Communications in Physics, Vol. 29, No. 3SI (2019), pp. 357-369
DOI:10.15625/0868-3166/29/3SI/14281

CONSTRAINTS FROM SPECTRUM OF SCALAR FIELDS IN THE 3-3-1 MODEL WITH CKM MECHANISM

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Received 20 August 2019

Accepted for publication 12 October 2019

Published 22 October 2019

Abstract. *We explore constraints from positivity of scalar mass spectra in the 3-3-1 model with CKS mechanism. The conditions for positivity of the diagonal elements are most important since other constraints are followed from the first ones.*

Keywords: extensions of electroweak gauge sector, extensions of electroweak Higgs sector, electroweak radiative corrections.

Classification numbers: 12.60.Cn; 12.60.Fr; 12.15.Lk.

I. INTRODUCTION

It is well known that the Higgs mechanism plays a very important role for production of particle masses. In general, the Higgs potential has to be bounded from below to ensure its stability [1]. In the Standard Model (SM) it is enough to have a positive Higgs boson quartic coupling $\lambda > 0$. In the extended models with more scalar fields, the potential should be bounded from below in all directions in the field space as the field strength approaches infinity. It is interesting to note that the square scalar mass matrix is associated with the Hessian matrix H_{ij} determined at the vacuum

$$(H_0)_{ij} = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\bigg|_{\phi=min}. \quad (1)$$

The condition for the potential to be bounded from below also leads to positivity of the above matrix [2]. The mentioned condition practically is the positivity of the principal minors. In this paper we focus our attention on positivity of scalar mass spectra and intend to get the constraints from it.

Let us remind the useful definition. A symmetric matrix M^2 of quadric form $x^T M^2 x$ for all vector x in $\mathbb{R}^n$ with the following properties

$$\begin{cases} x^T M^2 x \geq 0, & M^2 \quad \text{is called positive } \textit{semidefinite}, \\ x^T M^2 x > 0, & M^2 \quad \text{is called positive } \textit{definite}. \end{cases} \tag{2}$$

If M^2 is 2×2 matrix with elements being $M^2_{ij}, i,j = 1,2$ then Eq.(2) leads to the following conditions

$$M^2_{11} > 0, \quad M^2_{22} > 0, \tag{3}$$

$$M^2_{12} + \sqrt{M^2_{11} M^2_{22}} > 0. \tag{4}$$

For 3×3 matrix we have [1]

$$M^2_{11} > 0, \quad M^2_{22} > 0, \quad M^2_{33} > 0, \tag{5}$$

$$M^2_{12} + \sqrt{M^2_{11} M^2_{22}} > 0, \tag{6}$$

$$M^2_{13} + \sqrt{M^2_{11} M^2_{33}} > 0, \tag{7}$$

$$M^2_{23} + \sqrt{M^2_{22} M^2_{33}} > 0, \tag{8}$$

and

$$\sqrt{M^2_{11} M^2_{22} M^2_{33}} + M^2_{12}\sqrt{M^2_{33}} + M^2_{13}\sqrt{M^2_{22}} + M^2_{23}\sqrt{M^2_{11}} > 0, \tag{9}$$

$$det M^2 = M^2_{11} M^2_{22} M^2_{33} - (M^4_{12} M^2_{33} + M^4_{13} M^2_{22} + M^4_{23} M^2_{11}) + 2 M^2_{12} M^2_{13} M^2_{23} > 0. \tag{10}$$

For the matrices of rank 4 or 5. the reader is referred to Refs. [3,4].

One of the main purposes of the models based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$ (for short, 3-3-1 model) [5,6] is concerned with the search of an explanation for the number of generations of fermions. Combined with the QCD asymptotic freedom, the 3-3-1 models provide an explanation for the number of fermion generations. To provide an explanation for the observed pattern of SM fermion masses and mixings, various 3-3-1 models with flavor symmetries [7–9]and radiative seesaw mechanisms [7,12] have been proposed in the literature. However, some of them involve non-renormalizable interactions [10], others are renormalizable but do not address the observed pattern of fermion masses and mixings due to the unexplained huge hierarchy among the Yukawa couplings [8] and others are only focused either in the quark mass hierarchy [8,11], or in the study of the neutrino sector [12,13], or only include the description of SM fermion mass hierarchy, without addressing the mixings in the fermion sector [14].

