

NUCLEAR SYMMETRY ENERGY IN CHIRAL MODEL OF NUCLEAR MATTER

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Abstract. *The physical properties of asymmetric nuclear matter are studied in the Extended Nambu-Jona-Lasinio (ENJL) model formulated directly in the nucleon degrees of freedom. It results that the density dependence of the nuclear symmetry energy and its related quantities are basically in good agreement with data of recent analyses.*

I. INTRODUCTION

In recent years many radioactive beam facilities have been established in the world, such as FAIR/GSI in Germany, SPIRAL2/GANIL in France, RIB/RIKEN in Japan and so on. Experimental realizations using radioactive beams with large neutron or proton excess have created a breakthrough for exploring the role of isospin degree of freedom in modern nuclear physics. It was shown that most observed phenomena are mainly determined by the in-medium nucleon-nucleon potentials and the isospin-dependent nuclear equation of state (EOS), especially the density dependence of the nuclear symmetry energy. As was known, the latter quantity plays a crucial role for understanding not only a lot of important issues in nuclear physics [1-3], but also many critical problems in astrophysics [4, 5]. There have been so far a great deal of investigations on this subject based on both nonrelativistic [6-9] and relativistic [10-17] nuclear theories. The present paper is devoted to the study of the isospin-dependent equation of state, in particular, the nuclear symmetry energy and its related quantities. Our starting point is the Extended Nambu-Jona-Lasinio (ENJL) model, which is able to remove some drawbacks of both nonrelativistic theories (where the causality is violated at high densities) and relativistic theories (where the chiral symmetry is absent). In our previous article [18] the chiral phase transition and liquid-gas phase transition at subsaturation density were scrutinized for symmetric nuclear matter

by means of the Lagrangian of the ENJL model given as

$$\begin{aligned}\mathcal{L}_{NJL} = & \bar{\psi}(i\hat{\partial} - m_0)\psi + \frac{G_s}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \frac{G_v}{2}(\bar{\psi}\gamma_\mu\psi)^2 \\ & + \frac{G_{sv}}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2](\bar{\psi}\gamma_\mu\psi)^2.\end{aligned}$$

Now \mathcal{L}_{NJL} is generalized to the case of asymmetric nuclear matter by incorporating rho meson into consideration, namely,

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\hat{\partial} - m_0 + \gamma_0\mu)\psi + \frac{G_s}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \frac{G_v}{2}[(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\psi)^2] \\ & + \frac{G_{sv}}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2][(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\psi)^2] - \frac{G_r}{2}[(\bar{\psi}\gamma^\mu\frac{\vec{\tau}}{2}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\frac{\vec{\tau}}{2}\psi)^2],\end{aligned}\quad (1)$$

where $\mu = \text{diag}(\mu_p, \mu_n)$, $\mu_{p,n} = \mu_B \pm \mu_I/2$; m_0 is the ‘‘bare’’ mass of the nucleon, $\vec{\tau}$ is the isospin Pauli matrices, and G_s , G_v , G_{sv} , and G_r are coupling constants.

In the mean-field approximation we replace

$$\begin{aligned}(\bar{\psi}\Gamma_i\psi)^2 &= 2\bar{\psi}\Gamma_i\psi[\bar{\psi}\Gamma_i\psi] - [\bar{\psi}\Gamma_i\psi]^2, \\ (\bar{\psi}\Gamma_i\psi\bar{\psi}\Gamma_j\psi)^2 &= ([\bar{\psi}\Gamma_i\psi]\bar{\psi}\Gamma_j\psi)^2 + (\bar{\psi}\Gamma_i\psi[\bar{\psi}\Gamma_j\psi])^2 - ([\bar{\psi}\Gamma_i\psi][\bar{\psi}\Gamma_j\psi])^2 \\ &= [\bar{\psi}\Gamma_i\psi]^2(2\bar{\psi}\Gamma_j\psi[\bar{\psi}\Gamma_j\psi] - [\bar{\psi}\Gamma_j\psi]^2) + (2\bar{\psi}\Gamma_i\psi[\bar{\psi}\Gamma_i\psi] - [\bar{\psi}\Gamma_i\psi]^2)[\bar{\psi}\Gamma_j\psi]^2 \\ &\quad - [\bar{\psi}\Gamma_i\psi]^2[\bar{\psi}\Gamma_j\psi]^2,\end{aligned}\quad (2)$$

with $\Gamma = \{1, i\gamma_5\vec{\tau}, \gamma_\mu, \gamma_5\gamma_\mu\}$, and the averaging at finite density and temperature denoted by angular brackets.

