

## NUCLEAR MEAN-FIELD DESCRIPTION OF PROTON ELASTIC SCATTERING BY $^{12,13}\text{C}$ AT LOW ENERGIES

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**Abstract.** *Nuclear reactions of proton by light nuclei at low energies play a key role in the study of nucleosynthesis which is of interest in nuclear astrophysics. The most fundamental process which is very necessary is the elastic scattering. In this work, we construct a microscopic proton-nucleus potential in order to describe the differential cross-sections over scattering angles of the proton elastic scattering by  $^{12}\text{C}$  and  $^{13}\text{C}$  in the range of available energies 14 - 22 MeV. The microscopic optical potential is based on the folding model using the effective nucleon-nucleon interaction CDM3Yn. The results show the promising use of the CDM3Yn interactions at low energies, which were originally used for nuclear reactions at intermediate energies. This could be the premise for the study of nuclear reactions using CDM3Yn interaction in astrophysics at low energies.*

Keywords: elastic scattering, folding model, CDM3Yn.

Classification numbers: 24.10.Cn, 24.10.Ht, 25.40.Cm.

## I. INTRODUCTION

When a proton beam interacts with a target nucleus, there are a variety of reaction channels occurring such as elastic scattering, inelastic scattering, or nucleon rearrangement collision processes. The elastic scattering channel, however, plays the most vital role and is fundamental to describe other reaction channels. One of the simplest and the most effective ways to describe the nuclear elastic scattering is the optical model approach in which a complicated many-body problem of proton elastic scattering is reduced to a one-body problem with a complex single-particle potential. In particular, the real part of the potential describes the elastic scattering channel, while the imaginary part takes account of all the non-elastic channels.

An optical potential can be constructed phenomenologically with the parameters of a potential form or microscopically starting from nucleon interactions. For the phenomenological approach, the nuclear potential is commonly of the Woods-Saxon (WS) form, which can be easy in the calculation and in the description of measured data. This approach, however, cannot provide further physical information as well as cannot predict for the description of unavailable experimental data of elastic scattering on a wide range of unstable nuclei. Meanwhile, a microscopic nucleon-nucleus optical model could be constructed to overcome this problem, in which the folding model using an effective nucleon-nucleon (NN) interaction has been widely and successfully used to generate the optical potential [1]. In particular, the potential of the nucleon-nucleus system can be defined by summing over all NN interactions between the incident nucleon and each nucleon binding in the target nucleus. The important inputs of calculation in the folding model are therefore the nucleon density of target and a version of effective NN interaction.

In this work, we focus on the effective NN interaction CDM3Yn which is a version of density dependent NN interaction of M3Y interaction (Michigan 3 Yukawa). Particularly, the M3Y interaction is obtained from G-matrix elements of bare NN interaction related to M3Y-Reid [2] and M3Y-Paris [3]. The parameters of density dependence of CDM3Yn were determined in 1900s in Hartree-Fock (HF) calculations to describe the properties of saturation of symmetric nuclear matter, which was performed by Khoa *et al.* [4]. It has been used to describe not only the equation of state (EOS) of asymmetric nuclear matter but also the isovector of nucleon optical potential [5]. Recently, the density dependence of the CDM3Yn interaction was included on the HF level by the rearrangement term (RT) of the nucleon optical potential derived from the Hugenholtz-van Hove (HvH) theorem [6, 7]. The addition of the RT into the folding calculation of the nucleon optical potential was shown to be necessary for the overall good optical model description of elastic nucleon scattering at low energies. Then CDM3Yn versions have been applied successfully to microscopic calculations of nucleon-nucleus and nucleus-nucleus scattering potentials in folding model [8, 9].

The folding calculation using CDM3Yn interaction focuses mainly on the scattering from zero-spin and zero-isospin target nuclei. Recent studies indicate that the CDM3Yn interaction can describe well proton-nucleus scattering with intermediate-mass and heavy targets ( $A \geq 40$ ) and  $E > 30$  MeV [6, 7]. As a result, the description of proton-nucleus scattering on light nuclei at low energies ( $E \lesssim 20$  MeV) using CDM3Yn is the subject of interest. In this energy range, using an optical model to describe scattering data is not simple due to the effects of strong coupling to non-elastic channel or non-locality. Moreover, the knowledge of elastic scattering at low energies also supports the studies of nuclear reactions in astrophysics. For instance, the elastic scattering

