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# Thermopower of a Luttinger-liquid-based two-channel charge Kondo circuit: nonperturbative solution

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**Abstract.** *Recently, the influence of electron-electron interactions on the thermoelectric transport in a two-channel charge Kondo circuit has been studied in [Phys. Rev. B* **105** *(2022) L121405]. In this paper, we revisit the Luttinger-liquid-based model and discuss in details the limit when the spin mode is noninteracting ($g_\sigma = 1$) while the interaction in the charge sector is repulsive ($g_\rho \leq 1$). The thermoelectric transport coefficients are computed nonperturbatively with respect to the reflection amplitude at the quantum point contact. At low temperatures the thermopower shows the non-Fermi liquid behavior in the vicinity of the Coulomb peaks. We also demonstrate that repulsive interaction results in the enhancement of the thermoelectric power.*

Keywords: thermoelectric transport; thermopower; two-channel charge Kondo effect.

Classification numbers: 73.23.Hk; 73.50.Lw; 72.15.Qm; 73.21.La.

## 1. Introduction

The study of quantum thermoelectric transport in nano-structured devices at low temperature has been an important and rapidly developing topic since the 1990s. Low-dimensional materials have been recognised to significantly enhance thermoelectric efficiency much better than bulk materials [1–5]. Thanks to the development of nanotechnology a large variety of quantum nanodevices has been created. Quantum dot (QD) is one of the most simple and significant nanostructures which exhibits most of the fundamental quantum properties [6]. Namely, a QD device can be controlled and fine-tuned by external electric and magnetic fields both in and out of equilibrium. Study of the thermoelectric transport through the QD devices provides important knowledge about the influence of strong electron-electron interactions, interference effects and resonance scattering on the quantum transport [7, 8]. Since the properties of the QD are fully determined by the Coulomb Blockade (CB) phenomenon [9–11] and the charge states are well quantized, a QD-based setup provides a perfect playground for studying the quantum impurity models. At sufficiently low temperature, a QD with an odd number of electrons behaves as a spin-$1/2$ impurity. Many-particle spin-flip scattering processes result in the effective 'impurity screening' at zero temperature. As a consequence, the transport in QD systems is featured by the strong correlations, and the Kondo effect [12] is observed [13].

Interestingly, phenomena similar to the conventional Kondo effect [14–18] may be observed in a variety of systems without magnetic degree of freedom [19–21]. For instance, the unconventional charge Kondo phenomenon concerns the degeneracy of the charge states. One of the most prominent charge Kondo setup is implemented by a single-electron transistor in which a large metallic QD is strongly coupled to one (or several) lead(s) through an (or several) almost transparent single-mode quantum point contact(s) [QPC(s)] [22–24]. This setup is first proposed in the pioneering theories of Flensberg, Matveev, and Furusaki (FMF). The recently raised interest in the FMF model is due to the understanding that it can be adapted to study the multi-channel Kondo (MCK ) effect. Indeed, the logic behind the mapping of FMF setup to a MCK problem is that the two degenerate charge states of the QD are considered as "a quantum impurity", while the electron location (namely, in or out of QD) is treated as an iso-spin variable. The additional internal degrees of freedom, such as spin projection quantum number of electrons or number of single-mode QPCs, determine the number of different channels in the Kondo problem [23–25]. The charge Kondo setup has been recently implemented in a breakthrough series of experiments [26, 27], which open an access to investigate MCK problem experimentally. The dominant characteristic of the MCK setups is known as non-Fermi liquid (NFL) behavior [28, 29].

The FMF model is especially interesting since electrons are transmitted between the leads and the QD through QPCs. The electrons at low energies are effectively described by a one-dimensional (1D) Hamiltonian. Although the FMF model has been studied for several years [22–25, 30], the effects of electron-electron interaction in the vicinity of QPC(s) have been studied only very recently by applying the Luttinger liquid (LL) model [31, 32]. Unlike the Fermi liquid (FL) description, where the charged excitations are represented by quasiparticles (electrons and holes), electrons do not propagate in the LL. According to the Tomonaga-Luttinger theory at low energies, the spectrum of the LL consists of gapless bosonic excitations (charge and spin density waves) [33, 34]. The advantage of the LL model is that all interaction processes (namely, the forward and the backward scatterings) [37] can be described by applying the Abelian bosonization technique [35–37]. The LL Hamiltonian in the bosonic representation is thus modified from the Hamiltonian of the FL through additional dimensionless interaction parameters $g_\rho$, $g_\sigma$ (so-called Luttinger parameters) and the renormalization of the Fermi velocity $v_\rho$, $v_\sigma$ (the indices $\rho, \sigma$ stand for the charge and spin modes, respectively) [37].