Eqs. (2) combines with the bosonization

$$\sigma = \bar{\psi}\psi, \quad \vec{\pi} = \bar{\psi}i\gamma_5\vec{\tau}\psi, \quad \omega_\mu = \bar{\psi}\gamma_\mu\psi, \quad \phi_\mu = \bar{\psi}\gamma_5\gamma_\mu\psi, \quad \vec{\varrho}_\mu = \bar{\psi}\gamma_\mu\frac{\vec{\tau}}{2}\psi, \quad \vec{\chi}_\mu = \bar{\psi}\gamma_5\gamma_\mu\frac{\vec{\tau}}{2}\psi,$$

yielding

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\hat{\partial} - m_0 + \gamma_0\mu)\psi + [G_s + G_{sv}(\omega^2 + \phi^2)]\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\psi \\ & - [G_v - G_{sv}(\sigma^2 + \pi^2)]\bar{\psi}\gamma^\mu(\omega_\mu + \gamma_5\phi_\mu)\psi - G_r\bar{\psi}\gamma^\mu\frac{\vec{\tau}}{2}(\vec{\varrho}_\mu + \gamma_5\vec{\chi}_\mu)\psi \\ & - \frac{G_s}{2}(\sigma^2 + \pi^2) + \frac{G_v}{2}(\omega^2 + \phi^2) + \frac{G_r}{2}(\varrho^2 + \chi^2) - 3\frac{G_{sv}}{2}(\sigma^2 + \pi^2)(\omega^2 + \phi^2).\end{aligned}\quad (3)$$

In (3) we impose $\mu_I = 0$ since we are not interested to the pion condensation in what follows.

This paper is structured as follows. Section II deals with the equations of state and the expression for nuclear symmetry energy. In Section III are presented the results of numerical computation for nuclear symmetry energy and isospin-dependent EOS. The conclusion and discussion are given in Section IV.

II. EQUATIONS OF STATE

Assume the sigma, pion, omega and rho fields develop the ground state expectation values

$$\langle\sigma\rangle = u, \quad \langle\pi_i\rangle = 0, \quad \langle\omega_\mu\rangle = \rho_B\delta_{0\mu}, \quad \langle\varrho_{\mu i}\rangle = \rho_I\delta_{\mu 0}\delta_{i3},\quad (4)$$

in cold nuclear matter. Inserting (4) into (3) leads to

$$\mathcal{L}_{MFT} = \bar{\psi}(i\hat{\partial} - M^* + \gamma_0\mu^*)\psi - U(\rho_B, u), \quad (5)$$

where

$$M^* = m_0 - \tilde{G}_s u, \quad \tilde{G}_s = G_s + G_{sv}\rho_B^2 = G_s[1 + \xi(\rho_B/\rho_0)^2], \quad \xi = \rho_0^2 G_{sv}/G_s, \quad (6)$$

$$\mu_{p,n}^* = \mu_B^* \pm \mu_I^*/2, \quad \mu_B^* = \mu_B - \Sigma_v = \mu_B - [G_v - G_{sv}u^2]\rho_B, \quad (7)$$

$$\mu_I^* = -G_r\rho_I, \quad (8)$$

$$U(\rho_B, u) = \frac{1}{2}[\tilde{G}_s u^2 - 2\Sigma_v\rho_B + G_v\rho_B^2 - G_r\rho_I^2]. \quad (9)$$

The solution M^* of Eq. (7) is the effective mass of nucleon, which reduces to the nucleon mass in vacuum.