of proton from  $^{12}\text{C}$  and  $^{13}\text{C}$  targets at astrophysical energies is related to the first proton radiative capture reactions ( $p, \gamma$ ) occurring in the carbon-nitrogen-oxygen (CNO) cycle in asymptotic giant branch (AGB) stars after the nucleus  $^{12}\text{C}$  is formed by the  $3\alpha$  process. The mean-field potential models applied to these elastic scattering  $^{12}\text{C}(p, p)$  and  $^{13}\text{C}(p, p)$  are regularly of the WS form [10, 11] with the parameters adjusted to obtain best fits in comparison with experimental data. Nevertheless, these phenomenological potential models are still relatively simple and depend on the parameters of a function instead of starting from NN interactions. Thus, in the present work the data of elastic scattering of  $^{12,13}\text{C}(p, p)$  are analyzed, which is based on the microscopic calculation in folding model with the chosen effective NN interaction CDM3Y6.

The construction of the folded scattering potential using CDM3Y6 interaction is represented in Sec. II. Finally, the theoretical results of elastic scattering of proton on  $^{12,13}\text{C}$  in the range of available energies 14-22 MeV are analyzed and discussed in Sec. III.

## II. PROTON-NUCLEUS OPTICAL POTENTIAL IN FOLDING MODEL

The optical potential contains the nuclear central (mean-field) part, the spin-orbit (s.o.), and Coulomb (coul.) potentials as

$$V = V_{\text{central}} + V_{\text{s.o.}} + V_{\text{coul.}} \quad (1)$$

The Coulomb potential of a uniformly charged sphere is used in the optical model calculation of the elastic proton scattering

$$V_{\text{coul.}}(r) = \begin{cases} Ze^2(3 - (r/R_0)^2)/(2R_0) & ; r \leq R_0 \\ Ze^2/r & ; r > R_0 \end{cases} \quad (2)$$

where the Coulomb radius is taken as  $R_{\text{coul.}} = R_0 = 1.25 \times A^{1/3}$  (fm) in which  $A$  is the mass number of the target.

The spin-orbit is of the Thomas form defined by

$$V_{\text{s.o.}} = -2V_S \left( \frac{\hbar}{m_\pi c} \right)^2 \frac{1}{r} \frac{df_S(r)}{dr} \mathbf{S} \cdot \mathbf{L}, \quad (3)$$

where the factor  $(\hbar/(m_\pi c))^2$  is the squared pion Compton wavelength,  $f_S(r)$  is the WS form factor defined as  $f_S(r) = [1 + \exp((r - R_S)/a_S)]^{-1}$ , in which  $a_S$ ,  $R_S$  and  $V_S$  are the diffuseness, the radius, and the real depth of the spin-orbit potential, respectively.

In the phenomenological model, the nuclear central potential is of conventional type with standard radial dependence, using the real strength  $V$  and the imaginary strength  $W$  in MeV [12,13]

$$V_{\text{central}}(r) = -[V_R f_R(r) + i(W_V f_V(r) - 4a_D W_D (d/dr) f_D(r))]. \quad (4)$$

Such a macroscopic optical potential consists of the set of parameters depending on not only the mass and charge number of the target nucleus but also the energy and charge of the projectile. The phenomenological optical model potentials for neutrons and protons with bombarding energies from a few keV up to 200 MeV for the nuclei in the mass range  $24 \leq A \leq 209$  had been recently reported in Ref. [14, 15]. However, for the energies below 10 MeV for lighter nuclei, there are a wide diversity of difficulties in consistent study of dispersion relation between the real and imaginary parts. With the use of absorbed surface potential, Nodvik *et al.* [11] reported the sets of

parameters of WS and Gaussian potentials to analyze the differential cross sections at the bombarding energies from about 12 MeV to 20 MeV. Recently, the WS parameters have been updated via nucleon scattering upon light nuclei ( $A < 13$ ) from 65 MeV to 75 MeV [10].

In the microscopic approach the optical potential can be calculated as the superposition of interactions of incident proton and each target nucleon, namely folding model. In this approximation the incident proton does not perturb the density distribution of the target. The optical potential of elastic scattering of nucleon upon the target  $A$  can be determined as

$$V_{\text{fold}}(\mathbf{r}) = \int v_{\text{NN}}(\rho, |\mathbf{R} - \mathbf{r}|) \rho(\mathbf{R}) d\mathbf{R}, \quad (5)$$

where  $\rho$  is the nucleon density distribution of the ground-state (g.s.) target and  $v_{\text{NN}}$  is the effective interaction of incident proton and each target nucleon separated by the distance  $s = |\mathbf{R} - \mathbf{r}|$ , with  $\mathbf{r}$  and  $\mathbf{R}$  are the positions of the proton and the nucleon in the target, respectively; with the origin placed at the center of the target. In the context of a complex folded optical potential it is necessary to renormalize (scale) slightly the real and imaginary part. A folded optical potential with CDM3Y6 interaction used in the analysis of data of elastic scattering at a certain incident energy  $E$  can be written as