In this work, we reconsider the effects of the electron-electron interactions on the thermoelectric properties in the FMF setup, which is mapped onto the two-channel charge Kondo (2CK) problem, as discussed in Ref. [32]. The perturbative results obtained in [32] are applicable in the finite temperature interval $|r^*|^2 T_K^* \ll T \ll T_K^*$, where $|r^*|$ is the parameter of the perturbative expansion and $T_K^*$ is the effective Kondo temperature. In the current manuscript, we investigate a special limit where the spin modes are noninteracting ($g_\sigma = 1$). It allows us to go beyond the perturbation theory. The thermoelectric coefficients, namely, electric conductance $G$, thermoelectric coefficient $G_T$, and thermopower (TP) $S$ are computed nonperturbatively with respect to the reflection amplitude of the QPC without the condition that this reflection amplitude is small. The behavior of the TP is investigated below the temperature $|r^*|^2 T_K^*$. The logarithmic temperature scaling of the TP around the Coulomb peaks demonstrates the NFL picture of the 2CK, while electron-electron interactions in the LL results in enhancement of the TP.

The paper is organized as follows. We describe the theoretical model in Sec. II. Equations for the thermoelectric coefficients are presented and discussed in Sec. III. We conclude our work in Sec. IV.

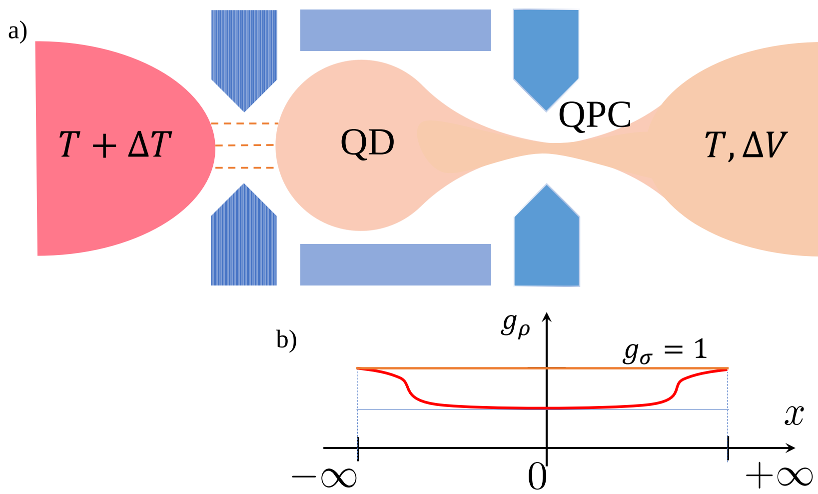


**Fig. 1.** a) Schematic of a single-electron transistor device in which a quantum dot (QD) is weakly coupled to the left electrode and strongly coupled to the right one through a quantum point contact (QPC) (see text for the details). The QD and the right electrode (orange color) are at the reference temperature $T$ while the left electrode (red color) is at higher temperature $T+\Delta T$. The effects of electron-electron interaction in the QD-QPC structure are discussed in this work. b) The charge Luttinger parameter $g_\rho$ is assumed to asymptotically equal to 1 at the position of the tunnel barrier ($x = -\infty$) and away from the QPC ($x = +\infty$) while the spin Luttinger parameter is assumed to satisfy $g_\sigma = 1$.

## 2. Model

We consider a single-electron transistor device as shown in Fig. 1. The central part is a large metallic QD in the weak CB regime. It is strongly coupled to the right lead through an almost transparent single-mode QPC. This QD-QPC structure is formed in a two-dimensional electron gas (2DEG). We assume that QD-QPC part is in thermal equilibrium at temperature $T$ (orange part (right electrode) in Fig. 1). The electron-electron interaction in the vicinity of the QPC are controlled by applying an external gate voltage [38–40]. The interacting electrons in quasi-one dimensional constriction (QPC) are thus described by the LL model [37]. We assume that the interaction disappears in the vicinity of the tunnel junction at ($x = -\infty$) and far away from the QPC, in the right 2DEG electrode ($x = \infty$). The left lead, also formed in 2DEG, is coupled to the QD through a tunnel barrier. It is considered at higher temperature $T + \Delta T$ (red part in Fig. 1) in order to investigate the Seebeck effect at this barrier. Without any loss of generality, the left lead is described by conventional FL theory. In the linear response theory the TP is determined through the ratio of the thermoelectric coefficient $G_T = I_e/\Delta T$ and electric conductance $G = I_e/\Delta V$ as $S = G_T/G|_{I_e=0} = -\Delta V/\Delta T$ where a thermovoltage $\Delta V$ is applied between the left lead and the QD-QPC structure to implement a zero-current condition for the electric current. The temperature difference across the tunnel barrier $\Delta T$ is controlled by using a current heating technique [13]. It is assumed to be small in comparison with the reference temperature $T$ to guarantee the linear response regime.