Based on (5) the grand partition function Z is established

$$Z = \int D\bar{\psi}D\psi D\sigma D\vec{\pi}D\omega_\mu \exp \int_0^\beta d\tau \int_V d^3x i\mathcal{L}_{MFT}.$$

Integrating out the nucleon degrees of freedom we arrive at

$$Z = \exp\left(-i\frac{VU}{T}\right) \det S^{-1},$$

in which V is the volume of system and

$$\det S^{-1}(k; \rho_B, u) = (k_0 - E_-^-)(k_0 + E_+^-)(k_0 - E_-^+)(k_0 + E_+^+), \quad (10)$$

with

$$E_\mp^\pm = E_k^\pm \mp \mu_B^*, \quad E_k^\pm = E_k \pm \mu_I^*/2, \quad E_k = \sqrt{k^2 + M^{*2}}.$$

Then the effective potential Ω is derived

$$\begin{aligned} \Omega(\rho_B, u) &= i\frac{T}{V} \ln Z = U(\rho_B, u) + i\text{Tr} \ln S^{-1}(\rho_B, u) \\ &= U(\rho_B, u) + 2 \int \frac{d^3k}{(2\pi)^3} [2E_k + T \ln(n_-^- n_+^-) + T \ln(n_-^+ n_+^+)], \end{aligned} \quad (11)$$

with n_\mp^\pm being the Fermi distribution function, $n_\mp^\pm = [e^{E_\mp^\pm/T} + 1]^{-1}$.

The pressure P is defined by means of (11)

$$P = -\Omega_{\text{taken at minimum}}.$$

The energy density is obtained by the Legendre transform of P ,

$$\begin{aligned} \mathcal{E}(\rho_B, u) &= \Omega(\rho_B, u) + T\zeta + \mu_B\rho_B + \mu_I\rho_I \\ &= \frac{1}{2}[\tilde{G}_s u^2 + G_v\rho_B^2 + G_r\rho_I^2] + 2 \int \frac{d^3k}{(2\pi)^3} E_k \left[(n_-^- + n_+^- - 1) + (n_-^+ + n_+^+ - 1) \right], \end{aligned} \quad (12)$$

with the entropy density defined by

$$\begin{aligned} \varsigma = & -2 \int \frac{d^3k}{(2\pi)^3} \left[n_-^- \ln n_-^- + (1 - n_-^-) \ln(1 - n_-^-) + n_+^- \ln n_+^- + (1 - n_+^-) \ln(1 - n_+^-) \right. \\ & \left. + n_-^+ \ln n_-^+ + (1 - n_-^+) \ln(1 - n_-^+) + n_+^+ \ln n_+^+ + (1 - n_+^+) \ln(1 - n_+^+) \right]. \end{aligned}$$

The ground state of nuclear matter is determined by the minimum conditions

$$\frac{\partial \Omega}{\partial \sigma} = 0, \quad \frac{\partial \Omega}{\partial \omega_0} = 0, \quad \frac{\partial \Omega}{\partial \rho_{03}} = 0,$$

or

$$u = 2 \int \frac{d^3k}{(2\pi)^3} \frac{M^*}{E_k} \left[(n_-^- + n_+^- - 1) + (n_-^+ + n_+^+ - 1) \right], \quad (13)$$

$$\rho_B = 2 \int \frac{d^3k}{(2\pi)^3} \left[(n_-^- - n_+^-) + (n_-^+ - n_+^+) \right], \quad (14)$$

$$\rho_I = \int \frac{d^3k}{(2\pi)^3} \left[(n_-^- + n_+^- - 1) - (n_-^+ + n_+^+ - 1) \right], \quad (15)$$

which are usually called the gap equations.

In terms of the baryon density (14) the expression for P is rewritten as

$$P = -\frac{(m_0 - M^*)^2}{2\tilde{G}_s} - \frac{G_v}{2} \rho_B^2 + \frac{G_r}{2} \rho_I^2 + (\mu_B - \mu_B^*) \rho_B - 2 \int \frac{d^3k}{(2\pi)^3} [2E_k + T \ln(n_-^- n_+^-) + T \ln(n_-^+ n_+^+)], \quad (16)$$

and the energy density takes the form

$$\mathcal{E} = \frac{(m_0 - M^*)^2}{2\tilde{G}_s} + \frac{G_v}{2} \rho_B^2 + \frac{G_r}{2} \rho_I^2 + 2 \int \frac{d^3k}{(2\pi)^3} E_k \left[(n_-^- + n_+^- - 1) + (n_-^+ + n_+^+ - 1) \right], \quad (17)$$

