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INVESTIGATION OF THE FCNC PROCESSES IN THE 3-4-1-1 MODEL

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Abstract. *We study the FCNC problems in 3-4-1-1 model in a way different from the previous work. The sources of FCNC at the tree-level in the 3-4-1-1 model come from both the gauge and scalar sectors. We show that the most stringently bound on the tree-level FCNC interactions comes from the meson oscillations. The lower bound on the new physics scale is imposed more tightly than in the previous work, $M_{new} > 22$TeV. On the allowed value domain of the new physical scale, we show that the contribution of the tree-level FCNC interactions to the $Br(B_s \to \mu^+\mu^-)$ is negligible.*

Keywords: Dark matter; standard model; models beyond the standard model.

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I. INTRODUCTION

The standard model (SM) is incomplete since it cannot solve many crucial questions of nature: the neutrino masses, dark matter, cosmic inflation, matter-antimatter asymmetry... The answer to these questions, we have to study beyond the SM. A common feature of the electroweak extension of the SM is an extension of the Higg sector and gauge symmetry. Among a class of models based on the extended gauge symmetry, the models that include B-L as a gauge charge have intriguing features. The models contain non-commutative B-L, the dark and normal-matters non-trivially unified in gauge multiplets. The discrete symmetry, which protected stability of the dark matter (DM), appears naturally as the residual $B-L$ charge [1–8]. The $U(1)_{B-L}$ breaking field successfully inflates the early universe, cosmic inflation [2, 8] and its vacuum expectation value yields a mass for right-handed neutrinos. Thus, this kind of extended SM can explain small neutrinos masses through the exchange of heavy right-handed neutrino, well-known as seesaw mechanisms [8]. The model having a higher weak isospin symmetry $SU(P)_L$ can offer both natural solutions for the questions of generation number [1, 9] and multicomponent DM for $P \geq 4$ [9, 10] due to a non-trivial structure of the residual gauge symmetry relevant to the existence of multi B-L charges. In this direction of expansion, the minimal realization of multicomponent DM corresponds to the gauge symmetry, $SU(3)_C \times SU(4)_L \times U(1)_X \times U(1)_N$ (3-4-1-1). In this model, the fermion generations transform differently under the gauge symmetry. Thus it creates the tree-level FCNC.

The new physics contribution to the FCNC processes is bounded strongly. Especially the processes related to B$-$physics. SM contributions to the meson mixing systems are almost matching the observed quantities. Thus, such studying gives stringent constraints on the FCNC interactions. Bound on new physics scale from FCNC interactions has been studied previously in the 3-4-1-1 model [9]. The result was insufficient because they have studied only the $B_s^0 - \bar{B}_s^0$ mixing from contributions of new neutral gauge bosons. Source of new scalar sectors and SM contributions were not considered. Therefore, the lower bound on the new physics scale indicated in [9] is a few TeV. In this work, we study the FCNC problems in the 3-4-1-1 model in a way different from previous works [9]. We will include both SM and all the tree-level contributions (both gauge and scalar fields) to study some FCNC processes, namely the meson mixing and rare decay of B-meson. We can obtain new results.

We organize our paper as follows. In Sec. II, we give a brief overview of the 3-4-1-1 model. The FCNC interactions in the 3-4-1-1 model, including both scalar and gauge fields, are derived in Sec. III. In Sec. IV, we concentrate on studying the mass difference of mesons and the rare decay $B_s \to \mu\mu$. Our conclusions are given in Sec. V.

II. A SUMMARY OF THE 3-4-1-1 MODEL

The model naturally provides multi-component dark matter is based on the gauge symmetry $SU(3)_C \times SU(P)_L \times U(1)_X \times U(1)_N$, where $SU(3)_C$ is the color group, $SU(P)_L$ is an extension of the $SU(2)_L$ weak-isospin. The minimal realization of multicomponent dark matter corresponds to $P = 4$, called the 3-4-1-1 model. The last two factors, $U(1)_X, U(1)_N$ determine the electric charge Q and $B-L$ as follows [10]

$$Q = T_3 + \beta T_8 + \gamma T_{15} + X, \quad B-L = bT_8 + cT_{15} + N, \tag{1}$$

where the coefficients have been determined in [9, 10] as

$$\beta=-\frac{1}{\sqrt{3}}\left(2q+1\right),\quad \gamma=\frac{1}{\sqrt{6}}\left(q-3p-1\right), b=-\frac{2}{\sqrt{3}}\left(n+1\right),\quad c=\frac{1}{\sqrt{6}}\left(n-3m-2\right). \tag{2}$$

The fermion and scalar contents under gauge symmetry are given as

$$\begin{aligned}
\psi_{aL} =&(\nu,e,E,F)^T_{aL}\sim(1,4,\frac{p+q-1}{4},\frac{m+n-2}{4}),\\
&\nu_{aR}\sim(1,1,0,-1),\quad e_{aR}\sim(1,1,-1,-1),\\
Q_{\alpha L} =&(d,-u,J,K)^T\sim(3,4^*,-\frac{p+q+\frac{1}{3}}{4},-\frac{n+m+\frac{2}{3}}{4})_{\alpha L},\\
&u_{aR}\sim\left(3,1,\frac{2}{3},\frac{1}{3}\right), d_{aR}\sim\left(3,1,-\frac{1}{3},\frac{1}{3}\right)\\
Q_{3L} =&(u,d,J,K)^T_{3L}\sim(3,4,\frac{q+p+\frac{5}{3}}{4},\frac{n+m+\frac{10}{3}}{4}),\quad J_{\alpha R}\sim\left(3,1,-q-\frac{1}{3},-n-\frac{2}{3}\right),\\
J_{3R}\sim&\left(3,2,q+\frac{2}{3},n+\frac{4}{3}\right), K_{\alpha R}\sim\left(3,1,-p-\frac{1}{3},-m-\frac{2}{3}\right), K_{3R}\sim\left(3,1,p+\frac{2}{3},m+\frac{4}{3}\right),\\
E_{aR}\sim&(1,1,q,n),\quad F_{aR}\sim(1,1,p,m),
\end{aligned}\tag{3}$$

