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Finite-difference time-domain simulation of THz light pulse with orbital angular momentum produced using a spiral phase plate

K. Alaba[†], **Hannah R. Bardolaza**, **Nathaniel P. Hermosa II**, **Elmer S. Estacio**

National Institute of Physics, University of the Philippines, Diliman, Philippines

E-mail: [†]kalaba@nip.upd.edu.ph

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Abstract. *A silicon spiral phase plate was proposed for a pulsed terahertz beam to produce orbital angular momentum with a topological charge of $l = 1$. The resulting field distribution after the terahertz pulse traverses the silicon spiral phase plate was simulated using finite-difference time-domain method and was observed at different propagation distances. The simulation shows that a helical spatial structure and optical vortex can be observed in the pulse due to the silicon spiral phase plate and is best observed at the farthest distance of 7.5 cm. Additionally, the cross-section field distribution of the pulse at different times as it passes through the silicon spiral phase plate indicates a helical wavefront structure. Moreover, the output of a source beam with different central frequencies was observed when using the same silicon spiral phase plate where it similarly showed a hollow distribution, indicating a vortex albeit being located off-center. This demonstrates the generation of a structured beam with orbital angular momentum of electromagnetic waves using silicon spiral phase plate for a broadband terahertz single pulse.*

Keywords: orbital angular momentum; spiral phase plate; terahertz; vortex.
Classification numbers: 07.05.Tp; 42.79.-e; 78.20.Bh; 78.20.-e.

1. Introduction

1.1. Structured light and orbital angular momentum

Electromagnetic (EM) waves or light have an inherent structure as sinusoidal electric and magnetic fields that are propagating perpendicular to each other. A seemingly unstructured plane wave with sinusoidal fringes can be considered to have structured light intensity which can be seen in the formation of fringes. Even a plane wave can be considered to be structured with a uniform phase gradient, despite being only visible using an interferometer. Traditionally, optics used to be mainly interested in Gaussian beams where unwanted transverse modes are typically unwanted and removed, despite their numerous possible applications in observing the behavior of light beams [1, 2]. The advancements in optical devices such as liquid-crystal, spatial light, modulators, and digital micromirror devices, opened up possibilities for reshaping light. The produced light is generally referred to as structured light and has several possible degrees of freedom and dimensions such as amplitude, phase, and polarization like with spatiotemporal structured light [1]. One important interest brought about by the developments in structured light is enabling light beams to carry orbital angular momentum [1, 3, 4].

Light is said to carry an orbital angular momentum (OAM) when it has an azimuthal phase dependence. Another way for light to carry angular momentum is through azimuthal polarization dependence which introduces spin angular momentum (SAM). However, the momentum due to OAM is significantly greater than with SAM [5, 6]. In this study, a broadband terahertz pulse with orbital angular momentum by a spiral phase plate will be simulated and the resulting beam patterns will be investigated.

1.2. Terahertz wave

Most of the studies on structured light still focus on a single wavelength or frequency in the optical range [6]. However, the structuring of light is not limited to the optical region and can be applied to other types of EM waves such as terahertz waves. Terahertz wave spans the frequency range from 0.1 to 30 THz, between microwave and infrared. Terahertz bridges the domains of electronics and photonics and holds significant applications in various fields, including high-speed data communications, radar imaging, nondestructive testing, and biological detection [7, 8]. Current methods of terahertz generation produce a pulse that is a superposition of plane waves propagating at different frequencies and amplitudes, whose convolution produces the terahertz time domain waveform [7–9]. This combination of different frequencies outside the visible range can be a challenge in producing terahertz pulses with a desired structure since studies on structured light are concentrated on monochromatic or single-frequency visible light sources [1, 6].
Current terahertz applications mainly utilize parameters such as amplitude, phase, and polarization. Producing structured terahertz pulses can expand the current applications by introducing more degrees of freedom. Terahertz with OAM has great potential in multiplexing and quantum cryptography in communications due to the possible unlimited number of states. A kernel function in real space that arises from structured terahertz can be used in post-processing when taking its convolution with imaging signals which can produce intensity maxima at amplitude or phase edges that improve imaging contrast, which can be very helpful when imaging samples such as biological tissues, which terahertz can penetrate and image non-invasively [8].