Starting from (16) and (17) we have that

- (1) The binding energy per nucleon

$$\mathcal{E}_{\text{bin}} = -M_N + \mathcal{E}/\rho_B \quad (18)$$

- (2) The nuclear symmetry energy

$$E_{\text{sym}} = \left. \frac{\rho_B}{8} \frac{\partial^2 \mathcal{E}}{\partial \rho_I^2} \right|_{\rho_I=0}. \quad (19)$$

- (3) The compressibility

$$K(\rho_B, \alpha) = 9 \frac{\partial P}{\partial \rho_B}, \quad (20)$$

where α is isospin asymmetry, $\alpha = (\rho_n - \rho_p)/\rho_B$.

It obvious that all the thermodynamic properties of our system are governed by the equations of state (16) and (17). However, in this article we restrict ourselves to consider only cold nuclear matter, $T = 0$, in which case Eqs. (13) - (20) are simplified to

$$u = -\frac{1}{\pi^2} \left[\int_{k_{F_p}}^{\Lambda} k^2 dk \frac{M^*}{E_k} + \int_{k_{F_n}}^{\Lambda} k^2 dk \frac{M^*}{E_k} \right], \quad (21)$$

$$\rho_B = \frac{1}{3\pi^2} [k_{F_p}^3 + k_{F_n}^3], \quad (22)$$

$$\rho_I = \frac{1}{6\pi^2} [k_{F_p}^3 - k_{F_n}^3], \quad (23)$$

and

$$\mathcal{E} = \frac{(m_0 - M^*)^2}{2\tilde{G}_s} + \frac{G_v}{2}\rho_B^2 + \frac{G_r}{2}\rho_I^2 - \frac{1}{\pi^2} \left[\int_{k_{F_p}}^{\Lambda} k^2 dk E_k + \int_{k_{F_n}}^{\Lambda} k^2 dk E_k \right]. \quad (24)$$

III. NUMERICAL COMPUTATIONS AND RESULTS

In order to proceed to the numerical computation the in-vacuum mass of nucleon is chosen to be $M_N = 939$ MeV. Five parameters m_0 , G_s , G_v , ξ , and Λ were determined together with the in-medium mass of nucleon M^* and the incompressibility K_0 in Ref. [18] for symmetric nuclear matter ($G_r = 0$). Their values are listed in Table 1.

Table 1. Values of parameters and physical quantities.

Λ (MeV)	G_s (fm ²)	G_v/G_s	m_0 (MeV)	ξ	M^*/M_N	K_0 (MeV)
400	8.507	0.933	41.264	0.032	0.684	285.91

As to fixing G_r let us employ the expansion of nuclear symmetry energy around ρ_0

$$E_{\text{sym}} = a_4 + \frac{L}{3} \left(\frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left(\frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \dots$$

with a_4 being the bulk symmetry parameter of the Weizsaecker mass formula. Experimental data provide $a_4 = 30 - 35$ MeV; L and K_{sym} are related to slope and curvature of NSE at saturation density ρ_0

$$L = 3\rho_0 \left(\frac{\partial E_{\text{sym}}}{\partial \rho_B} \right)_{\rho_B = \rho_0}, \quad (25a)$$

$$K_{\text{sym}} = 9\rho_0^2 \left(\frac{\partial^2 E_{\text{sym}}}{\partial \rho_B^2} \right)_{\rho_B = \rho_0} \quad (25b)$$

Then G_r is fitted to give $a_4 = 32$ MeV, its value is $G_r = 0.417G_s$. Thus, all of the model parameters are fixed, they allow us to calculate the isospin-dependent EOSs of asymmetric nuclear matter.

In Fig. 1 we show the dependence of binding energy $\mathcal{E}_{\text{bin}}(\rho_B, \alpha)$ on density ρ_B/ρ_0 and α . For comparison with the results of the relativistic Brueckner-Hartree-Fock approach with and without the momentum-dependent self-energies [11] given in Fig. 2 we plot in Fig. 3 the density dependence of $\mathcal{E}_{\text{bin}}(\rho_B, \alpha)$ at several values of α . From these figures we deduce that in our model the asymmetric matter is less stiff and the isospin dependence of saturation density is strong enough.