where $a=1,2,3,\ \alpha=1,2$ are the generation index. The scalar sector, which is necessary for realistic symmetry breaking and mass generation, consists of the following Higgs fields [9, 10]

$$\begin{aligned}
\eta^T &=(\eta_1,\eta_2,\eta_3,\eta_4)\sim\left(1,4,\frac{p+q-1}{4},\frac{m+n+2}{4}\right),\\
\chi^T &=(\chi_1,\chi_2,\chi_3,\chi_4)\sim\left(1,4,\frac{p-3q-1}{4},\frac{m-3n-2}{4}\right),\\
\rho^T &=(\rho_1,\rho_2,\rho_3,\rho_4)\sim\left(1,4,\frac{q+p+3}{4},\frac{m+n+2}{4}\right),\\
\Xi^T &=(\Xi_1,\Xi_2,\Xi_3,\Xi_4)\sim\left(1,4,\frac{q-3p-1}{4},\frac{n-3m-2}{4}\right),\\
\phi &=(1,1,0,2).
\end{aligned}\tag{4}$$

The electrically-neutral scalars can develop vacuum expectation values (VEVs),

$$<\eta_1^0>=\frac{u}{\sqrt{2}},\quad <\rho_2^0>=\frac{v}{\sqrt{2}},\quad <\chi_3^0>=\frac{w}{\sqrt{2}},\quad <\Xi_4^0>=\frac{V}{\sqrt{2}},\quad <\phi>=\frac{\Lambda}{\sqrt{2}}. \tag{5}$$

The scheme of the gauge symmetry breaking is summarized

$$\begin{gathered}
SU(3)_C\otimes SU(4)_L\otimes U(1)_X\otimes U(1)_N\\
\downarrow w,\Lambda,V\\
SU(3)_C\otimes SU(2)_L\otimes U(1)_Y\otimes P\\
\downarrow u,v\\
SU(3)_C\otimes U(1)_Q\otimes P.
\end{gathered}$$

The multiple matter parity takes the form as

$$P=P_n\otimes P_m,\quad P_n^{\pm}=(-1)^{\pm(3n+1)},\quad P_m=(-1)^{\pm(3m+1)}. \tag{6}$$

This symmetry makes "wrong $B-L$ particles" become stabilized, see [10]. The consistent condition for the VEVs is [10] $\Lambda \gg w, V \gg u, v$, $u^2+v^2 = 246^2$ GeV^2. The scalar potential is given [10] as follows

$$\begin{aligned}V_{\text{Higgs}} =& \mu_1^2\eta^\dagger\eta + \mu_2^2\rho^\dagger\rho + \mu_3^2\chi^\dagger\chi + \mu_4^2\Xi^\dagger\Xi + \lambda_1(\eta^\dagger\eta)^2 + \lambda_2(\rho^\dagger\rho)^2 + \lambda_3(\chi^\dagger\chi)^2 + \lambda_4(\Xi^\dagger\Xi)^2 \\ &+ (\eta^\dagger\eta)(\lambda_5\rho^\dagger\rho + \lambda_6\chi^\dagger\chi + \lambda_7\Xi^\dagger\Xi) + (\rho^\dagger\rho)(\lambda_8\chi^\dagger\chi + \lambda_9\Xi^\dagger\Xi) + \lambda_{10}(\chi^\dagger\chi)(\Xi^\dagger\Xi) \\ &+ \lambda_{11}(\eta^\dagger\rho)(\rho^\dagger\eta) + \lambda_{12}(\eta^\dagger\chi)(\chi^\dagger\eta) + \lambda_{13}(\eta^\dagger\Xi)(\Xi^\dagger\eta) + \lambda_{14}(\rho^\dagger\chi)(\chi^\dagger\rho) \\ &+ \lambda_{15}(\rho^\dagger\Xi)(\Xi^\dagger\rho) + \lambda_{16}(\chi^\dagger\Xi)(\Xi^\dagger\chi) + (\lambda_{17}\eta\rho\chi\Xi + H.c.) + V(\phi), \end{aligned} \tag{7}$$

where the last term is the potential of ϕ plus the interactions of ϕ with η, ρ, χ, and Ξ,

$$V(\phi) = \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2 + (\phi^*\phi)(\lambda_{18}\eta^\dagger\eta + \lambda_{19}\rho^\dagger\rho + \lambda_{20}\chi^\dagger\chi + \lambda_{21}\Xi^\dagger\Xi). \tag{8}$$

We expand the neutral scalar fields as

$$\begin{aligned}&\eta = \left(\frac{1}{\sqrt{2}}(u+S_1+iA_1), \eta_2^-, \eta_3^q, \eta_4^p\right)^T, \quad \rho = \left(\rho_1^+, \frac{1}{\sqrt{2}}(v+S_2+iA_2), \rho_3^{q+1}, \rho_4^{p+1}\right)^T, \\ &\chi = \left(\chi_1^{-q}, \chi_2^{-q-1}, \frac{1}{\sqrt{2}}(w+S_3+iA_3), \chi_4^{p-q}\right)^T, \Xi = \left(\Xi_1^{-p}, \Xi_2^{-p-1}, \Xi_3^{q-p}, \frac{1}{\sqrt{2}}(V+S_4+iA_4)\right)^T, \\ &\phi = \frac{1}{\sqrt{2}}(\Lambda + H_N + iG_N). \end{aligned} \tag{9}$$

