-zero values of p and ρ. In FRW models in general relativity the negative pressure corresponds to a repulsive gravity and is associated with the cosmological constant Λ. In the same footing we consider this false vacuum state in order to explore some of the interesting features of the model in bimetric theory.

From (16) and (17) we get

$$\alpha' - \beta' = \dot{\nu} - \dot{\mu} \tag{19}$$

The possible implications of this equation are

$$\text{(i)} \qquad \alpha'(z) = \beta'(z) = \dot{\nu}(t) = \dot{\mu}(t) = 0 \tag{20}$$

$$\text{(ii)} \qquad \alpha'(z) = \beta'(z) \tag{21}$$

$$\dot{\nu}(t) = \dot{\mu}(t) \tag{22}$$

$$\text{(iii)} \qquad \alpha'(z) - \beta'(z) = \dot{\nu}(t) - \dot{\mu}(t) = c = \text{constant} \tag{23}$$

III. INFLATIONARY COSMOLOGICAL MODELS

Case-I

It is evident from (20) that α, β, ν and $\mu$are constant quantities. Let us suppose that $\alpha = k$, $\beta = l$,$\nu = m$ and $\mu = n$ where k, l, m, n are constants of integration. Integration of these relations immediately yield

$$A = A_0 e^{kz+mt} \tag{24}$$

$$B = B_0 e^{lz+nt} \tag{25}$$

With the convenient choice $A_0 = B_0 = 1$, the metric for this model can be expressed as,

$$ds^2 = e^{kz+mt}(-dt^2 + dz^2) + e^{lz+nt}(dx^2 + dy^2) \tag{26}$$

From equations (17)and (18), we can have the physical properties of the model, expressed in general, as

$$p = -\rho = \frac{1}{16\pi k}\left(\beta' - \dot{\mu}\right) \tag{27}$$

Since β and μ are constant quantities for the present model (23), the proper pressure$p$ and the energy density ρ assume null values i.e $p = \rho = 0$. For all negative values of k and l, the local inhomogeneity vanishes for large values of z.

The volume scale factor of the model can be expressed by

$$\tau = AB = \exp[(k+l)z + (m+n)t] \tag{28}$$

which clearly represents an inflationary vacuum universe.

Case-II

In view of (21) and (22), we may consider

$$\alpha(z) = \beta(z) \tag{29}$$

and

$$\nu(t) = \mu(t)(z). \tag{30}$$

Baring the discrepancies in the proportionality constants, from (29) and (30) it can be ascertained that $A \propto B$.

Integration of (29) and (30) yield

$$A = A_1 \exp\left(\int \beta dz + \int \mu\, dt\right) \tag{31}$$

and

$$B = B_1 \exp\left(\int \beta\, dz + \int \mu\, dt\right). \tag{32}$$

The metric for this model can now be expressed as

$$ds^2 = \left[A_1(-dt^2 + dz^2) + B_1(dx^2 + dy^2)\right] \exp\left(\int \beta dz + \int \mu dt\right). \tag{33}$$

The present model (33) is more involved and the metric potentials are expressed in the quadrature form. This is because of the nature of the metric chosen to describve the model, and since $A \propto B$, $p = -\rho$, all the field equations(7)-(**??**) reduce to a single equation. In such a situation it is not easy to get a particular solution for the equations and the metric potentials are to taken in quadrature form or else chosen arbitrarily to satisfy the physical situations of the universe. Consequently, the properties of this model can not be directly ascertained and they depend upon the choice of the functionals β and μ.

The interesting feature of the model is that if we chose $\beta = z$ and $\mu = t$, we get the same result as earlier i.e. $p = -\rho = 0$. It may be noted here that linear functions of β and μ do not lead to the survival of the model in Rosen's bimetric theory. Any other convenient choices of the functional β and μ may provide some determinate solutions to the model.

The volume scale factor for the model can be expressed as

$$\tau = A_1 B_1 \exp\left[2\left\{\int \beta\, dz + \int \mu\, dt\right\}\right]. \tag{34}$$

The inflationary nature of the model depends upon the functionalμ. For a choice of $\beta = z$ and$\mu = t$, $\tau = A_1 B_1 \exp[z^2 + t^2]$, which represents an accelerating universe.