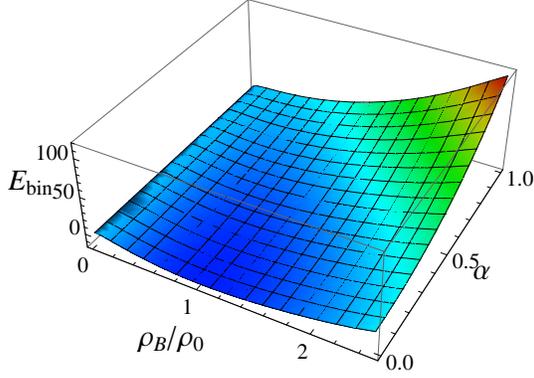


Fig. 1. The dependence of binding energy on nuclear density and isospin asymmetry α .

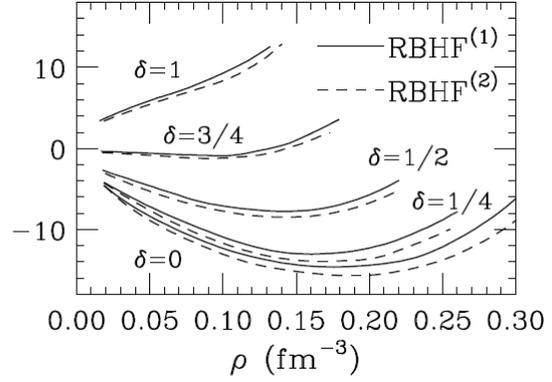


Fig. 2. The equation of state of asymmetric nuclear matter from the relativistic Brueckner-Hartree-Fock calculations. Taken from Ref. [11].

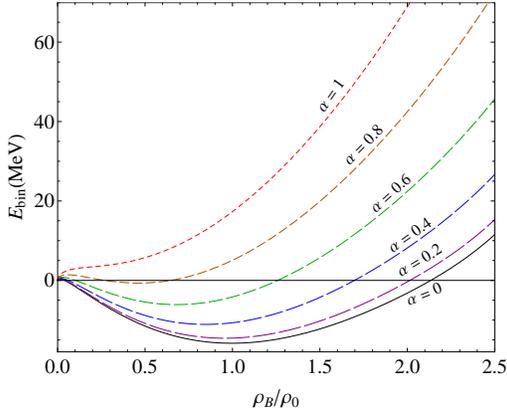


Fig. 3. The equation of state of asymmetric nuclear matter derived from our chiral model.

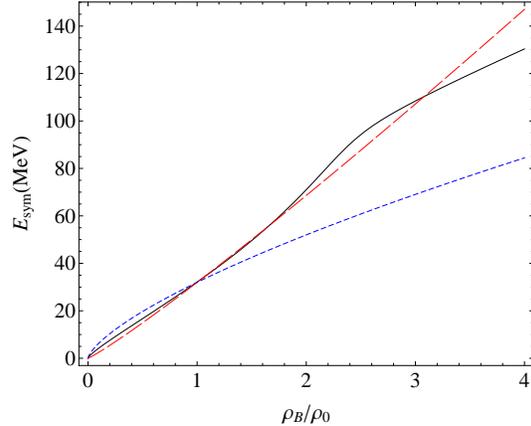


Fig. 4. The ρ_B/ρ_0 dependence of E_{sym} (solid line), $E_1 = 32(\rho_B/\rho_0)^{0.7}$ (dotted line), and $E_2 = 32(\rho_B/\rho_0)^{1.1}$ (dashed line).

Now the density dependence of nuclear symmetry energy is concerned. It is known that besides the constraints extracted from astrophysical observation [19] we get significant progress in constraining the behavior of NSE around and below the nuclear saturation

density from various experiments [20]. However, the predictions for behavior of NSE at supra-saturation densities are quite divergent. In this regard, the prediction made by our model could provide some insight into this issue. Taking into account Eqs. (13), (14), (15) and (19) altogether and the implementing the numerical computation with the aid of MATHEMATICA [21] we arrive at Fig. 4 describing the density dependence of nuclear symmetry energy. Here, we also plot the graphs of the functions $E_1 = 32(\rho_B/\rho_0)^{0.7}$ and $E_2 = 32(\rho_B/\rho_0)^{1.1}$. It is evident that

- Around and below the nuclear saturation densities our predicted graph is close to the Brueckner-Hartree-Fock prediction [22].
- $E_1 < E_{\text{sym}} < E_2$ for $\rho_B < 2\rho_0$ and $\rho_B > 3\rho_0$,
- $E_{\text{sym}} > E_2 > E_1$ for $2\rho_0 < \rho_B < 3\rho_0$.