Hamiltonian describing the QD coupled weakly to the left lead and strongly to the right one, has form

$$H = H_0 + H_C + H_L + H_R, \tag{1}$$

where $H_0$ characterizes electrons in QD and in two ($L/R$) leads,

$$H_0 = \sum_{k,\lambda} \varepsilon_k a^\dagger_{k,\lambda} a_{k,\lambda} + \sum_{p,\lambda} \varepsilon_p d^\dagger_{p,\lambda} d_{p,\lambda} + H_\rho + H_\sigma. \tag{2}$$

Here, $a_{k,\lambda}$ and $d_{p,\lambda}$ denote the electrons in the left lead and in the QD at the left tunnel barrier, correspondingly. $\lambda =\uparrow,\downarrow$ stands for the spin projection quantum number. The third and fourth terms in Eq.(2) describe the charge and spin modes of the 1D interacting electrons (Luttinger liquid) in the right contact (QPC) (in $\hbar = k_B = 1$ units):

$$H_\rho = \frac{v_\rho}{2\pi} \int_{-\infty}^{+\infty} \{ g_\rho \pi^2 \Pi_\rho^2(x) + g_\rho^{-1} [\partial_x \phi_\rho(x)]^2 \} dx, \tag{3}$$

$$H_\sigma = \int_{-\infty}^{+\infty} \frac{v_\sigma}{2\pi} \left[ g_\sigma \pi^2 \Pi_\sigma^2(x) + g_\sigma^{-1} [\partial_x \phi_\sigma(x)]^2 \right], \tag{4}$$

with the charge $\phi_\rho = (\phi_\uparrow + \phi_\downarrow)/\sqrt{2}$ and spin $\phi_\sigma = (\phi_\uparrow - \phi_\downarrow)/\sqrt{2}$ degrees of freedom are separated. Their conjugate momentums $\Pi_\rho$ and $\Pi_\sigma$ satisfy the canonical commutation relation $[\phi_\alpha(x), \Pi_\alpha(x')] = i\delta(x - x')$, with $\alpha = \rho, \sigma$. The interactions in the charge and spin sectors are characterized by the dimensionless Luttinger parameters, $g_\rho$ and $g_\sigma$ [35–37]. In this work, we assume that the interaction in the spin modes is absent $g_\sigma = 1$ (it corresponds to a system with spin rotational symmetry [37]), while the interaction in the charge modes is characterized by the parameter $g_\rho = 1/\sqrt{1 + U/\pi v_F}$ with $U$ being a strength of electron-electron interaction in the vicinity of QPC. Thus, $g_\rho > 1$ corresponds to the case of attractive interaction, and $0 < g_\rho < 1$ corresponds to the repulsive interaction. $v_\rho = v_F/g_\rho$ is the interaction renormalized Fermi velocity, while $v_\sigma = v_F$ at $g_\sigma = 1$.

Second term in the Eq.(1) describes CB phenomenon in the QD,

$$H_C = E_C \left[ \hat{n} + \frac{\sqrt{2}}{\pi} \phi_\rho(0) - N \right]^2, \tag{5}$$

where $E_C = e^2/2C$ is the charging energy ($C$ is the QD capacitance), $\hat{n}$ is the electron operator of electrons entered through the left (tunnel) contact, while $\sqrt{2}\phi_\rho(0)/\pi$ is number of electrons entered through the QPC (from the right lead) [41], $N$ is dimensionless parameter controlled by the gate voltage $V_g$ as $N = CV_g/e$.

The tunnel Hamiltonian, which describes the weak coupling between the left lead and the QD, reads

$$H_L = \sum_{k,p,\lambda} (t a^\dagger_{k,\lambda} d_{p,\lambda} \hat{F} + h.c.), \tag{6}$$

where $t \ll 1$ is hopping amplitude and $\hat{F}$ is the charge-lowering operator introduced in explicit form, which obeys the commutation relation $\left[\hat{F}, \hat{n}\right] = \hat{F}$. We notice that the operator $d_{p,\lambda}$ can be expressed through the fermionic operator in the 1D system as $d_{p,\lambda} \to \psi_\lambda(-\infty)$ with $\psi_\lambda(x) \sim e^{i\phi_\lambda(x)}$.

The Hamiltonian $H_R$ demonstrating the backward scattering in the QPC with the small reflection amplitudes $|r_\uparrow| = |r_\downarrow| = |r| \ll 1$ is written as

$$H_R = -\frac{2D}{\pi}|r| \cos\left[\sqrt{2}\phi_\rho(0)\right] \cos\left[\sqrt{2}\phi_\sigma(0)\right], \tag{7}$$

where $D$ is the bandwidth.

## 3. Thermoelectric coefficients

In this Section, the electric conductance $G$ and thermoelectric coefficient $G_T$ are computed nonperturbatively with respect to the reflection amplitude and at low energy. The nonperturbative treatment goes beyond the condition that $|r| \ll 1$. Moreover, this enables us to obtain a nonperturbative expression for the TP at arbitrarily low temperatures.