We assume $\Lambda \gg w, V$ such that the scalar field ϕ is decoupled. Therefore, H_N is identified as the heavy Higgs, which can play a role of the inflaton field [2], while the pseudo-scalar field, G_N, is a massless Goldstone bosons associated with $U(1)_N$ breaking. After integrating ϕ out, the remaining scalar quadruplets are mixed. The physical scalar states are related to the gauge states as given in [10]

$$\begin{aligned}&H_1 \simeq c_{\alpha_2}S_1 + s_{\alpha_2}S_2, \quad H_2 \simeq s_{\alpha_2}S_1 - c_{\alpha_2}S_2, \quad H_3 \simeq c_{\alpha_1}S_3 - s_{\alpha_1}S_4, \quad H_4 \simeq s_{\alpha_1}S_3 + c_{\alpha_1}S_4, \\ &\mathscr{A} \simeq s_{\alpha_2}A_1 + c_{\alpha_2}A_2, \quad G_Z \simeq c_{\alpha_2}A_1 - s_{\alpha_2}A_2, \quad G_{Z_2} \simeq s_{\alpha_3}A_3 - c_{\alpha_3}A_3, \\ &G_{Z_3} \simeq -c_{\alpha_3}A_3 - s_{\alpha_3}A_4, \quad G_W^\pm \simeq s_{\alpha_2}\rho_1^\pm - c_{\alpha_2}\eta_2^\pm, \quad \mathscr{H}_1^\pm \simeq c_{\alpha_2}\rho_1^\pm + s_{\alpha_2}\eta_2^\pm, \\ &\mathscr{H}_6^{\pm(q-p)} \simeq s_{\alpha_3}\Xi_3^{\pm(q-p)} + c_{\alpha_3}\chi_4^{\pm(q-p)}, \quad G_{W_{13}}^{\pm q} \simeq \chi_1^{\pm q}, \quad G_{W_{14}}^{\pm p} \simeq \Xi_1^{\pm p}, \quad G_{W_{23}}^{\pm(q+1)} \simeq \chi_2^{\pm(q+1)}, \\ &G_{W_{24}}^{\pm(p+1)} \simeq \Xi_2^{\pm(p+1)}, \quad G_{W_{34}}^{\pm(q-p)} \simeq c_{\alpha_3}\Xi_3^{\pm(q-p)} - s_{\alpha_3}\chi_4^{\pm(q-p)}, \quad \mathscr{H}_2^{\pm q} \simeq \eta_3^{\pm q}, \quad \mathscr{H}_3^{\pm p} \simeq \eta_4^{\pm p}, \\ &\mathscr{H}_4^{\pm(q+1)} \simeq \rho_3^{\pm(q+1)}, \quad \mathscr{H}_5^{\pm(p+1)} \simeq \rho_4^{\pm(p+1)}, \end{aligned} \tag{10}$$

where, the $\alpha_i, i = 1,2,3$ are defined by $\tan 2\alpha_1 = \frac{\lambda_{10}wV}{\lambda_4V^2 - \lambda_3w^2}$, $\tan\alpha_2 = \frac{v}{u}$, $\tan\alpha_3 = \frac{w}{V}$. The fields G_X are identified to the Goldstone bosons that are eaten by gauge bosons X. The remaining scalar fields are physical state and their masses are given in [10]. The mass spectrum of gauge bosons including the kinetic mixing term was considered in [9]. If assuming the hierarchies, $u, v \ll w, V \ll \Lambda$, the kinetic mixing term between two $U(1)$ gauge bosons can be ignored. The physical neutral gauge bosons are related to the gauge states as follows

$$A_\mu = s_W A_{3\mu} + c_W \left(\beta t_W A_{8\mu} + \gamma t_W A_{15\mu} + \frac{t_W}{t_X} B_\mu\right), \quad (11)$$

$$Z_{1\mu} = c_W A_{3\mu} - s_W \left(\beta t_W A_{8\mu} + \gamma t_W A_{15\mu} + \frac{t_W}{t_X} B_\mu\right), \quad (12)$$

$$Z'_{2\mu} = \frac{1}{\sqrt{1-\beta^2 t_W^2}} \left[(1-\beta^2 t_W^2) A_{8\mu} - \beta\gamma t_W^2 A_{15\mu} - \frac{\beta t_W^2}{t_X} B_\mu\right], \quad (13)$$

$$Z'_{3\mu} = \frac{1}{\sqrt{1+\gamma^2 t_X^2}} \left(A_{15\mu} - \gamma t_X B_\mu\right), \quad (14)$$

with the corresponding masses are given in [10]. The non-Hermitian gauge bosons are denoted as

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(A_{1\mu} \mp iA_{2\mu}\right), \quad W^{\pm q}_{13\mu} = \frac{1}{\sqrt{2}}\left(A_{4\mu} \pm A_{5\mu}\right), \quad W^{\pm p}_{14\mu} = \frac{1}{\sqrt{2}}\left(A_{9\mu} \pm iA_{10\mu}\right),$$
$$W^{\pm(q+1)}_{23\mu} = \frac{1}{\sqrt{2}}\left(A_{6\mu} \pm iA_{7\mu}\right), W^{\pm(p+1)}_{24\mu} = \frac{1}{\sqrt{2}}\left(A_{11\mu} \pm iA_{12\mu}\right), W^{\pm(q-p)}_{34\mu} = \frac{1}{\sqrt{2}}\left(A_{13\mu} \mp iA_{14\mu}\right).$$