2. Design and simulation

2.1. Orbital angular momentum and spiral phase plates

Light with an orbital angular momentum can also be observed to have a helical wavefront. There are several ways to induce a helically varying phase and amplitude. One such way is utilizing diffractive optical elements typically using either spiral Fresnel lenses or forked diffraction gratings such as in Vijayakumar and Bhattacharya's (2012) study [10]. There they explored spiral-phase Fresnel zone plate and obtained asymmetrical donut-shaped beam from continuous optical source [10]. However, in this method, a designed refractive element works only for a single frequency [6, 11]. One alternative method involves the use of spiral phase plates.

A spiral phase plate (SPP) is ideally composed of a homogeneous material with an index of refraction n and is highly transparent in the frequency range of interest. Geometrically, it has an azimuthally varying thickness which can be expressed as

$$h(\theta) = \frac{l\lambda\theta}{(n-1)(2\pi)}. \tag{1}$$

Here, θ is the azimuthal angle and λ is the wavelength of the illuminating beam and l is the topological charge [8, 11]. This study will focus solely on the simplest and most common case $l = 1$. Additionally, since an SPP completes only 1 rotation, there is a jump in the thickness of the SPP between angles $\theta = 2\pi$ and $\theta = 0$, as shown in Fig. 1.

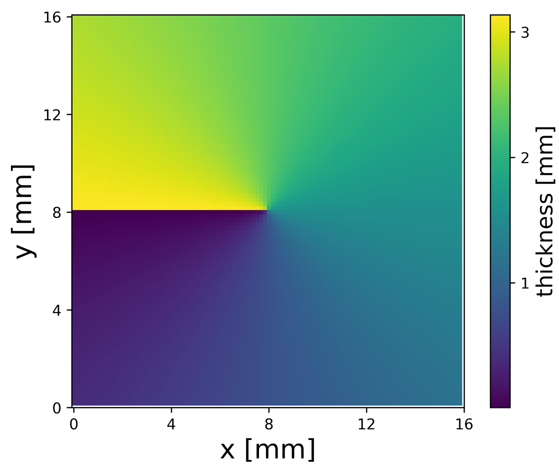

Fig. 1. Representation of a spiral phase plate with a uniform index of refraction and azimuthally varying thickness.

After passing through SPP, the output beam will have a vortex in the middle characterized by a hollow amplitude distribution where the maximum amplitude is distributed in a ring surrounding the vortex [8].

Although only a single wavelength can be accounted for, in calculating the thickness of the SPP as shown in Eq. (1), the vortex THz beams are still ideal in other frequencies due to the limited bandwidth of the radiated beam. However, if the bandwidth is big enough, the frequencies far from the center frequency will generate imperfect vortex THz beams [8].

In this study, the output light beam of a pulsed terahertz light source, composed of different frequencies, after propagating through an SPP will be investigated using finite-difference time-domain simulations.

To investigate the output beam, finite-difference time-domain simulations were performed using FDTD: 3D Electromagnetic Simulator of Ansys Optics 2023 R1 from Lumerical Inc. [12]. FDTD is a 3D full-wave electromagnetic solver in the time domain. It is commonly used for modeling nanophotonic devices, processes, and materials. FDTD works by discretizing both the space and time of the simulation domain. Discretization of space into mesh cells allows the modeling of complex geometries while the discretization of time into time steps captures the temporal evolution of EM fields. The electromagnetic fields are calculated from Maxwell's equations in every mesh cell and the solutions are repeatedly time-stepped [12,13].