$$V_{\text{central}}(r) = N_R \text{Re}V_{\text{fold}}(r) + iN_I \text{Im}V_{\text{fold}}(r), \quad (6)$$

where  $N_R$  and  $N_I$  are the renormalization factors for the real and imaginary folded potentials, respectively. Due to the anti-symmetry of proton-nucleus system, the folded potential includes the direct term  $V_{\text{dir.}}$  and exchange term  $V_{\text{exc.}}$ , in which the exchange potential is non-local in coordinate space. Recently, by  $R$ -matrix method, the nucleon-nucleus scattering equation was solved directly with the non-local potential [7]. While, using the Wentzel-Kramers-Brillouin (WKB) approximation for the shift of the scattering wave by the spatial exchange of the incident nucleon and that bound in the target, the exchange term becomes localized [16], the proton optical potential depends explicitly on energy via the proton relative momentum  $k(E, r)$  which is determined self-consistently from the folded proton-nucleus potential as

$$k^2(E, r) = \frac{2\mu}{\hbar^2} [E - \text{Re}V_{\text{fold}}(E, r) - V_{\text{coul.}}(r)], \quad (7)$$

with  $V_{\text{fold}}(E, r) = V_{\text{dir.}}(r) + V_{\text{exc.}}(E, r)$ . More details of the folding model calculation can be found in Ref. [6].

The important ingredient in the folding model is the density distribution of the nucleus. In this study, the g.s. densities of targets  $^{12}\text{C}$  and  $^{13}\text{C}$  were given by the independent particle model (IPM) [17]. Each single-nucleon wave function is then determined using a separate WS potential. The WS depth was adjusted in each case to give the required binding energy using a fixed WS diffuseness and the WS radius was fine-tuned for the proton g.s. density to have the root-mean-squared (rms) radius close to that deduced from the measured charge radius. The nucleon binding energies for the bound nucleons of the target were taken from the shell-model results. Fig. 1 illustrates the nucleon density distributions of the considered g.s. targets calculated using IPM.

The central part of the CDM3Yn interaction was used in the HF results explicitly as

$$v_{\text{NN}}(\rho, s) = F_{00}(\rho)v_{00}(s) + F_{01}(\rho)v_{01}(s), \quad (8)$$

with  $v_{00}$  and  $v_{01}$  being the isoscalar (IS) and isovector (IV) of the interaction, respectively. The density dependence  $F_i(\rho)$  ( $i = 00, 01$ ) is parameterized as the combination of the interactions

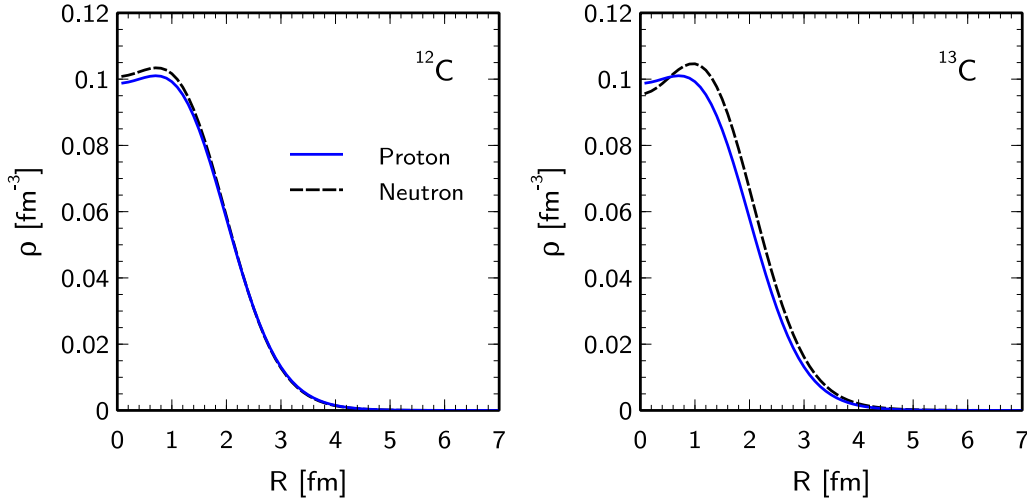
BDM3Y and DDM3Y [18]

$$F_i(\rho) = C_i[1 + \alpha_i \exp(-\beta_i \rho) + \gamma_i \rho]. \quad (9)$$

The radial parts were derived from the M3Y-Paris interaction in terms of three Yukawa strengths [18]