### *3.1. General formulas*

The electric current through the tunnel barrier reads as

$$I_e = -2\pi e |t|^2 \int_{-\infty}^{\infty} d\epsilon \nu_L(\epsilon)\nu_D(\epsilon)\left[f_L(\epsilon) - f_D(\epsilon)\right], \tag{8}$$

where the density of states (DoS) of the dot at the weak barrier is given by equation:

$$\nu_D(\epsilon) = -\frac{1}{\pi}\cosh\left(\frac{\epsilon}{2T}\right)\int_{-\infty}^{\infty} \mathscr{G}_D\left(\frac{1}{2T} + it\right) e^{i\epsilon t} dt, \tag{9}$$

and $\mathscr{G}_D(1/2T + it)$ is the Green's Function (GF) of electrons in the dot at the tunnel barrier. DoS of electrons in the left electrode $\nu_L(\epsilon) = \nu_L$ is assumed to be energy independent, $f_L(\epsilon) = f(\epsilon, T + \Delta T)$, $f_D(\epsilon) = f(\epsilon + e\Delta V, T)$ are Fermi distribution functions of the left lead and the dot, respectively. The current in linear response regime is given by

$$I_e = 2\pi e^2 \nu_L |t|^2 \frac{\Delta V}{4T}\int_{-\infty}^{\infty} d\epsilon \frac{\nu_D(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)} - 2\pi e \nu_L |t|^2 \frac{\Delta T}{4T^2}\int_{-\infty}^{\infty} d\epsilon \frac{\epsilon \nu_D(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)}. \tag{10}$$

We calculate the thermoelectric coefficients as follows:

$$G = \left.\frac{\partial I_e}{\partial \Delta V}\right|_{\Delta T = 0} = \frac{\pi e^2 \nu_L |t|^2}{2T}\int_{-\infty}^{\infty} d\epsilon \frac{\nu_D(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)}, \tag{11}$$

$$G_T = \frac{\partial I_e}{\partial \Delta T}\bigg|_{\Delta V=0} = -\frac{\pi e \nu_L |t|^2}{2T^2}\int_{-\infty}^{\infty} d\epsilon \frac{\epsilon \nu_D(\epsilon)}{\cosh^2\left(\frac{\epsilon}{2T}\right)}. \tag{12}$$

The GF's in imaginary time representation is parametrized as $\mathscr{G}_D(\tau) = -\nu_D \pi T [\sin(\pi T \tau)]^{-1} \times K(\tau)$, where $\nu_D$ is the DoS in the dot. We assume that all effects of CB in the QD, electron-electron interactions in the vicinity of the QPC and scattering at the QPC are accounted by the correlator $K(\tau) = \langle T_\tau \hat{F}(\tau)\hat{F}^\dagger(0)\rangle$ ($T_\tau$ is the time-ordering operator, the imaginary time $\tau$ runs from 0 to $\beta = 1/T$) [24,25]. It is convenient to introduce a notation $G_L = 2\pi e^2 \nu_L \nu_D |t|^2$ for a tunnel conductance. Performing integration over the energy in Eqs. (11) and (12) with the notice of Eq. (9), we obtain the general formulas for the electric conductance and thermoelectric coefficient as:

$$G = G_L \frac{\pi T}{2}\int \frac{1}{\cosh^2(\pi T t)} K\left(\frac{1}{2T} + it\right) dt, \tag{13}$$

$$G_T = -\frac{i\pi^2}{2}\frac{G_L T}{e}\int \frac{\sinh(\pi T t)}{\cosh^3(\pi T t)} K\left(\frac{1}{2T} + it\right) dt. \tag{14}$$

In order to compute the thermoelectric coefficients in Eqs.(13)-(14) one essentially needs to calculate the electron correlator $K(1/2T + it)$ explicitly.

### *3.2. Correlation function $K(\tau)$*

We compute the time-ordered correlation function $K(\tau)$ through the functional integration over the bosonic fields $\phi_{\rho(\sigma)}(x,t)$ similarly to the method of Matveev-Andreev theory [25]:

$$K(\tau) = Z(\tau)/Z(0), \tag{15}$$

$$Z(\tau) = \prod_{\alpha=\rho,\sigma}\int \mathscr{D}\phi_\alpha \exp\left[-\mathscr{S}_0 - \mathscr{S}_C(\tau) - \mathscr{S}'\right], \tag{16}$$

where $\mathscr{S}_0$, $\mathscr{S}_C$, and $\mathscr{S}'$ are Euclidean actions describing the free Luttinger liquid, CB in the QD and the backscattering at the QPC, respectively. The action $\mathscr{S}_0$ includes two independent actions [35–37] $\mathscr{S}_0 = \mathscr{S}_0^{(\rho)} + \mathscr{S}_0^{(\sigma)}$, where

$$\mathscr{S}_0^{(\rho)} = \frac{v_\rho}{2\pi g_\rho}\int dx \int_0^\beta dt \left[\frac{(\partial_t \phi_\rho)^2}{v_\rho^2} + (\partial_x \phi_\rho)^2\right], \tag{17}$$

$$\mathscr{S}_0^{(\sigma)} = \frac{v_F}{2\pi}\int dx \int_0^\beta dt \left[\frac{(\partial_t \phi_\sigma)^2}{v_F^2} + (\partial_x \phi_\sigma)^2\right]. \tag{18}$$

Since $F^\dagger(0)$ increases number of electrons from 0 to 1 at time $t = 0$, whereas $F(\tau)$ changes it back at $t = \tau$, one can replace $\hat{n}$ by $n_\tau(t) = \theta(t)\theta(\tau - t)$. Therefore, the CB action $\mathscr{S}_C$ in bosonic representation reads [22–25, 30, 41]

$$\mathscr{S}_C = E_C \int_0^\beta dt [n_\tau(t) + \frac{\sqrt{2}}{\pi}\phi_\rho(0,t) - N]^2. \tag{19}$$