This behavior of E_{sym} at supra-saturation densities is basically in agreement with the analysis of Refs. [23-25].

Next the theory is highlighted by studying the behaviors of the pressure P and effective nucleon mass M^* at high density. In Fig. 5 we show the ρ_B/ρ_0 dependence of pressure P at different isospin asymmetry and in Fig. 6 is shown the high-density behavior of the pressure for neutron matter. The shaded region in Fig. 5 (Fig. 6) denotes the constraint on high-density behavior of the pressure corresponding to symmetric nuclear matter derived from the simulations of flow data in heavy-ion collision experiments [2]. The density dependence of M^* is represented in Fig. 7 which manifests the chiral restoration at high density.

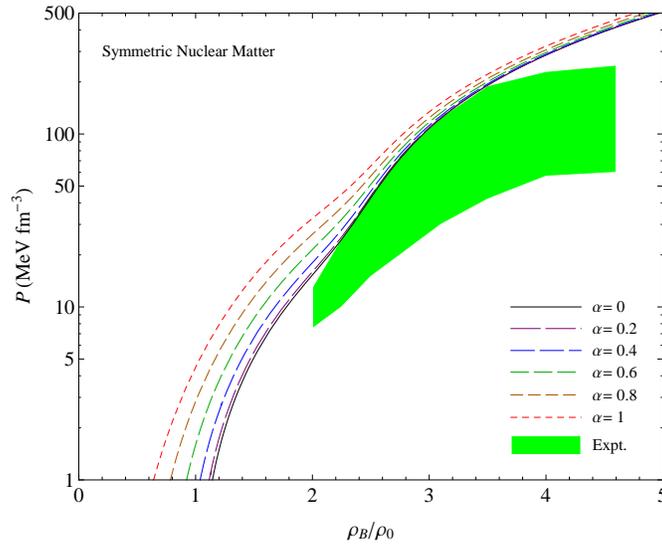


Fig. 5. The EoS of cold asymmetric nuclear matter at high baryon density at several isospin asymmetry α . The shaded area means a constraint on the behavior of the pressure of symmetric nuclear matter consistent with the experimental flow data [2].

Finally let us turn to the determination of several interesting quantities.

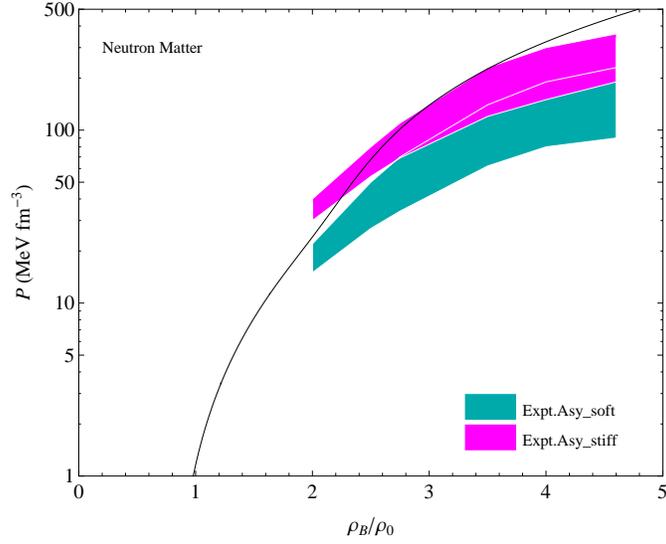


Fig. 6. The EoS and constraint (shaded region) on the high-density behavior of the EoS for neutron matter [2].