A simulation region with dimensions of approximately 16 mm by 16 mm in the $x-$ and $y-$ directions and 10 cm in the z-direction was created. A broadband light source with a central frequency of 0.4 THz and a Gaussian profile was placed at one end of the $z-$axis and the SPP was placed close to the source to provide enough distance for the light to propagate after impinging through the SPP. The simulation time was set so that the beam could have enough time to propagate from one end to the other end of the simulation region.

The height as a function of the azimuthal angle was calculated using Eq. (1) for a central frequency of 0.4 THz and a silicon material with an index of refraction of $n = 3.4$ in the terahertz range with a negligible extinction coefficient [14]. This central frequency value is a common frequency achieved with terahertz emitters.

Finally, using the same designed SPP, the output beams for different central frequencies were investigated.

3. Results and Discussion

3.1. Propagation distance

Using a SPP with a topological charge of $l = 1$, designed for a central frequency of 0.4 THz, the resulting electric field distribution was observed at different distances from the SPP, as shown in Fig. 2, with the subfigures following the same color scale. Most notably, a hole or small region in the center can be observed where the magnitude of the electric field is close to 0. This is a characteristic of the vortex in the middle where different phases of light interfere destructively resulting in a dark region [15]. These observations are similar to when a single frequency optical beam is used [1,3,4].

At distances near the SPP, despite the presence of the vortex, the structured light is not properly formed as indicated by the non-uniform radial distribution of the light. However, at distances farther from the SPP, the distribution of light is radially uniform and the vortex is almost perfectly circular.

3.2. Propagation time

The propagation of the single pulse at different times was also observed by taking snapshots of the pulse at different time intervals on a plane parallel to the propagation axis as shown in Fig. 3, with the subfigures following the same color scale. This shows a single pulse of a terahertz signal with a central frequency of 0.4 THz where the blue and red bands represent the positive and negative peaks of the signal, respectively. It can be observed that before passing through the SPP, at 11 ns, the signal follows a Gaussian distribution where the peak intensity is located at the center and has the same axis as the designed SPP. After passing through the SPP, it can be observed that the positive peak on the positive $x-$position is now ahead of the positive peak at the negative $x-$position. Moreover, the negative peak of the signal at the positive $x-$position is now aligned with the positive peak of the signal at the negative $x-$position. This indicates that the signal at the azimuthal position of θ is ahead of the signal in the azimuthal position $\theta + \pi$ by half of a wavelength.