Lastly, the weak backscattering at the QPC is described by the action contribution $\mathscr{S}'$ as

$$\mathscr{S}' = -\frac{2D}{\pi}|r|\int_0^\beta dt\cos[\sqrt{2}\phi_\rho(0,t)]\cos[\sqrt{2}\phi_\sigma(0,t)]. \tag{20}$$

In the limit $r=0$ (no backscattering at the QPC), the functional integral Eq. (16) is Gaussian. The correlator $K_0(\tau)\equiv K(\tau)|_{r=0}=K_\rho(\tau)$ is calculated at low temperature $T\ll E_C$ and at $\tau\gg E_C^{-1}$:

$$K_\rho(\tau)=\left(\frac{\pi^2 T}{2\gamma g_\rho E_C}\frac{1}{\sin(\pi T\tau)}\right)^{1/g_\rho}, \tag{21}$$

where $\gamma=e^C$, $C\approx 0.577$ is Euler's constant. Plugging Eq. (21) into Eq. (14), we find that the thermoelectric coefficient $G_T$ vanishes at $|r|=0$. It is explained as follows. The Hamiltonian (or action) of the system in the absence of backscattering possesses electron-hole symmetry. It is invariant when the sign of the gate voltage is changed: $N\to -N$. Both the current $I_e$ and bias voltage $\Delta V$ are changed to opposite but the temperature difference $\Delta T$ does not depend on the charge of the current carriers. Therefore, $G(N)=G(-N)$ and $G_T(N)=-G_T(-N)$. A finite contribution to $G_T$ has been obtained in the perturbation theory over the small reflection amplitude $r\ll 1$ in Ref. [32].

It was shown that the thermoelectric properties of the system (for instance, the logarithmic enhancement of TP, see Eq. (13) in Ref. [32]) are controlled by the charge and spin fluctuations at low temperatures $T<g_\rho E_C$. However, due to the CB in the QD, the effect of small but finite $|r|$ on the charge modes is negligible. It allows one to integrate out the charge degree of freedom. Meanwhile, a small backscattering $|r|$ at the QPC pins the fluctuations of the spin modes, thus changing their low frequency dynamics dramatically. Therefore, the correlation function can be split into charge and spin components as $K(\tau)=K_\rho(\tau)K_\sigma(\tau)$, where $K_\sigma(\tau)=Z_\sigma(\tau)/Z_\sigma(0)$ is the functional integral of the spin degrees of freedom averaged over the fast charge modes. In this paper we compute $K_\sigma(\tau)$ nonperturbatively, which allows us to go beyond the perturbation theory as represented in Ref. [32], and obtain the expression for the TP at arbitrarily low temperatures.

We simply replace the $\cos[\sqrt{2}\phi_\rho(0,t)]$ in action Eq. (20) by its value averaged over the charge field $\langle\cos[\sqrt{2}\phi_\rho(0,t)]\rangle_\tau=\left(\frac{2\gamma g_\rho E_C}{\pi D}\right)^{\frac{g_\rho}{2}}\cos[\pi N-\chi(t)]$, with $\chi(t)=\pi n_\tau(t)+\delta\chi(t)$, and obtain the effective action for the spin degrees of freedom in the form [25, 30]

$$\mathscr{S}_\tau = \int dx\int_0^\beta dt\frac{v_F}{2\pi}\left[\frac{(\partial_t\phi_\sigma)^2}{v_F^2}+(\partial_x\phi_\sigma)^2\right]-\frac{2D}{\pi}\int_0^\beta dt\tilde{\lambda}(t)\cos[\sqrt{2}\phi_\sigma(0,t)], \tag{22}$$

where

$$\tilde{\lambda}(t)=|r|\left(\frac{2\gamma g_\rho E_C}{\pi D}\right)^{\frac{g_\rho}{2}}(-1)^{n_\tau(t)}\cos[\pi N-\delta\chi_\tau(t)], \tag{23}$$

$$\delta\chi_\tau(t)\approx\frac{\pi^2 T}{2g_\rho E_C}\left[\cot(\pi T(t-\tau)-\cot(\pi Tt)]\right). \tag{24}$$

One should notice that the model described by the action Eq.(22) is a boundary sine-Gordon model, similar to an impurity in a 1D electron liquid [42] with the Luttinger parameter $1/2$. Although the coefficient in front of cosine term in Eq.(22) is time-dependent, the model is expected to be solved nonperturbatively.

After performing a simple transformation $\sqrt{2}\phi_\sigma \to \phi$ and refermionization procedure [42], our model [as shown in Eq. (22)] is mapped onto resonant scattering model, which is described by Hamiltonian

$$H_\tau(t) \quad = \quad \int \left[ v_F k c_k^\dagger c_k - \sqrt{\frac{2v_F}{\pi D}} \tilde{\lambda}(t)(c+c^\dagger)\left(c_k - c_k^\dagger\right)\right] dk, \tag{25}$$

in which the operators $c_k^\dagger$ and $c_k$ satisfying the anticommutation relations $\left\{c_k, c_{k'}^\dagger\right\} = \delta\left(k-k'\right)$ create and destroy chiral fermions; $c$ is a fermion annihilation operator anticommuting with $c_k^\dagger$ and $c_k$. Our solution, being nonperturbative in $|r|$ accounts for low energy dynamics of the spin modes.