The Yukawa interactions have a following form

$$\begin{aligned}\mathscr{L}_{\text{Yukawa}} =& \frac{1}{2} f^\nu_{ab} \bar{\nu}^c_{aR} \nu_{bR} \phi + h^\nu_{ab} \bar{\psi}_{aL} \eta \nu_{bR} + h^e_{ab} \bar{\psi}_{aL} \rho e_{bR} + h^E_{ab} \bar{\psi}_{aL} \chi E_{bR} + h^F_{ab} \bar{\psi}_{aL} \Xi F_{bR} \\ &+ h^u_{3a} \bar{Q}_{3L} \eta u_{aR} + h^u_{\alpha a} \bar{Q}_{\alpha L} \rho^* u_{aR} + h^d_{3a} \bar{Q}_{3L} \rho d_{aR} + h^d_{\alpha a} \bar{Q}_{\alpha L} \eta^* d_{aR} \\ &+ h^J_{33} \bar{Q}_{3L} \chi J_{3R} + h^K_{33} \bar{Q}_{3L} \Xi K_{3R} + h^J_{\alpha\beta} \bar{Q}_{\alpha L} \chi^* J_{\beta R} + h^K_{\alpha\beta} \bar{Q}_{\alpha L} \Xi^* K_{\beta R} + H.c. \end{aligned} \quad (15)$$

After symmetry breaking, the fermion fields receive mass. The up-and down-quarks mass matrices derived from the Yukawa (15)

$$m^u_{\alpha a} = \frac{1}{\sqrt{2}} h^u_{\alpha a} v, \quad m^u_{3a} = -\frac{1}{\sqrt{2}} h^u_a u, \quad m^d_{\alpha a} = -\frac{1}{\sqrt{2}} h^d_{\alpha a} u, \quad m^d_{3a} = -\frac{1}{\sqrt{2}} h^d_{3a} v. \quad (16)$$

The exotic quarks do not mix with the SM quarks, and their mass matrix has a form

$$m^J_{33} = -h^J_{33} \frac{w}{\sqrt{2}}, \quad m^J_{\alpha\beta} = -h^J_{\alpha\beta} \frac{w}{\sqrt{2}}, \quad m^K_{33} = -h^K_{33} \frac{V}{\sqrt{2}}, \quad m^K_{\alpha\beta} = -h^K_{\alpha\beta} \frac{V}{\sqrt{2}}. \quad (17)$$

The charged leptons have a Dirac mass $m^e_{ab} = -h^e_{ab} \frac{v}{\sqrt{2}}$ while the neutrinos have both Dirac and Majorana masses $m^D_{\nu ab} = -h^\nu_{ab} \frac{u}{\sqrt{2}}$, $m^M_{\nu ab} = f^\nu_{ab} \Lambda$. Thus, the mall neutrino mass can be explained via a see-saw mechanism.

To close this section, we would like to note that the up-quark and down-quark mass matrices are not flavor-diagonal. They can be diagonalized by the matrices $V_{u_{L,R}}, V_{d_{L,R}}$ according to

$$V^\dagger_{u_L} m^u V_{u_R} = \mathscr{M}_u = \text{Diag}(m_{u_1}, m_{u_2}, m_{u_3}), \quad V^\dagger_{d_L} m^d V_{d_R} = \mathscr{M}_d = \text{Diag}(m_{d_1}, m_{d_2}, m_{d_3}), \quad (18)$$

and the mass eigenstates are related to the flavor states by

$$\begin{aligned} u'_{L,R} =& (u'_{1L,R}, u'_{2L,R}, u'_{3L,R})^T = V^\dagger_{u_{L,R}} (u_{1L,R}, u_{2L,R}, u_{3L,R})^T, \\ d'_{L,R} =& (d'_{1L,R}, d'_{2L,R}, d'_{3L,R})^T = V^\dagger_{d_{L,R}} (d_{1L,R}, d_{2L,R}, d_{3L,R})^T. \end{aligned} \quad (19)$$

The CKM matrix is defined as $V_{\rm CKM} = V^\dagger_{u_L} V_{d_L}$. The flavor states e_a are related to the physical states e'_a by using two unitary matrices $U^l_{L,R}$ as

$$e_{aL} = (U^l_L)_{ab} e'_{bL}, \quad e_{aR} = (U^l_R)_{ab} e'_{bR}. \tag{20}$$

III. THE FCNC INTERACTIONS

Because the third generation of quark transforms differently from the two first generations, it causes FCNC interactions at the tree-level. The scalar neutral current arises from the Yukawa interactions (15)

$$\begin{aligned}\mathscr{L}^{NC}_{scalar} \supset & \frac{h^u_{3a}}{\sqrt{2}} \bar{u}_{3L} \left(u + S_1 + iA_1\right) u_{aR} - \frac{h^u_{\alpha a}}{\sqrt{2}} \bar{u}_{\alpha L} \left(v + S_2 - iA2\right) u_{aR} \\ & + \frac{h^d_{3a}}{\sqrt{2}} \bar{d}_{3L} \left(v + S_2 + iA2\right) d_{aR} + \frac{h^d_{\alpha a}}{\sqrt{2}} \bar{d}_{\alpha L} \left(u + S_1 - iA_1\right) d_{aR}.\end{aligned} \tag{21}$$

Changing Eq.(21) to the physical eigenstates, we obtain the scalar FCNC interactions as follows

$$\mathscr{L}^{FCNC}_{scalar} = \frac{g}{2m_W} \left(\bar{d}'_L \Gamma^d d'_R + \bar{u}'_L \Gamma^u u'_R\right) H_1 + \frac{ig}{2m_W} \left(\bar{d}'_L \Gamma^d d'_R - \bar{u}'_L \Gamma^u u'_R\right) \mathscr{A} + H.c., \tag{22}$$

where $\Gamma^{u,d}_{ij}$ are given as

$$\begin{aligned}\Gamma^u_{ij} &= -\frac{2}{s_{2\alpha_2}} (V^\dagger_{u_L})_{i3} (V_{u_L})_{3k} m_{u_k} (V^\dagger_{u_R})_{ka} (V_{u_R})_{aj}, \\ \Gamma^d_{ij} &= -\frac{2}{s_{2\alpha_2}} (V^\dagger_{d_L})_{i3} (V_{d_L})_{3k} m_{d_k} (V^\dagger_{d_R})_{ka} (V_{d_R})_{aj}.\end{aligned} \tag{23}$$