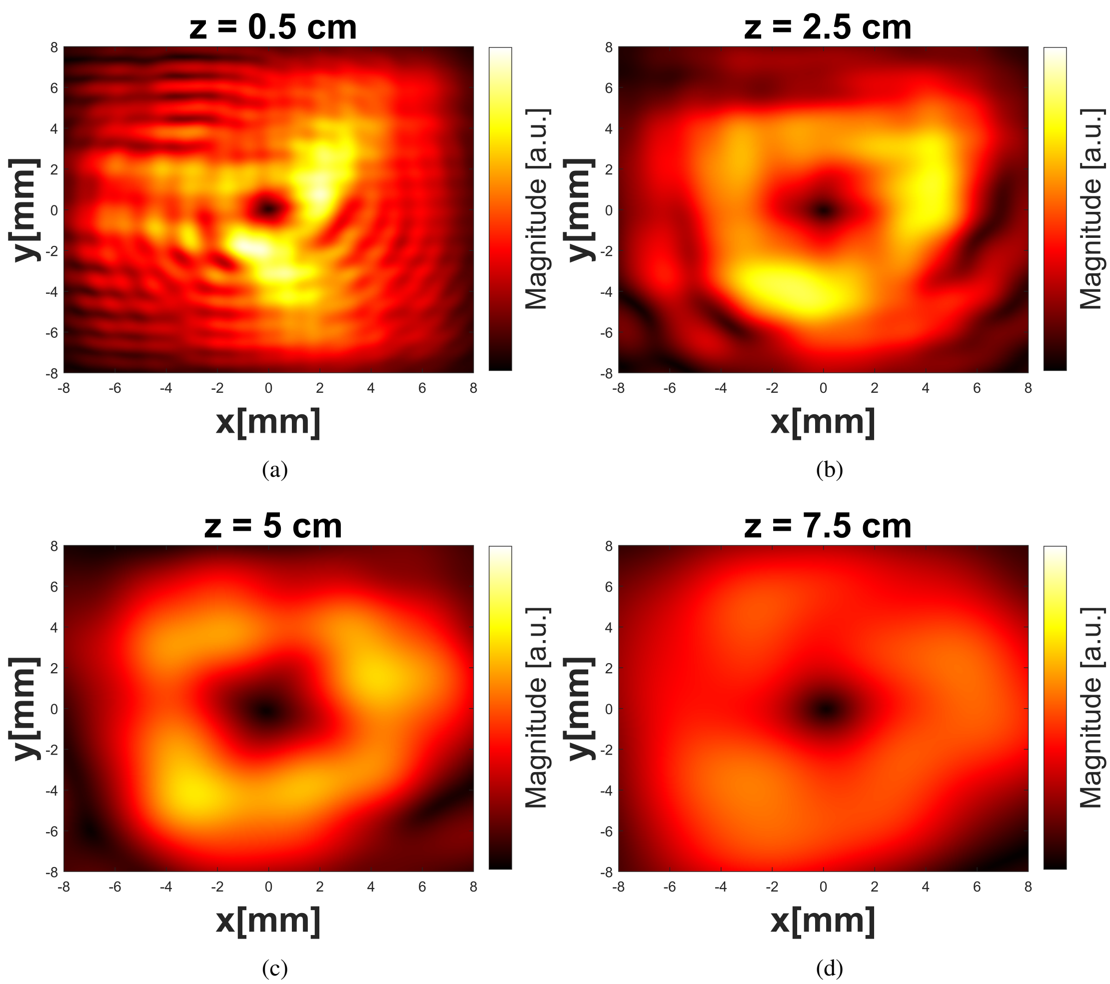

Fig. 2. Distribution of the output electric field magnitude for a terahertz source with a central frequency of 0.4 THz at distances of (a) 0.5 cm, (b) 2.5 cm, (c) 5.0 cm, and (d) 7.5 cm away from the SPP. The subplots are following the same color scale.

Moreover, a discontinuity in the $x = 0$ position with a finite width can be observed after passing through the SPP due to the vortex that is induced by the SPP.

This can be attributed to the non-uniform phase delay introduced by the non-uniform thickness of the SPP [4]. Since the thickness follows an azimuthally varying pattern as shown in Fig. 1, the phase delay also follows a similar profile where a singularity in the center, along the propagation axis, can be observed due to the different phases canceling each other [4]. Moreover, in contrast with the Gaussian profile of the input beam having its peak intensity in the center, the output beam has a peak intensity in a ring around the vortex which can also be observed in Figs. 2 and 4.