$$v_i(s) = \sum_{k=1}^3 Y_i(k) \frac{\exp(-\mu_k s)}{\mu_k s}. \quad (10)$$

In the present work, we use the version of CDM3Y6 interaction which was determined to reproduce the saturation properties of cold nuclear matter in the HF calculation. This density dependent interaction has been successfully used in the analysis of folding model of nucleon-nucleus and nucleus-nucleus scattering [6, 9, 19]. The imaginary density dependence of the CDM3Y6 interaction was determined using the similar density dependence as those used in the real interaction, with the parameters determined at each energy to reproduce on the HF level the energy dependent imaginary nucleon optical potential given by Jeukenne-Lejeune-Mahaux (JLM) parametrization of the Brueckner-Hartree-Fock (BHF) results for nuclear matter [20]. The parameters of density dependence, Yukawa strengths, and ranges can be found in Ref. [6]. Moreover, the nucleon mean-field potential has been thoroughly investigated in a recent extended HF study of the single-particle potential in nuclear matter using the CDM3Y6 density dependent version [6]. In this work, the folding model of the nucleon optical potential of finite nuclei has been extended to take into account the rearrangement term (RT), and applied to the study of the elastic proton scattering on the  $^{12}\text{C}$  and  $^{13}\text{C}$  targets at the energies from 14 MeV to 22 MeV.



**Fig. 1.** Nucleon density distributions of g.s.  $^{12}\text{C}$  and  $^{13}\text{C}$  based on IPM.

### III. RESULT AND DISCUSSION

The CDM3Y6 version of the folding model for the local nucleon optical potential discussed above has been adopted in this work to calculate the nucleon optical potential for the study of the elastic proton scattering on  $^{12}\text{C}$  and  $^{13}\text{C}$  targets. In the previous results [7], the

elastic scattering on intermediate-mass and heavy nuclei can be analyzed successfully without renormalization of the strength of the real folded potential. To evaluate the validity of this argument for the case of light nuclei  $^{12,13}\text{C}$ , we remained unchanged  $N_R = 1$  and only adjusted slightly the strength of imaginary potentials, while the phenomenological spin-orbit potential was used [11]. From the optical model results obtained with the CDM3Y6 interaction shown in Fig. 2 for elastic scattering  $^{12}\text{C}(p, p)$ , one can see that there is not a good optical model description of the data [21] at several energies from 14.0 MeV to 19.4 MeV. The optical-model description of the elastic  $^{12}\text{C}(p, p)$  and  $^{13}\text{C}(p, p)$  scattering data is given by the complex folded optical potential obtained with the CDM3Y6 interaction with the best fits represented in Table 1.