The vector and axial-vector FCNC interactions were studied in [9]. The FCNCs couple only the ordinary quarks to T_8, T_{15}. In the mass basis, the tree-level FCNC interactions have a form as follows

$$\mathscr{L}^{gauge}_{FCNC} = -\bar{q}'_{iL} \gamma^\mu q'_{jL} (V^*_{qL})_{3i} (V_{qL})_{3j} \left(g_1 Z_{1\mu} + g_2 Z_{2\mu} + g_3 Z_{3\mu} + g_4 Z_{4\mu}\right), \quad i \neq j. \tag{24}$$

with, q' denoting $u' = (u'_1, u'_2, u'_3)$ either $q' = d' = (d'_1, d'_2, d'_3)$. In the limit, $\Lambda \gg w, V$, the couplings, $g_i, i = 1, ..., 4$, can be written as follows

$$g_1 = 0, \quad g_2 = \frac{g}{\sqrt{6}} \left(\frac{\sqrt{2}}{\sqrt{1 - \beta^2 t_W^2}} c_\varphi - \frac{1 + \gamma(\sqrt{2}\beta + \gamma) t_X^2}{\sqrt{1 + \gamma^2 t_X^2}} s_\varphi \right), g_3 = g_2 \left(c_\varphi \to s_\varphi, s_\varphi \to -c_\varphi\right), \tag{25}$$

where $c_\varphi = \cos\varphi, s_\varphi = \sin\varphi$ and φ is determined by

$$\tan 2\varphi = \frac{4\sqrt{2} w^2 \left[1 - \gamma\left(2\sqrt{2}\beta - \gamma\right) t_X^2\right] \sqrt{1 + (\beta^2 + \gamma^2) t_X^2}}{w^2 \left[7 - \gamma^2 (2\sqrt{2}\beta - \gamma)^2 t_X^4 + (8\beta^2 + 4\sqrt{2}\beta\gamma + 6\gamma^2) t_X^2\right] - 9V^2 (1 + \gamma^2 t_X^2)^2}. \tag{26}$$

IV. CONSTRAINS ON NEW PHYSICS FROM FCNC INTERACTIONS AT THE TREE-LEVEL

IV.1. Meson mixing at the tree-level

At the tree-level, both new scalar and gauge boson fields mediate the FCNC interactions. From Eqs. (22, 24), after integrating the heavy fields out, we obtain the effective interactions describing the meson mixing as follows

$$\begin{aligned}\mathcal{L}^{eff}_{\mathrm{FCNC}} = & -\left[(V^*_{qL})_{3i}(V_{qL})_{3j}\right]^2\left(\frac{g_2^2}{m^2_{Z_2}}+\frac{g_3^2}{m^2_{Z_3}}\right)\left(\bar{q'_{iL}}\gamma^\mu q'_{jL}\right)^2 \\ & +\frac{g^2}{4m_W^2}\left\{(\Gamma^q_{ij})^2\left(\frac{1}{m^2_{H_1}}-\frac{1}{m^2_{\mathscr{A}}}\right)\left(\bar{q}'_{iL}q'_{jR}\right)^2+(\Gamma^{q*}_{ji})^2\left(\frac{1}{m^2_{H_1}}-\frac{1}{m^2_{\mathscr{A}}}\right)\left(\bar{q}'_{iR}q'_{jL}\right)^2\right\} \\ & +\frac{g^2}{4m_W^2}\left\{\Gamma^{q*}_{ji}\Gamma^q_{ij}\left(\frac{1}{m^2_{H_1}}+\frac{1}{m^2_{\mathscr{A}}}\right)(\bar{q}'_{iL}q'_{jR})(\bar{q}'_{iR}q'_{jL})+\Gamma^{q*}_{ji}\Gamma^q_{ij}\left(\frac{1}{m^2_{H_1}}+\frac{1}{m^2_{\mathscr{A}}}\right)(\bar{q}'_{iR}q'_{jL})(\bar{q}'_{iL}q'_{jR})\right\},\end{aligned} \tag{27}$$

The above effective Lagrangian has a form as given in [5, 11], thus the mass difference of the meson systems has the same general form as follows

$$\begin{aligned}(\Delta m_K)_{3411} = & \Re\left\{\frac{2}{3}\Theta^2_{12}\left(\frac{g_2^2}{m^2_{Z_2}}+\frac{g_3^2}{m^2_{Z_3}}\right)+\frac{5g^2}{48m_W^2}\left((\Gamma^d_{12})^2+(\Gamma^{d*}_{21})^2\right)\left(\frac{1}{m^2_{H_1}}-\frac{1}{m^2_{\mathscr{A}}}\right)\left(\frac{m_K}{m_s+m_d}\right)^2\right\}m_K f_K^2 \\ & -\Re\left\{\frac{g^2\Gamma^{d*}_{21}\Gamma^d_{12}}{4m_W^2}\left(\frac{1}{m^2_{H_1}}+\frac{1}{m^2_{\mathscr{A}}}\right)\left(\frac{1}{6}+\frac{m_K^2}{(m_s+m_d)^2}\right)\right\}m_K f_K^2, \\ (\Delta m_{B_d})_{3411} = & \Re\left\{\frac{2}{3}\Theta^2_{13}\left(\frac{g_2^2}{m^2_{Z_2}}+\frac{g_3^2}{m^2_{Z_3}}\right)+\frac{5g^2}{48m_W^2}\left((\Gamma^d_{13})^2+(\Gamma^{d*}_{31})^2\right)\left(\frac{1}{m^2_{H_1}}-\frac{1}{m^2_{\mathscr{A}}}\right)\left(\frac{m_{B_d}}{m_b+m_d}\right)^2\right\}m_{B_d} f_{B_d}^2 \\ & -\Re\left\{\frac{g^2\Gamma^{d*}_{31}\Gamma^d_{13}}{4m_W^2}\left(\frac{1}{m^2_{H_1}}+\frac{1}{m^2_{\mathscr{A}}}\right)\left(\frac{1}{6}+\frac{m_{B_d}^2}{(m_b+m_d)^2}\right)\right\}m_{B_d} f_{B_d}^2, \\ (\Delta m_{B_s})_{3411} = & \Re\left\{\frac{2}{3}\Theta^2_{23}\left(\frac{g_2^2}{m^2_{Z_2}}+\frac{g_3^2}{m^2_{Z_3}}\right)+\frac{5g^2}{48m_W^2}\left((\Gamma^{d*}_{32})^2+(\Gamma^d_{23})^2\right)\left(\frac{1}{m^2_{H_1}}-\frac{1}{m^2_{\mathscr{A}}}\right)\left(\frac{m_{B_s}}{m_s+m_b}\right)^2\right\}m_{B_s} f_{B_s}^2 \\ & -\Re\left\{\frac{g^2\Gamma^{d*}_{32}\Gamma^d_{23}}{4m_W^2}\left(\frac{1}{m^2_{H_1}}+\frac{1}{m^2_{\mathscr{A}}}\right)\left(\frac{1}{6}+\frac{m_{B_s}^2}{(m_s+m_b)^2}\right)\right\}m_{B_s} f_{B_s}^2.,\end{aligned} \tag{28}$$