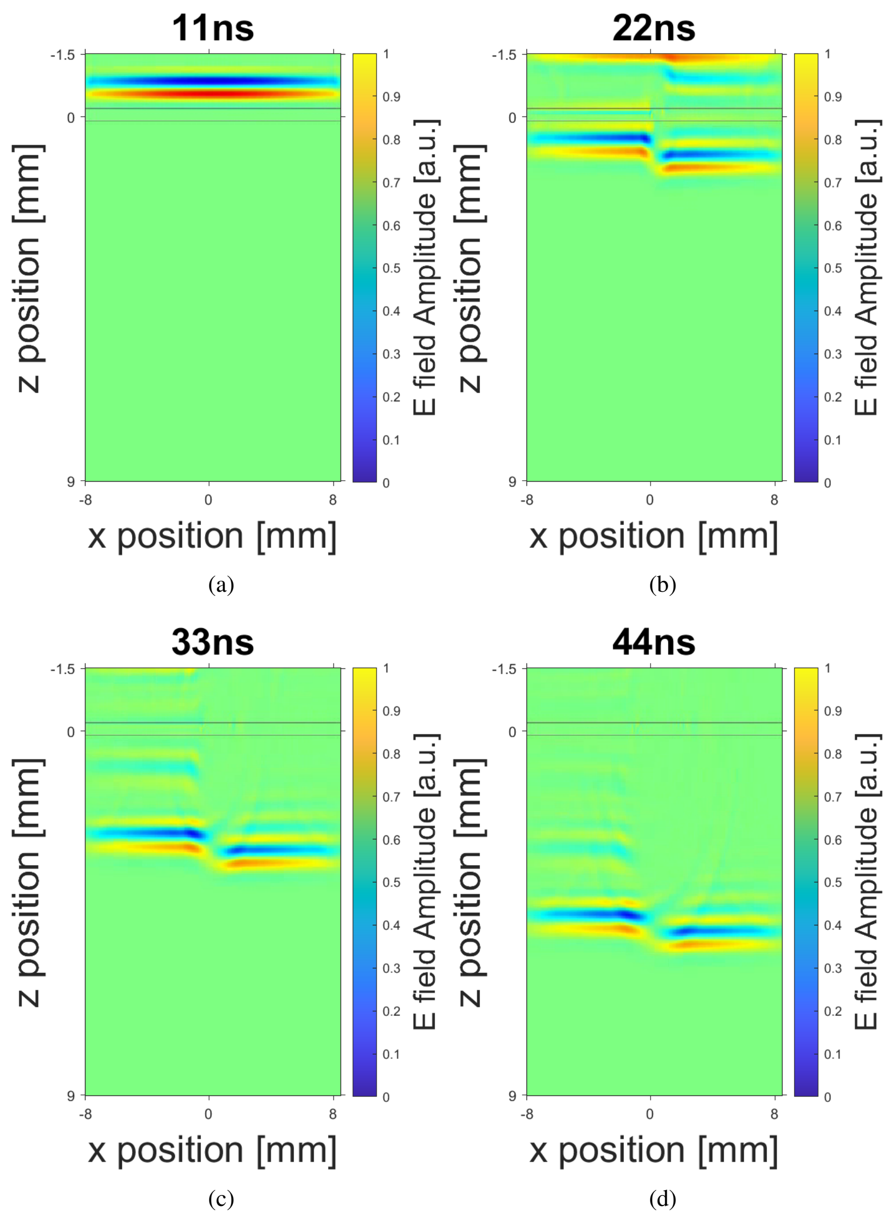

Fig. 3. Electric field amplitude distribution at a plane perpendicular to the propagation at different elapsed time of (a) 11 ns, (b) 22 ns, (c) 33 ns, and (d) 44 ns. The subplots are following the same color scale.

3.3. Different frequencies

Since a terahertz pulse is a superposition of waves with different frequencies, it is important to investigate the behavior of the other frequencies using the SPP, despite the SPP being designed for a different frequency. Shown in Fig. 4 is the electric field distribution of different frequencies when passing through an SPP designed for a central frequency of 0.4 THz.