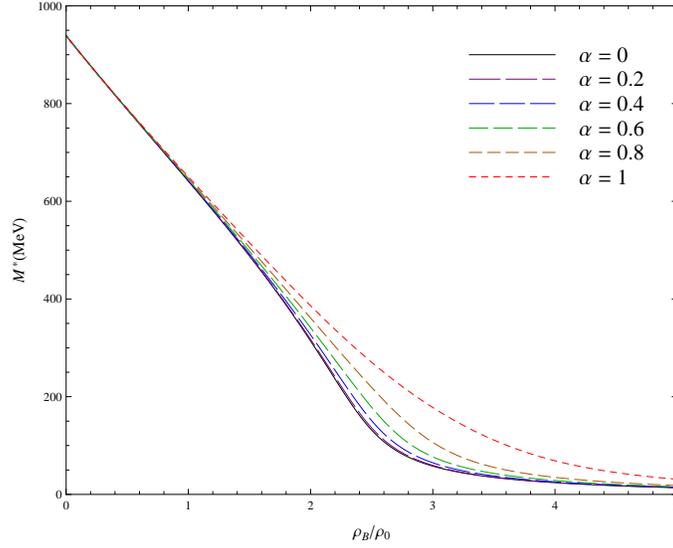


Fig. 7. The density dependence of effective nucleon mass M^* .

- (1) Eqs. (25) provide immediately the values of slope and curvature of NSE: $L = 96.732$ MeV and $K_{\text{sym}} = -347.786$ MeV. It is clear that L is consistent with the constraint $46 \text{ MeV} \leq L \leq 111 \text{ MeV}$ obtained from heavy-ion data and the analyses of other models [24, 26] and K_{sym} is also within the analyses based on the in-medium NN cross section in the IBUU04 model [27]: $K_{\text{sym}} = -500 \pm 50$ MeV.

- (2) At the nuclear saturation density ρ_0 and around $\alpha = 0$, the isobaric incompressibility of asymmetric nuclear matter can be expressed to the second-order of alpha as

$$K(\alpha) = K_0 + K_{\text{asy}}\alpha^2,$$

here, K_{asy} is the isospin-dependent part

$$K_{\text{asy}} = K_{\text{sym}} - 6L$$

characterizing the density dependence of NSE. The information on this quantity can be extracted experimentally by measuring the giant monopole resonance of neutron-rich nuclei. Our calculation yields $K_{\text{asy}} = -928.18$ MeV. This value is different from the recently experimental measurements [28] that give $K_{\text{asy}} = -550 \pm 100$ MeV.

- (3) The symmetry pressure and the shift of nuclear saturation density with asymmetry at lowest order in alpha are obtained, respectively

$$P_{\text{sym}}(\rho_0) = \rho_0 L / 3 = 5.482 \text{ MeV}/\text{fm}^3,$$

$$\Delta\rho_0 = -\frac{3\rho_0 L}{K_0} = -0.173 \text{ fm}^{-3}.$$

The calculated values of the model parameters and physical quantities are listed in Table 2 and 3

Table 2. Values of the model parameters.

$\Lambda(\text{MeV})$	$G_s(\text{fm}^2)$	G_v/G_s	G_r/G_s	$m_0(\text{MeV})$	ξ
400	8.507	0.933	0.417	41.264	0.032

Table 3. Values of the model physical quantities.

M^*/M_N	$K_0(\text{MeV})$	$a_4(\text{MeV})$	$L(\text{MeV})$	$K_{\text{sym}}(\text{MeV})$	$K_{\text{asy}}(\text{MeV})$	$P_{\text{sym}}(\text{MeV}/\text{fm}^3)$	$\Delta\rho_0(\text{fm}^{-3})$
0.684	285.91	32	96.732	-347.786	-928.181	5.482	-0.173

IV. CONCLUSION AND DISCUSSION

Developing the previous work [18] we have carried out in this article a more realistic study of nuclear asymmetric matter, where it is found that

- Most calculated physical quantities listed in Table 2 are in agreement with the recent analyses derived from various models as well as experimental constraints.

- Most calculated physical quantities, in particular, the quantities K_0 , K_{sym} , and L , listed in the Table 3 are in agreement with experimental constraints as well as recent analyses of various models except for K_{asy} , where the theoretical value much differs from the experimental one. The formulae $K_{\text{asy}} = K_{\text{sym}} - 6L$ indicates that the experimental measurement for K_{asy} is probably not in accordance with those obtained for K_{sym} and L .

Experimental realizations at high energies in radioactive beam facilities are expected to create a good chance to explore the high density-behavior of NSE and other EoSs.

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