where, $\Theta_{ij} = (V^*_{qL})_{3i}(V_{qL})_{3j}$. Noting that the constraint for contribution of the new interactions to the meson systems must be determined by

$$(\Delta m_M)_{3411} = (\Delta m_M)_{\mathrm{Exp}} - (\Delta m_M)_{\mathrm{SM}}\,. \tag{29}$$

The SM contributions to the meson mass differences $(\Delta m_M)_{\mathrm{SM}}$ and their experimental values $(\Delta m_M)_{\mathrm{Exp}}$ can be found in [12,13]. We emphasize that the previous studies [9] were insufficient in the sense that only the new physics from gauge bosons analyzed, that sources of the scalar sector and SM have not been included. Hence, the bound on the new physics scale is low. Let us consider the role of each contribution. The new physics contributions to the meson mass differences can be divided into two parts: $(\Delta m_{K,B_s,B_d})_{3411} = \Delta m^{Z_2,Z_3}_{K,B_s,B_d} + \Delta m^{H_1,A}_{K,B_s,B_d}$. The first term is the contributions from the new gauge bosons while the second one comes from the contribution of the new scalars. From Eq.(28), we obtain

$$\frac{\Delta m^{H_1,\mathscr{A}}_{K,B_s,B_d}}{\Delta m^{Z_2,Z_3}_{K,B_s,B_d}} \simeq \frac{m_b^2}{m_W^2}\frac{m^2_{Z_{2,3}}}{m^2_{\mathscr{A},H_1}}. \tag{30}$$

If the new scalar fields have a mass of the same order as that of the gauge bosons particles, the contribution from the scalar fields $H_1, \mathscr{A}$ to the meson mass differences is insignificant. The constraint on the new physics scale from the contribution of the new gauge bosons considered as follows

$$\Delta m^{Z_2,Z_3}_{K,B_s,B_d} \leq (\Delta m_{K,B_s,B_d})_{\text{Exp}} - (\Delta m_{K,B_s,B_d})_{\text{SM}} \tag{31}$$

As mentioned in [12, 13], the strongest bound comes from $B_s^0 - \bar{B}_s^0$ mixing. The Eqs. (28), (29) lead to the bound as follows

$$\left[(V^*_{dL})_{32}\,(V_{dL})_{33}\right]^2 \left(\frac{g_2^2}{m^2_{Z_2}} + \frac{g_3^2}{m^2_{Z_3}}\right) \leq \frac{3.2}{10^6\,(\text{TeV})^2}. \tag{32}$$

By taking $V_{\text{CKM}} = V^\dagger_{uL}V_{dL} = V_{dL}$, we draw contour condition (32) in Fig. 1. The allowed region is blue. The new physics scales require the following constraints: $w > 25\text{TeV}, V > 20\text{TeV}$. This bound is more constrained than the previously known one [9].

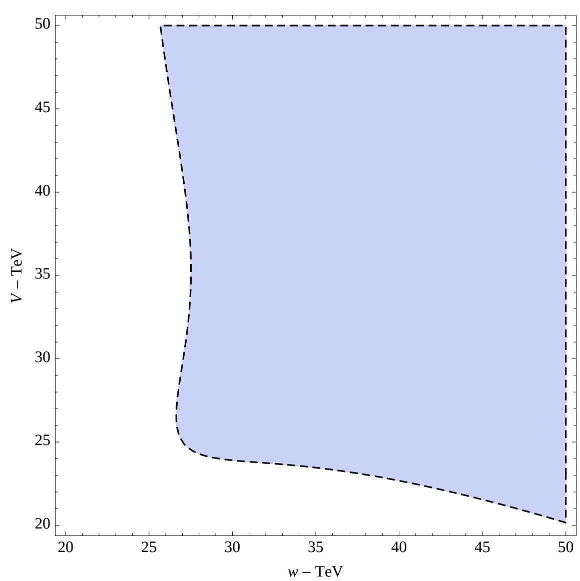


Fig. 1. The constraints for w and V obtained from the condition given in Eq.(32).