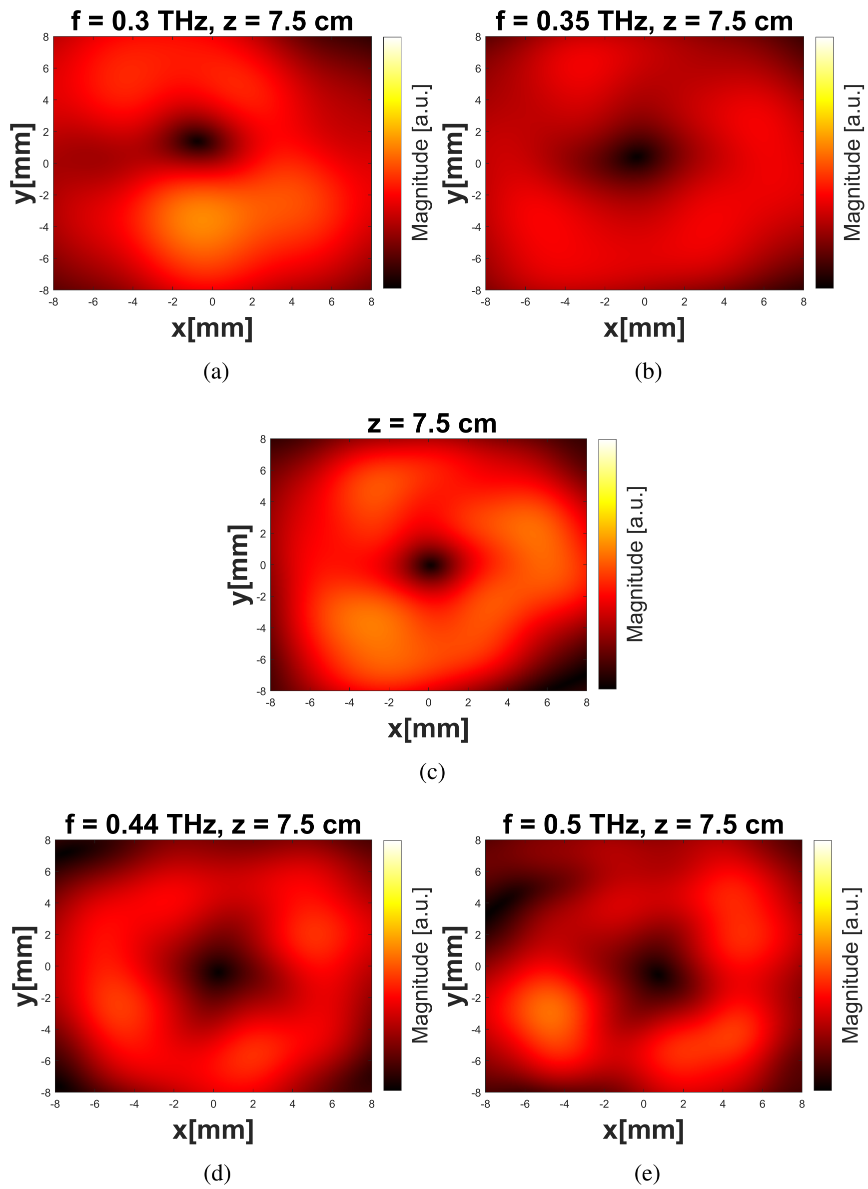

Fig. 4. Electric field magnitude distribution for frequencies of (a) 0.3 THz, (b) 0.35 THz, (c) 0.4 THz, (d) 0.44 THz, and (e) 0.5 THz. The subplots are following the same color scale.

Moreover, looking at the positions in the observation plane where the real and imaginary parts of the electric field are equal to zero as shown in Fig. 5, we can see the approximate position