We calculate the correlation function $K_\sigma(\tau)$ performing the perturbation theory over the $\delta\chi_\tau$. Spin contribution to the correlation function in $0^{\text{th}}$ order over $\delta\chi_\tau$ may be straightforwardly obtained from Eq. (25) [25]

$$K_\sigma^{(0)}(\tau) = \frac{2\Gamma_R}{\pi}\int_{-\infty}^{\infty} \frac{e^{\epsilon\tau} d\epsilon}{(\epsilon^2+\Gamma_R^2)(e^{\beta\epsilon}+1)}, \tag{26}$$

where

$$\Gamma_R(N) = \frac{4g_\rho E_C}{\pi}\left(\frac{2\gamma}{\pi}\right)^{g_\rho} |r^*|^2 \cos^2(\pi N) \tag{27}$$

is a new energy scale originated in the problem. Here, $|r^*| = |r|(g_\rho E_C/D)^{(g_\rho-1)/2}$ is an renormalized reflection amplitude taking into account the electron-electron interactions [42]. It is easy to check that in the limit $\Gamma_R(N) \to 0$, we reproduce perturbative result $K_\sigma(\tau) = 1$ from [32]. In the opposite limit $T \ll \Gamma_R$, the spin contribution to the correlation function $K(\tau)$ reads $K_\sigma(\tau) = (2T/\Gamma_R)\sin^{-1}(\pi T\tau)$. However, the result is an even function of the gate voltage $N$, and therefore $G_T = 0$.

The leading contribution to the thermoelectric coefficient can be obtained in the first order of perturbation theory over $\delta\chi_\tau$. The correction to $K_\sigma^{(0)}$ with logarithmic accuracy reads

$$K_\sigma^{(1)}(\tau) = \frac{4\sin(2\pi N)}{\pi}|r^*|^2\left(\frac{2\gamma}{\pi}\right)^{g_\rho}\log\left(\frac{g_\rho E_C}{T+\Gamma_R}\right)\int_{-\infty}^{\infty}\frac{\epsilon}{\epsilon^2+\Gamma_R^2}\frac{e^{\epsilon\tau}}{(e^{\beta\epsilon}+1)}d\epsilon. \tag{28}$$

### 3.3. Main results

The thermoelectric coefficients are computed nonperturbatively over the reflection amplitude $|r|$. Substituting Eqs. (21), (26) into Eq.(13) and Eqs. (21), (28) into Eq.(14), one can obtain the general expressions for the electric conductance $G$ and thermoelectric coefficient $G_T$, respectively. Performing integration over the time, we obtain

$$G = \frac{4g_\rho G_L}{\pi}\frac{\Gamma_R}{T}\left(\frac{\pi^2 T}{\gamma g_\rho E_C}\right)^{\frac{1}{g_\rho}}\int_{-\infty}^{\infty}\frac{d\tilde{\epsilon}}{\tilde{\epsilon}^2+\left(\frac{\Gamma_R}{T}\right)^2}\frac{1}{\cosh\left(\frac{\tilde{\epsilon}}{2}\right)}\mathrm{Re}\left\{\frac{{}_2F_1\left[2+\frac{1}{g_\rho}, w(1), \frac{1}{2}w(2), -1\right]}{1+g_\rho\left(2+i\frac{\tilde{\epsilon}}{\pi}\right)}\right\}, \tag{29}$$

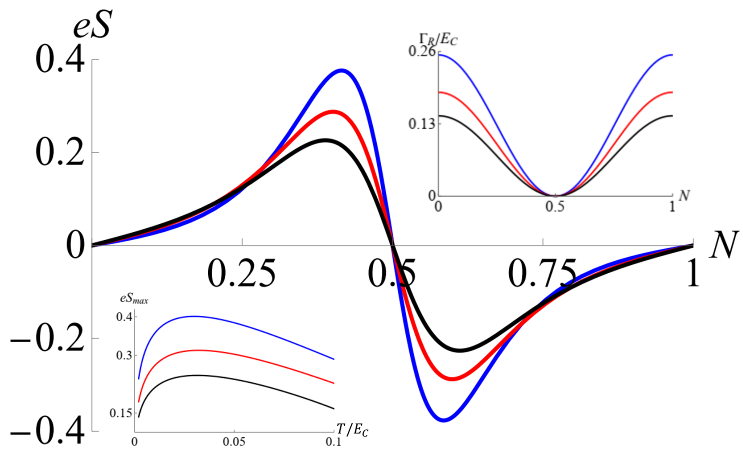


**Fig. 2.** (Color online) Plots of thermopower $eS$ as a function of dimensionless gate voltage $N$ for different values of the charge Luttinger parameter $g_\rho = 0.6$ (blue line), $g_\rho = 0.8$ (red line), and $g_\rho = 1$ (black line) with temperature $T/E_C = 0.01$. Top-right insert: $\Gamma_R/E_C$ as a function of $N$. Bottom-left insert: $eS_{max}$ as a function of $T/E_C$. Lines correspond to the same set of $g_\rho$ as on the main frame. Here the reflection amplitude satisfies $|r|^2 = 0.1$. We choose $E_C = 1$ and $E_C/D = 0.1$.