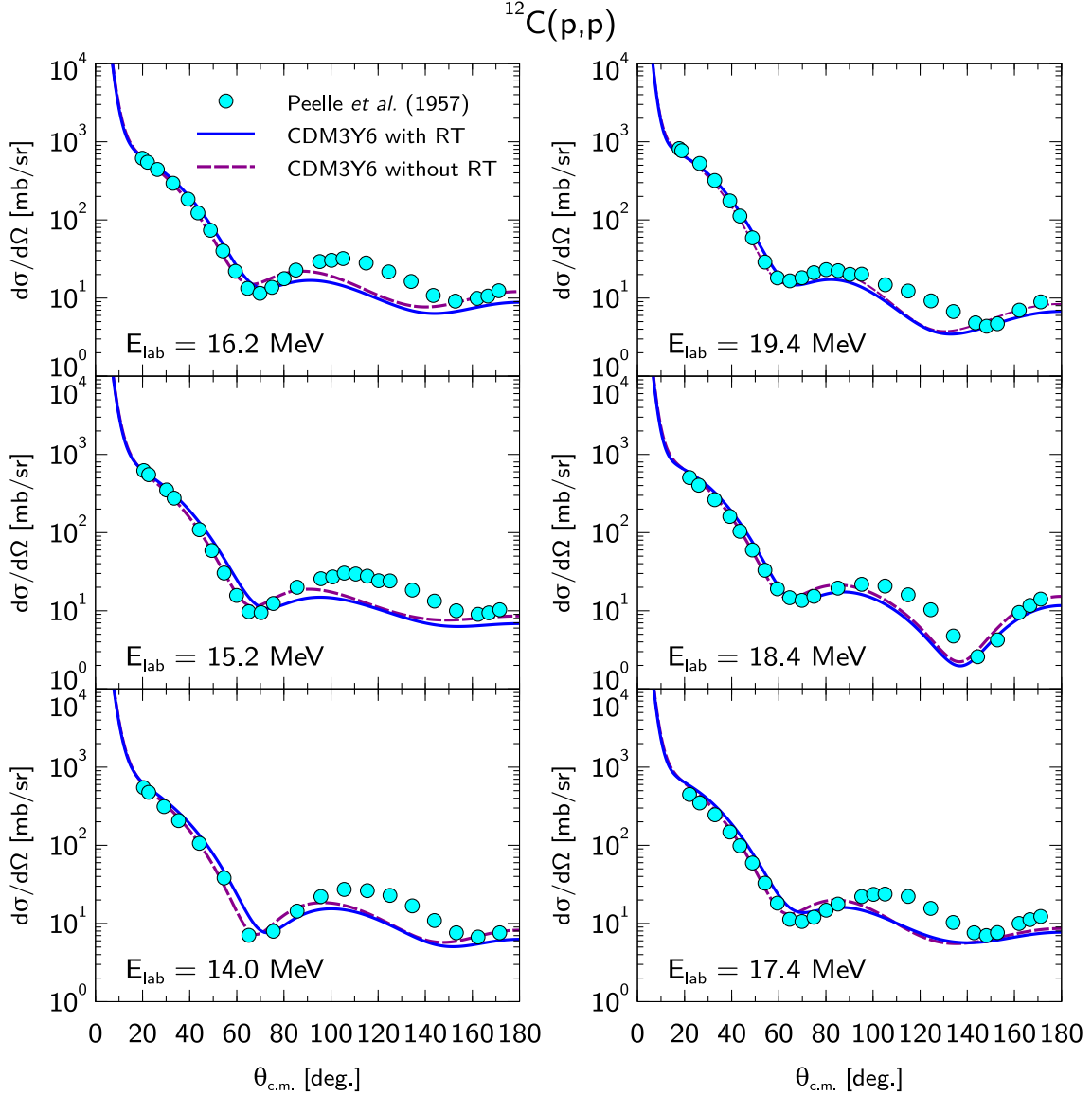
In this energy region, the differential cross section data for elastic  $^{13}\text{C}(p, p)$  scattering data are unavailable. While an overall renormalization of the imaginary proton folded potential without RT by a factor around 0.7 is needed for a optical model description of elastic proton scattering data at the considered energies, that with RT is lower, about 0.6. Although the impact by the RT of the local folding approach was pointed in the optical model results for elastic nucleon scattering on the medium-mass  $^{40,48}\text{Ca}$  and  $^{90}\text{Zr}$  targets [7], it fails to analyze the differential cross section of elastic scattering on lighter nuclei like  $^{12}\text{C}$  at low energies, one can see the similar scenario for the case without RT. At the forward angles one can see that the folded optical potential performs quite well, with the predicted elastic cross section. However, there are discrepancies between the folded potentials and experimental data at the angles larger than 70 degrees. It is pointed out that the limitation of the present microscopic

**Table 1.** Parameters of the renormalization factor of the imaginary folded potential fitted to JLM with RT and without RT used in the analysis of scattering.

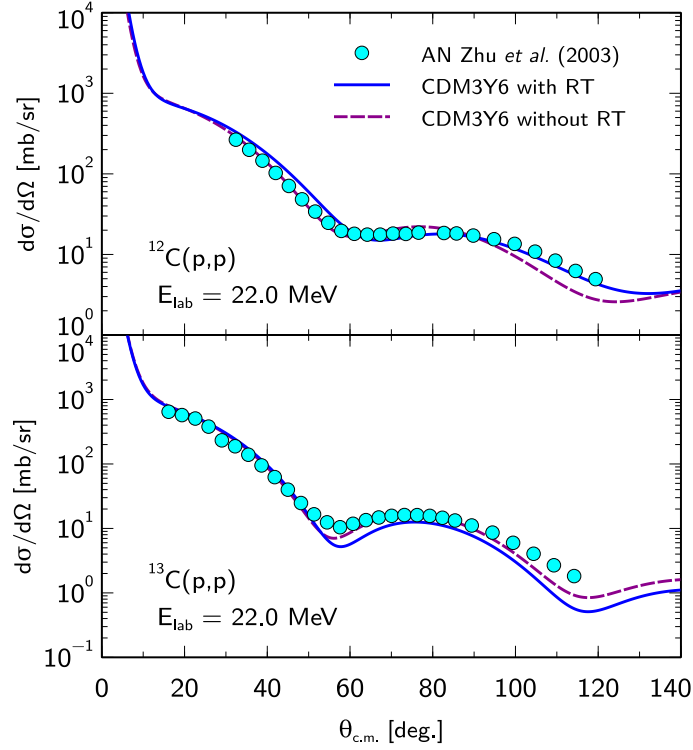
$E$ (MeV)	$N_I$ (without RT)	$N_I$ (with RT)
$^{12}\text{C}(p, p)$		
14.0	0.84	0.73
15.2	0.72	0.66
16.2	0.63	0.56
17.4	0.70	0.56
18.4	0.66	0.56
19.4	0.65	0.56
22.0	0.71	0.56
$^{13}\text{C}(p, p)$		
22.0	1.15	1.19