IV.2. $B_s \to \mu^+\mu^-$

Recent measurements at the LHC [14, 15] have given

$$\mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{exp}} = \left(3.2^{+1.5}_{-1.2}\right) \times 10^{-9}. \tag{33}$$

The SM prediction (including the effect of $B_s - \bar{B}_s$ oscillations) [16, 17] is

$$\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)_{\mathrm{SM}} = (3.56 \pm 0.18) \times 10^{-9}. \tag{34}$$

This decay channel leads to stringent constraints on physics beyond the standard model (BSM). Let us consider the new physics of the 3-4-1-1 model contributed to the rare $B_s \to \mu^+\mu^-$ decay. At tree-level, it is mediated by both new gauge bosons Z_2, Z_N and new scalars $H_1, \mathscr{A}$. Thus, the effective Hamiltonian for this process is presented via six operators as follows:

$$\mathscr{H}_{\mathrm{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=9,10,S,P} C_i(\mu) \mathscr{O}_i(\mu) + \sum_{i=S,P} C_i'(\mu) \mathscr{O}_i'(\mu) \right), \tag{35}$$

where the operators are defined by

$$\begin{aligned}
\mathscr{O}_9 &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l), \qquad & \mathscr{O}_{10} &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma^5 l), \\
\mathscr{O}_S &= \frac{e^2}{(4\pi)^2} (\bar{s} P_R b)(\bar{l} l), & \mathscr{O}_P &= \frac{e^2}{(4\pi)^2} (\bar{s} P_R b)\left(\bar{l}\gamma_5 l\right), \\
\mathscr{O}_S' &= \frac{e^2}{(4\pi)^2} (\bar{s} P_L b)(\bar{l} l), & \mathscr{O}_P' &= \frac{e^2}{(4\pi)^2} (\bar{s} P_L b)\left(\bar{l}\gamma_5 l\right).
\end{aligned} \tag{36}$$

The SM contribution to the Wilson coefficients can be found in [18] while the contributions coming from the 3-4-1-1 model have a following form

$$\begin{aligned}
C_9^{3411} =& \Theta_{23} \frac{m_W^2}{c_W V_{tb} V_{ts}^*} \frac{g^2}{e^2} \left(-\frac{g_2}{g} \frac{g_V^{Z_2}(f)}{m_{Z_2}^2} - \frac{g_3}{g} \frac{g_V^{Z_3}(f)}{m_{Z_3}^2} \right), \\
C_{10}^{3411} =& -\Theta_{23} \frac{m_W^2}{c_W V_{tb} V_{ts}^*} \frac{g^2}{e^2} \left(-\frac{g_2}{g} \frac{g_A^{Z_2}(f)}{m_{Z_2}^2} - \frac{g_3}{g} \frac{g_A^{Z_3}(f)}{m_{Z_3}^2} \right), \\
C_S^{3411} =& \frac{8\pi^2}{e^2} \frac{1}{V_{tb} V_{ts}^*} \frac{\Gamma_{23}^d \Gamma_{\alpha\alpha}^l}{m_{H_1}^2}, \qquad C_S'^{3411} = \frac{8\pi^2}{e^2} \frac{1}{V_{tb} V_{ts}^*} \frac{\left(\Gamma_{32}^d\right)^* \Gamma_{\alpha\alpha}^l}{m_{H_1}^2}, \\
C_P^{3411} =& -\frac{8\pi^2}{e^2} \frac{1}{V_{tb} V_{ts}^*} \frac{\Gamma_{23}^d \Delta_{\alpha\alpha}^l}{m_{\mathscr{A}}^2}, \qquad C_P'^{3411} = \frac{8\pi^2}{e^2} \frac{1}{V_{tb} V_{ts}^*} \frac{\left(\Gamma_{32}^d\right)^* \Delta_{\alpha\alpha}^l}{m_{\mathscr{A}}^2},
\end{aligned} \tag{37}$$

where $\Gamma_{\alpha\alpha}^l = \Delta_{\alpha\alpha}^l = \frac{u}{v} m_{l_\alpha}$, $g_{V,A}^{Z_{2,3}}$ are determined via the interactions of Z_2 and Z_3 with two charged leptons

$$-\frac{g}{2C_W} \bar{f}\gamma^\mu \left(g_V^{Z_2}(f) - g_A^{Z_2}(f)\gamma_5 \right) f Z_{2\mu} - \frac{g}{2C_W} \bar{f}\gamma^\mu \left(g_V^{Z_N}(f) - g_A^{Z_N}(f)\gamma_5 \right) f Z_{N\mu}. \tag{38}$$

The expressions of $g_{A,V}^{Z_{2,3}}$ can be found in [10]. The coefficient C_{10}^{SM} is predicted by the SM [18], $C_{10}^{\mathrm{SM}} = -4.103$. From the effective theory (35), we obtain the branching ratio of the $B_s \to l_\alpha^+ l_\alpha^-$

decay as follows

$$\mathrm{Br}(B_s \to l_\alpha^+ l_\alpha^-)_{\mathrm{theory}} = \frac{\tau_{B_s}}{64\pi^3}\alpha^2 G_F^2 f_{B_s}^2 |V_{tb}V_{ts}^*|^2 m_{B_s}\sqrt{1-\frac{4m_{l_\alpha}^2}{m_{B_s}^2}}$$
$$\times\left\{\left(1-\frac{4m_{l_\alpha}^2}{m_{B_s}^2}\right)\left|\frac{m_{B_s}^2}{m_b+m_s}\left(C_S-C_S'\right)\right|^2+\left|2m_{l_\alpha}C_{10}+\frac{m_{B_s}^2}{m_b+m_s}\left(C_P-C_P'\right)\right|^2\right\}, \quad (39)$$

where τ_{B_s} is the total lifetime of B_s meson. Due to the oscillations of the $B_s - \bar{B}_s$ system, the experimental result relates to the that of theory as follows [19]

$$\mathrm{Br}(B_s \to l_\alpha^+ l_\alpha^-)_{\mathrm{exp}} \simeq \frac{1}{1-y_s}\mathrm{Br}(B_s \to l_\alpha^+ l_\alpha^-)_{\mathrm{theory}}, \quad (40)$$

where $y_s = \frac{\Delta\Gamma_{B_s}}{2\Gamma_{B_s}} \simeq 0.061(9)$ [20]. For numerical study, we set all input parameters the same values as in Sec. III and define $\mathrm{Br}(B_s \to l_\alpha^+ l_\alpha^-)^{3411} = \mathrm{Br}(B_s \to l_\alpha^+ l_\alpha^-)_{\mathrm{theory}} - \mathrm{Br}(B_s \to l_\alpha^+ l_\alpha^-)_{\mathrm{SM}}$.