If we assume that the local inhomogeneity should be removed for large values of z and the universe be witnessed by an accelerating expansion phase it is worthwhile to chose , of course in an arbitrary manner, $\beta = -\frac{k_1}{z^2}$ and $\mu = 3k_2t^2$, for which the metric potentials can be expressed as $A = A_1 \exp\left[\frac{k_1}{z} + k_2t^3\right]$ and $B = B_1 \exp\left[\frac{k_1}{z} + k_2t^3\right]$, where k_1 and $k_2$are positive constants. The physical properties of the model for these convenient choice can be represented by $p = -\rho = \frac{1}{16\pi k}\left[\frac{2k_1}{z^3} - 6k_2t\right]$. With the growth of time, the inhomogeneity in the space time decreases whereas the properties of the model evolve gradually. At the beginning the values of p and $-\rho$ depend upon the local inhomogeneity of the space time.

Case-III

From (23), we get

$$a(z) = a_1 \exp\left(\frac{c}{2}z^2 + c_1 z + \int \beta\, dz,\right) \tag{35}$$

$$b(z) = b_1 \exp\left(\int \beta\, dz\right), \tag{36}$$

$$\lambda(t) = \lambda_1 \exp\left(\frac{c}{2}t^2 + c_2 t + \int \mu\, dt\right), \tag{37}$$

$$\theta(t) = \theta_1 \exp\left(\mu\, dt\right), \tag{38}$$

where $a_1, b_1, c_1, c_2, \lambda_1, \theta_1$ are constants. In view of (35)-(38) and (10)-(11), the metric potentials can be expressed as

$$A = A_2 \exp\left[\frac{c}{2}(z^2 + t^2) + c_1 z + c_2 t + \int \beta\, dz + \int \mu\, dt\right] \tag{39}$$

$$B = B_2 \exp\left(\int \beta \, dz + \int \mu \, dt\right) \tag{40}$$

where A_2, B_2, c_2 are constants.

The metric for this model can be written as

$$\begin{aligned} ds^2 = &\left\{A_2\left(-dt^2 + dz^2\right)\right\} \exp\left[\left(\frac{c}{2} + 1\right)(z^2 + t^2) + c_1 z + c_2 t + \int \beta \, dz + \int \mu \, dt\right] \\ &+ \left\{B_2\left(dx^2 + dy^2\right)\right\} \exp\left[\int \beta \, dz + \int \mu \, dt\right]. \end{aligned} \tag{41}$$

In this model also the metric potentials are expressed in quadrature forms and the properties of the model depend upon the choice of the functionals β and μ. For the convenient choice of $\beta = z$ and $\mu = t$, $p = -\rho = 0$ leading to a vacuum state. We may infer that any other convenient choices may lead to interesting features of the model.

With suitable functional forms i.e. $\beta(z) = z$ and $\mu(t) = t$, the metric can be expressed as

$$\begin{aligned} ds^2 = &\left[\left\{A_2\left(-dt^2 + dz^2\right)\right\} \exp\left[\left(\frac{c}{2} + 1\right)(z^2 + t^2) + c_1 z + c_2 t\right] + \left\{B_2\left(dx^2 + dy^2\right)\right\}\right] \\ &\times \exp\left[\frac{z^2 + t^2}{2}\right]. \end{aligned} \tag{42}$$

The volume scale factor of the model (10), can be expressed as

$$\tau = A_2 B_2 \exp\left[\frac{c}{2}(z^2 + t^2) + c_1 z + c_2 t + 2\int \beta \, dz + 2\int \mu \, dt\right] \tag{43}$$

which will represent an accelerating universe only if the contribution from $2\int \beta \, dz + 2\int \mu \, dt$ does not cancel out the contribution from $\frac{c}{2}(z^2+t^2)+c_1 z+c_2 t$. However, for any increasing functions β and μ, the model represents an inflationary model.

IV. CONCLUSION

In the present work, we have investigated the inhomogeneous plane symmetric models for perfect fluid distribution in the frame work of Rosen's Bimetric theory of gravitation. In order to get some determinate solution we have assumed the inflationary kind of solutions i.e $p = -\rho$, which represent a false vacuum state. The usual p=ρgquation of state provide only the vacuum solution i.e. p=gρ = wu The solutions to the field equations are obtained in quadrature form. Any convenient choice of the functional β and μ will lead interesting inflationary solutions of the model. However the choice, $\beta(z) = z$ and $\mu(t) = t$ reproduce the earlier vacuum models (i.e $p = -\rho = 0$) of Sahoo[10]. With certain convenient choice of the metric potentials corresponding to the physical situations of the universe, it can be ascertained that the local inhomogeneity is removed for large values of z and the properties of the model depend both upon the space and time coordinates of the space time. At the beginning of the universe, the properties depend upon the local inhomogeneity.

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