approach. Within the phenomenological optical model approach, Nodvik *et al.* [11] indicated that the absorbed (imaginary) surface potential at these energies is more dominant than the absorbed volume potential, while the imaginary folded potential based on JLM interaction is only defined as a central volume potential. The addition of imaginary surface potential is necessary due to the fact that it is characterized by the effect of strong coupling at low energies. As shown in Fig. 2, we can see that there is a considerable difference between the calculated theoretical results and measured data at the low energies. The deviation of the calculated values from the experimental data reduces gradually when the energy increases, which leads to that the coupling effect corresponding to imaginary surface potential decreases in the increasing energy. As shown in Fig. 3, the differential cross sections were calculated using optical folded potential for both  $^{12}\text{C}(p, p)$  and  $^{13}\text{C}(p, p)$  at 22 MeV in comparison with the experimental data in Ref. [22]. At this incident energy, the theoretical results with the folding model are improved better at larger angles. We used the best fits of the spin-orbit parameters are  $V_S = 5.0$  MeV,  $a_S = 0.60$  fm,  $R_S = R_0$  for both  $^{12}\text{C}(p, p)$  and

$^{13}\text{C}(p, p)$  at 22 MeV. We also tried to renormalize simultaneously the strengths of both the real and imaginary parts but there is no significant improvement on the description of elastic scattering.



**Fig. 2.** Optical model description of the elastic  $p + ^{12}\text{C}$  scattering data measured at the proton incident energies in the range of 14.0 - 19.4 MeV [21] given by the folded optical potential obtained with the CDM3Y6 interaction with RT and without RT, using the imaginary potential fitted to JLM interaction and the spin-orbit potential taken from Ref. [11]. The real folded potential was used with  $N_R = 1$ , while the imaginary folded potential was renormalized by the factor  $N_I$  given in Table 1.



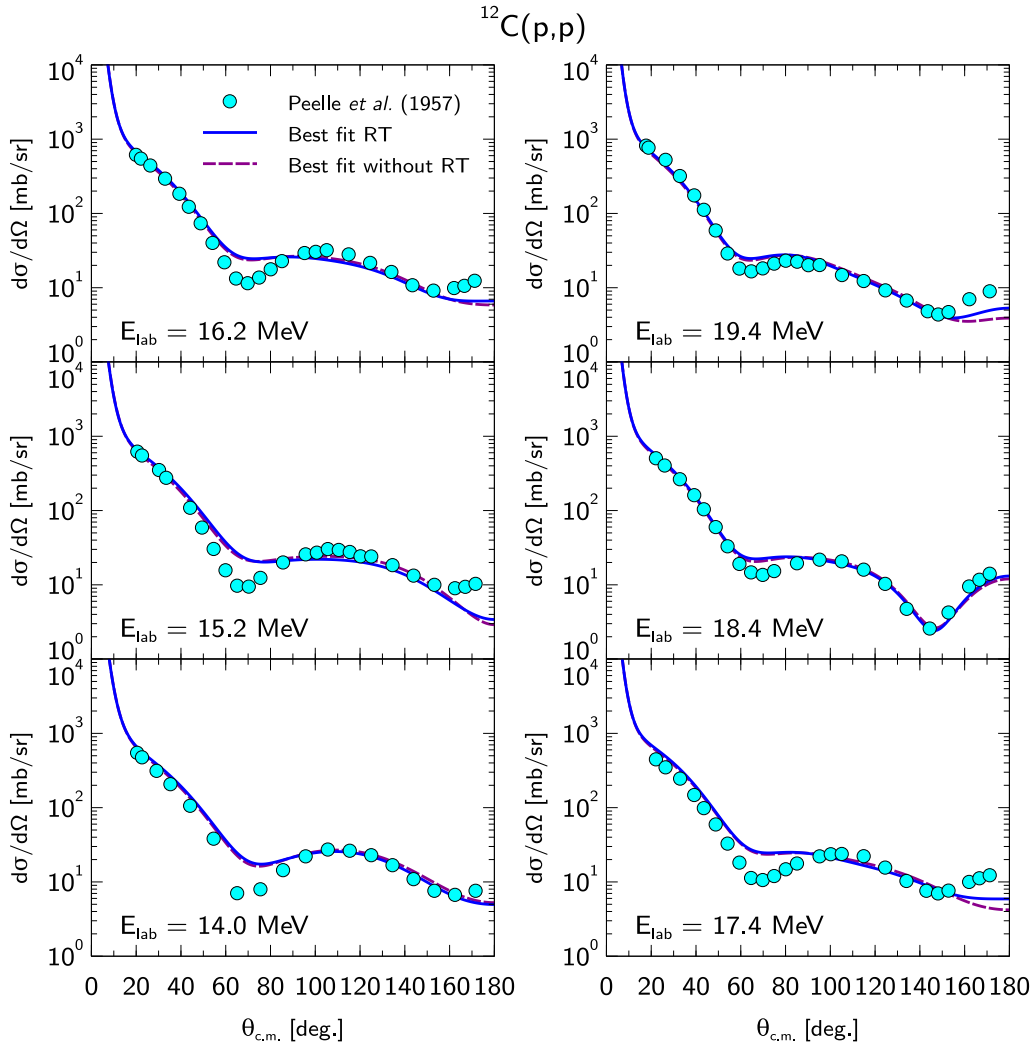
**Fig. 3.** Optical model description of the elastic  $p + ^{12,13}\text{C}$  scattering data measured at the bombarding energy of 22 MeV [22] given by the folded optical potential obtained with the CDM3Y6 interaction with RT and without RT, using the best-fit imaginary potential fitted to JLM interaction. The real folded potential was used with  $N_R = 1$ , while the imaginary folded potential was renormalized by the factor  $N_I$  given in Table 1.