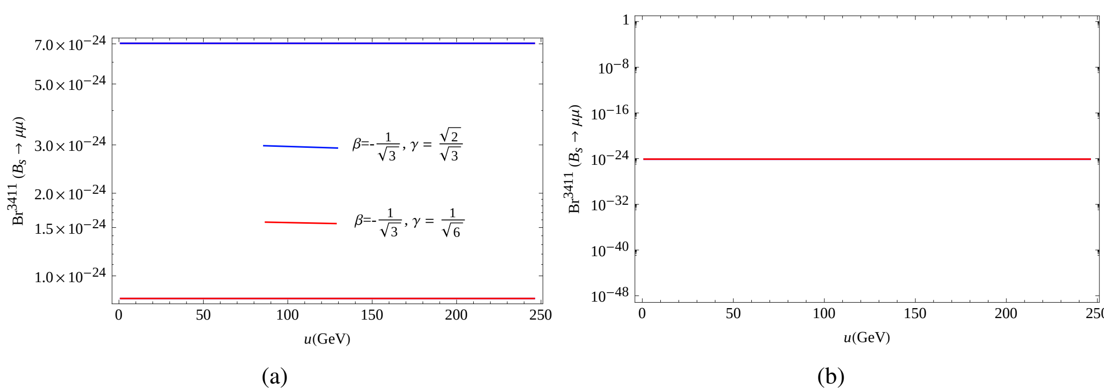
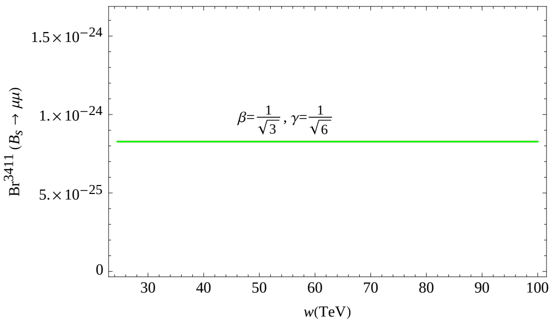
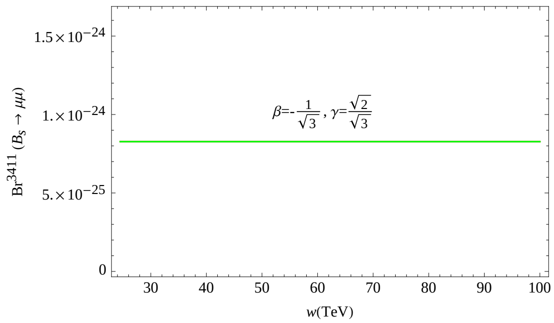


Fig. 2. The dependence of the branching ratio $\mathrm{Br}^{3411}(B_s \to \mu^+\mu^-)$ on the electroweak scale u with fixing $w = V = 30$TeV. (a) and (b) panels are plotted with $m_{\mathscr{A}} = m_{H_1} = 0.1 \times m_{Z_2}$,$m_{\mathscr{A}} = m_{H_1} = m_{Z_2}$ respectively. In the right-handed panel, we fix $\beta = \pm\frac{1}{\sqrt{3}}, \gamma = \frac{1}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{3}}$.

Figure 2 plots the branching ratio $\mathrm{Br}^{3411}(B_s \to \mu^+\mu^-)$ for different models through choosing different values of two parameters β, γ. If $m_{\mathscr{A},H_1} \simeq 0.1 \times m_{Z_2}$, the $\mathrm{Br}^{3411}(B_s \to \mu^+\mu^-)$ slightly depends on the value of γ while $m_{\mathscr{A},H_1} = m_{Z_2}$, it does not depend on the value of γ. Both figures predict that this branching does not depend on the values of u and β. For fixing $u = v = \frac{246}{\sqrt{2}}GeV, w = V$, we plot the dependence of the branching ratio $\mathrm{Br}^{3411}(B_s \to \mu^+\mu^-)$ on the new physics scale w in Fig. 3. In the space the parameter satisfies with the constraint $\bar{B}_s - B_s$ oscillations, all results show that branching ratio $\mathrm{Br}^{3411}(B_s \to \mu^+\mu^-)$ is much smaller compared to the SM contribution.



(a)



(b)

Fig. 3. The dependence of the branching ratio $\mathrm{Br}^{3411}(B_s \to \mu^+\mu^-)$ on the new physics scale. Both figures (a) and (b) are plotted with chosen values of $u = v = \frac{246}{\sqrt{2}}$, $m_{\mathscr{A}} = m_{H_1} = 0.1 \times m_{Z_2}$.

V. CONCLUSIONS

Due to the non-universal assignment of quark families, the FCNC interactions appear at the tree-level in both neutral scalar and gauge bosons sectors of the 3-4-1-1 model. If their masses are of the same order, the main contribution to the meson oscillations is the new gauge bosons mediation. The strongest constraint comes from the $B_s^0 - \bar{B}_s^0$ oscillation. We show that the lower limit of the new physics scale $w = V > 25$TeV, which is more stringent than the constraint given in [9]. This bound is since we have included both contributions of the SM and the new 3411 interactions while the previous work has ignored the contribution of the SM and the new neutral scalar Higgs bosons H_1 and $\mathscr{A}$. Based on the $B_s^0 - \bar{B}_s^0$ vertex and the parameter region consistent with the meson mass difference, we consider the branching ratio $B_s \to \mu^+\mu^-$ and realize that the new FCNC interactions give a small contribution to this branching ratio. This consistent with the recent measurements.

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