$$G_T = \frac{8g_\rho G_L}{e}\left(\frac{\pi^2 T}{\gamma g_\rho E_C}\right)^{\frac{1}{g_\rho}}\left(\frac{2\gamma}{\pi}\right)^{g_\rho}\log\left(\frac{g_\rho E_C}{T+\Gamma_R}\right)|r^*|^2\sin(2\pi N)\int_{-\infty}^{\infty}d\tilde{\epsilon}\frac{\tilde{\epsilon}}{\tilde{\epsilon}^2+\left(\frac{\Gamma_R}{T}\right)^2}\frac{1}{\cosh\left(\frac{\tilde{\epsilon}}{2}\right)}$$
$$\times\mathrm{Im}\left\{\frac{{}_2F_1\left(3+\frac{1}{g_\rho},w(1),w(2),-1\right)}{1+g_\rho\left(2+i\frac{\tilde{\epsilon}}{\pi}\right)}+\frac{{}_2F_1\left(3+\frac{1}{g_\rho},w(2),w(3),-1\right)}{1+g_\rho\left(4+i\frac{\tilde{\epsilon}}{\pi}\right)}\right\}, \tag{30}$$

where $\tilde{\epsilon} = \epsilon/T$, ${}_2F_1(a,b,c,d)$ is a hypergeometrical function, and $w(x) = x+(2g_\rho)^{-1}+i\tilde{\epsilon}/2\pi$. By plugging Eqs. (29) and (30) into the formula of the TP $S = G_T/G$ and performing the numerical calculations we plot the TP and maximum of TP as shown in Fig. 2. However, it is rather useful to write down analytical expressions for $G$, $G_T$, and $S$ in two separated temperature regimes $T \gg \Gamma_R$ and $T \ll \Gamma_R$.

For $T \gg \Gamma_R$, we re-obtain the perturbative results shown in Ref. [32]:

$$G = \frac{G_L}{2}\left(\frac{\pi^2}{2\gamma}\right)^{\frac{1}{g_\rho}}\frac{\sqrt{\pi}\Gamma\left(\frac{1}{2}+\frac{1}{2g_\rho}\right)}{\Gamma\left(\frac{3}{2}+\frac{1}{2g_\rho}\right)}\left(\frac{T}{g_\rho E_C}\right)^{\frac{1}{g_\rho}}, \tag{31}$$

$$G_T \sim -\frac{G_L}{e}|r^*|^2\sin(2\pi N)\log\left(\frac{g_\rho E_C}{T}\right)\left(\frac{T}{g_\rho E_C}\right)^{\frac{1}{g_\rho}}, \tag{32}$$

and

$$S \sim -\frac{|r^*|^2}{e}\sin(2\pi N)\log\left(\frac{g_\rho E_C}{T}\right), \tag{33}$$

where $\Gamma(x)$ is the gamma function. One should notice that the prefactors in Eqs. (32) and (33), which are complex function of $g_\rho$, are not shown [see Ref. [32] for their expressions]. Besides, there is a misprint in Eq. (10) of Ref. [32], we correct the expression for the conductance at $T \gg \Gamma_R$ in Eq. (31). Interestingly, when $N$ is half integer, $\Gamma_R(N)$ approaches zero, see the inset in Fig. 2. It means the perturbative results from Ref. [32] can be applied in the vicinity of Coulomb peaks even at $T \ll |r^*| g_\rho E_C$.

At $T \ll \Gamma_R$, the electric conductance is written as

$$G = G_L \left(\frac{\pi^2}{2\gamma}\right)^{\frac{1}{g_\rho}} \frac{\sqrt{\pi}\Gamma\left(\frac{3}{2}+\frac{1}{2g_\rho}\right)}{\Gamma\left(2+\frac{1}{2g_\rho}\right)} \frac{T}{\Gamma_R} \left(\frac{T}{g_\rho E_C}\right)^{\frac{1}{g_\rho}}. \tag{34}$$

It reproduces Eq. (69) in Ref. [24] when $g_\rho = 1$. The expression of the thermoelectric coefficient reads

$$G_T = -\frac{G_L \pi^4}{e} \left(\frac{T}{g_\rho E_C}\right)^{2+\frac{1}{g_\rho}} \frac{\sin(\pi N)}{2|r^*|^2 \cos^3(\pi N)} \log\left(\frac{\pi}{4|r^*|^2 (2\gamma/\pi)^{g_\rho} \cos^2(\pi N)}\right) \mathscr{C}_1(g_\rho), \tag{35}$$

where

$$\mathscr{C}_1(g_\rho) = \pi^{g_\rho^{-1}} \left(\frac{\pi}{2\gamma}\right)^{g_\rho + g_\rho^{-1}} \mathscr{C}(g_\rho), \tag{36}$$