**Table 2.** Parameters of the renormalization factor of real folded potential with RT and without RT used in the analysis of scattering.

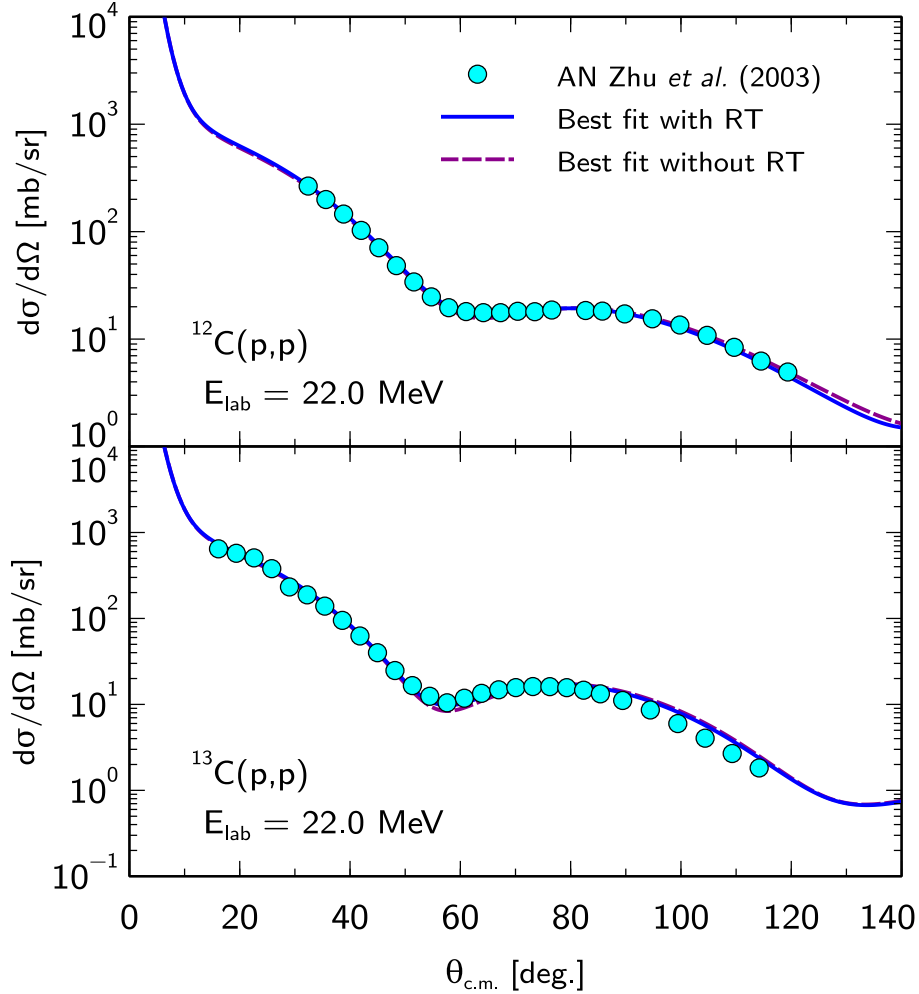
$E$ (MeV)	$N_R$ (without RT)	$N_R$ (with RT)
$^{12}\text{C}(p, p)$		
14.0	1.01	1.13
15.2	1.02	1.11
16.2	1.04	1.16
17.4	1.02	1.15
18.4	1.10	1.24
19.4	1.13	1.27
22.0	1.14	1.28
$^{13}\text{C}(p, p)$		
22.0	1.15	1.30