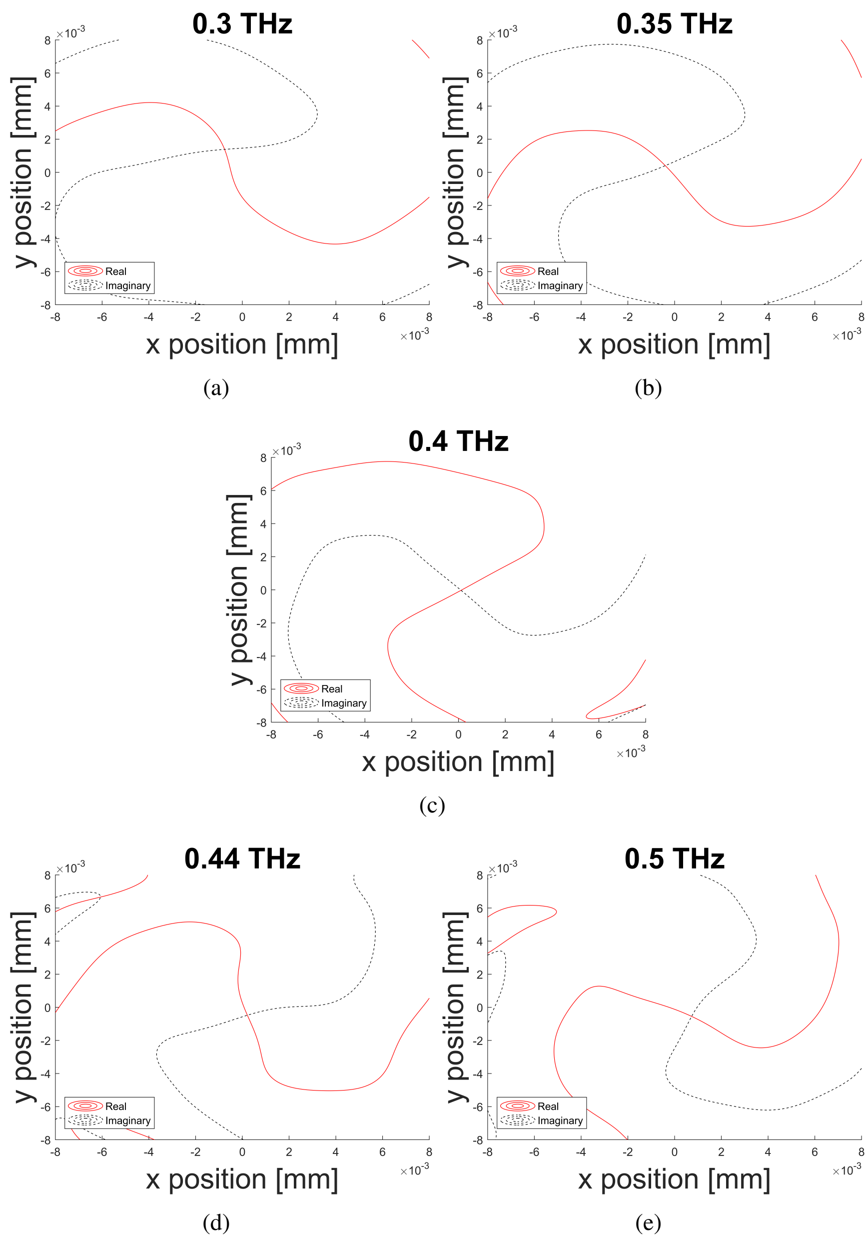

Fig. 5. Positions in the observation plane where the real and imaginary parts of the electric field are zero for frequencies of (a) 0.3 THz, (b) 0.35 THz, (c) 0.4 THz, (d) 0.44 THz, and (e) 0.5 THz.

of the vortex to be close to the center for each frequency. Since the real and imaginary parts of the electric field vanish, the intensity at this intersection is approximately equal to zero.

Figures 4 and 5 show that using a SPP designed for a frequency of 0.4 THZ can produce orbital angular momentum and optical vortex for other frequencies close to 0.4 THz. This is indicated by the annular electric field distribution with a vortex in the center whose position is approximated in Fig. 5.

4. Conclusions

A silicon SPP was proposed to induce orbital angular momentum on a terahertz pulse with a central frequency of 0.4 THz. Simulation results show that the proposed SPP was able to produce a helical wavefront and a hollow vortex in the propagation axis, indicating a structured light with orbital angular momentum and a helical wavefront. When the central frequency was changed while the same SPP was used, the field distribution was imperfect with a non-uniform azimuthal distribution of magnitude. However, a hole in the electric field distribution was still observed and the location where the electric field cancels out near the center, indicates an optical vortex and its position.

Acknowledgements

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Conflict of interest

The authors declare that they have no competing financial interests.

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