$$\begin{aligned}\mathscr{C}(g_\rho) = & -\frac{\sqrt{\pi}\Gamma\left(\frac{5}{2}+\frac{1}{2g_\rho}\right)}{2\Gamma\left(3+\frac{1}{2g_\rho}\right)} + \frac{g_\rho}{1+7g_\rho}\, {}_2F_1\left[1, -\frac{1+g_\rho}{2g_\rho}, \frac{1}{2}\left(9+\frac{1}{g_\rho}\right), -1\right] \\ & + \frac{g_\rho}{1+3g_\rho}\, {}_2F_1\left[1, -\frac{1+5g_\rho}{2g_\rho}, \frac{1}{2}\left(5+\frac{1}{g_\rho}\right), -1\right].\end{aligned} \tag{37}$$

Finally, we obtain the TP formula at low temperature limit $T \ll \Gamma_R$ as

$$S = -\frac{2\pi^2}{e} \tan(\pi N) \frac{T}{g_\rho E_C} \frac{1}{|r^*|^2} \log\left(\frac{\pi}{4|r^*|^2 (2\gamma/\pi)^{g_\rho} \cos^2(\pi N)}\right) \mathscr{C}_2(g_\rho), \tag{38}$$

where

$$\mathscr{C}_2(g_\rho) = \left(\frac{\pi}{2\gamma}\right)^{g_\rho} \frac{\Gamma(2+1/g_\rho)}{\sqrt{\pi}\Gamma(3/2+1/2g_\rho)} \mathscr{C}(g_\rho). \tag{39}$$

Interestingly, the logarithmic temperature scaling of TP $S \sim \log T$ in Eq. (33) is attributed to the NFL behavior of the 2CK effect. The linear temperature dependence of TP as shown in Eq. (38) is for the limit $T \ll \Gamma_R(N)$: it occurs at very low temperatures and/or in the vicinity of Coulomb valleys. It is out of our consideration since the 2CK regime in the FMF model only occurs in the vicinity of the charge degeneracy point ($N$ is half integer). The crossover between two regimes occurs at the values of $N_{max}$ where $T \sim \Gamma_R$. The distance of $N_{max}$ from the centers of the Coulomb peaks is $\delta N \sim \frac{1}{|r^*|}\sqrt{\frac{T}{g_\rho E_C}}$. At these $N_{max}$ points the TP reaches its maximum absolute value $S_{max}$, which can be estimated as

$$S_{max} \sim e^{-1} |r^*| \sqrt{\frac{T}{g_\rho E_C}} \log\left(\frac{g_\rho E_C}{T}\right). \tag{40}$$

We demonstrate our results in Figure 2. The energy scale $\Gamma_R$ (top-right insert) and TP $eS$ (main panel) are plotted as functions of dimensionless gate voltage $N$, while $eS_{max}$ is plotted as a function of $T$ (bottom-left insert) for different values of the charge Luttinger parameter $g_\rho = 0.6$ (blue line), $g_\rho = 0.8$ (red line), and $g_\rho = 1$ (black line). Here, we restrict ourselves to considering the minimal value of the Luttinger parameter $g_\rho^{min} = 0.6$. In principle, the nonperturbative treatment allows us to consider the case of stronger repulsive interaction. However, additional constraint associated with the condition $|r^*_{max}|^2 < 1$ should be accounted for. The main NFL behaviors of TP in the 2CK model remain unchanged for different values of the charge Luttinger parameter [25, 30]. We find that the maximal value of TP increases when electron-electron interaction becomes stronger, while its position in the $S(N)$ plot becomes closer to the Coulomb peak. The electron interaction in the LL manifests itself in the renormalization of both charging energy of the QD and backscattering amplitude at the QPC. The latter additionally contributes to the break of the particle-hole symmetry. We thus predict that the TP is enhanced by the electron-electron interaction, which is qualitatively consistent with the results obtained in Refs. [32, 43–45].

## 4. Conclusion

In this paper, we have re-investigated theoretically the thermoelectric transport through a single electron transistor device in which a QD is weakly coupled to one lead and strongly coupled to the other one through an almost transparent single-mode QPC. The QD-QPC structure of the system is mapped onto a two-channel charge Kondo model. The interacting electrons in the vicinity of the QPC are described by LL model and we only investigate a case of weakly repulsive interaction in the charge modes and absence of interaction in the spin modes. Applying the Abelian bosonization and refermionization techniques, we calculate the thermoelectric coefficients nonperturbatively with respect to the reflection amplitude of the QPC. We obtain the NFL behavior of the TP around the Coulomb peaks at very low temperature. Namely, the nonperturbative result not only covers the perturbative one, which is befitting in the temperature interval $|r^*|^2 g_\rho E_C \ll k_B T \ll g_\rho E_C$, but also is applicable in the lower temperature regime $k_B T \ll |r^*|^2 g_\rho E_C$. We predict that the TP is enhanced due to the repulsive electron-electron interaction, which opens an experimental access for investigation of Luttinger liquid properties in the two-channel Kondo regime.

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## Conflict of interest

The authors have no conflict of interest to declare.

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