As mentioned above, using imaginary potentials built from the JLM interaction leads to the failure of describing the elastic scattering on  $^{12,13}\text{C}$  by proton because of the absence of imaginary surface potential. In order to examine the effectiveness of real folded potentials in the description of elastic scattering, we use the phenomenologically imaginary potential including the volume and surface parts as well as the spin-orbit potential taken from Ref. [11], and change slightly in the strength of real folded potential. For the proton elastic scattering on  $^{12,13}\text{C}$  at 22 MeV, the parameters of imaginary surface potential were taken as  $W_D = 30$  MeV,  $a_D = 0.125$  fm,  $R_D = R_0$ . To have a good agreement in  $^{13}\text{C}(p, p)$  at 22 MeV, we adopted the weak imaginary volume potential with  $W_V = 5.0$  MeV,  $a_V = 0.6$  fm,  $R_V = R_0$ . The results are shown in Figs. 4 and 5.



**Fig. 4.** The same Fig. 2 but using the phenomenological imaginary surface potential and the spin-orbit potential taken from Ref. [11]. The real folded potential was renormalized by the factor  $N_R$  given in Table 2.



**Fig. 5.** The same Fig. 3 but using the phenomenological imaginary surface potential. The real folded potential was renormalized by the factor  $N_R$  given in Table 2.

One can see that the addition of imaginary surface potential and the slight renormalization of the real folded potential in this energy region are essential for the description of elastic scattering data; especially, the improvement on the large scattering angles. The volume absorption of an optical potential is unnecessary for the analysis of nucleon elastic scattering on  $1p$ -shell nuclei [12]. Indeed, the global optical potential of  $1p$ -shell nuclei shows that the volume absorption is effective at incident energies larger than 30 MeV [12]. The optical-model description of the elastic  $^{12}\text{C}(p,p)$  and  $^{13}\text{C}(p,p)$  scattering data is given by the complex folded optical potential obtained using the CDM3Y6 interaction with RT and without RT with the best-fit  $N_R$  shown in Table 2. The real folded potential without RT is renormalized by the factor  $N_R$  in the range of approximately 1.00 – 1.15, whereas one with RT is  $N_R \approx 1.1 - 1.3$ . In the restricted optical model calculation, one can see that the CDM3Y6 without RT gives  $N_R$  closer to unity compared to the case with RT. However, the strong coupling effect caused by inelastic channel as well as non-locality affect

significantly on the real potential. The reason why  $N_R$  for the potential with RT is larger than one without RT can be explained by these effects. Noticed that the parameters of the phenomenological imaginary potential and the Thomas form of the spin-orbit potential could be found in Ref. [11].

#### IV. CONCLUSION

The present analysis shows that the proton-nucleus optical potential calculated microscopically by the folding model using effective CDM3Y6 interaction with RT and without RT, whose density-dependence accounts for the saturation properties of the nuclear matter, can be used in the analysis of the elastic scattering of proton from light nuclei  $^{12}\text{C}$  and  $^{13}\text{C}$  in the energy range from 14 MeV to 22 MeV.

The imaginary folded potential fitted from JLM interaction gives a failed description of proton elastic scattering on  $^{12,13}\text{C}$ , which implies that the volume absorption is ineffective in this energy range. In order to improve the theoretical result, the addition of surface absorption and the best-fit normalization  $N_R > 1$  are essential for the elastic channel.

Also, the inclusion of the RT into the folding model calculation of the nucleon optical potential gives a similar trend in comparison with the case without RT, but the renormalization factor  $N_R$  in the case with RT is almost higher than one without RT whose  $N_R$  is closer to unity. It could be explained by the fact that strong coupling effects and non-locality can be found in the elastic channel.

The obtained results in this work are the critical potential of upcoming low-energy researches on nuclear astrophysics, where the nuclear reactions occur in the stellar environment, and the astrophysical quantities could be then analyzed and evaluated.

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