It is interesting to find an alternative explanation for the observed SM fermion mass and mixing pattern. The first renormalizable extension of the 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$, which explains the SM fermion mass hierarchy by a sequential loop suppression has been done in Ref. [15]. This model is called by the 3-3-1 model with Carcamo-Kovalenko-Schmidt (CKS) mechanism.

The aim of this paper is to apply the procedure in (2) for the recently proposed 3-3-1 model with CKS mechanism.

The further content of this paper is as follows. In Sect. **II**, we briefly present particle content of scalar sector and spontaneous symmetry breaking (SSB) of the model. The Higgs sector is considered in Sect. **III**. The Higgs sector consists of two parts: the first part contains lepton number conserving terms and the second one is lepton number violating. We study in details the first part and show that the Higgs sector has all necessary ingredients. We make conclusions in Sect. **IV**.

II. SCALAR FIELDS OF THE MODEL

In the model under consideration, the Higgs sector contains three scalar triplets: χ, η and ρ and seven singlets φ_1^0, φ_2^0, ξ^0, ϕ_1^+, ϕ_2^+, ϕ_3^+ and ϕ_4^+. Hence, the scalar spectrum of the model is composed of the following fields

$$\begin{aligned}
\chi &= \langle\chi\rangle+\chi' \sim \left(1,3,-\frac{1}{3}\right), \qquad (11)\\
\langle\chi\rangle &= \left(0,0,\frac{v_\chi}{\sqrt{2}}\right)^T, \quad \chi' = \left(\chi_1^0,\chi_2^-,\frac{1}{\sqrt{2}}(R_{\chi_3^0}-iI_{\chi_3^0})\right)^T,\\
\rho &= \left(\rho_1^+,\frac{1}{\sqrt{2}}(R_\rho - iI_\rho),\rho_3^+\right)^T \sim \left(1,3,\frac{2}{3}\right),\\
\eta &= \langle\eta\rangle+\eta' \sim \left(1,3,-\frac{1}{3}\right),\\
\langle\eta\rangle &= \left(\frac{v_\eta}{\sqrt{2}},0,0\right)^T, \quad \eta' = \left(\frac{1}{\sqrt{2}}(R_{\eta_1^0}-iI_{\eta_1^0}),\eta_2^-,\eta_3^0\right)^T,\\
\varphi_1^0 &\sim (1,1,0), \qquad \varphi_2^0 \sim (1,1,0),\\
\phi_1^+ &\sim (1,1,1), \quad \phi_2^+ \sim (1,1,1), \quad \phi_3^+ \sim (1,1,1), \quad \phi_4^+ \sim (1,1,1),\\
\xi^0 &= \langle\xi^0\rangle+\xi^{0'},\langle\xi^0\rangle = \frac{v_\xi}{\sqrt{2}},\xi^{0'} = \frac{1}{\sqrt{2}}(R_{\xi^0}-iI_{\xi^0}) \sim (1,1,0). \qquad (12)
\end{aligned}$$

The $Z_4\times Z_2$ assignments of the scalar fields are shown in Table 1.

Table 1. Scalar assignments under $Z_4\times Z_2$

	χ	η	ρ	φ_1^0	φ_2^0	ϕ_1^+	ϕ_2^+	ϕ_3^+	ϕ_4^+	ξ^0
Z_4	1	1	-1	-1	i	i	-1	-1	1	1
Z_2	-1	-1	1	1	1	1	1	-1	-1	1

The fields with nonzero lepton number are presented in Table 2. Note that the three gauge singlet neutral leptons N_{iR} as well as the elements in the third component of the lepton triplets, namely ν_{iL}^c have lepton number equal to -1.

Table 2. Nonzero lepton number L of fields

	$T_{L,R}$	$J_{1L,R}$	$J_{2L,R}$	ν^c_{iL}	$e_{iL,R}$	$E_{iL,R}$	N_{iR}	Ψ_R	χ^0_1	χ^+_2	η^0_3	ρ^+_3	ϕ^+_2	ϕ^+_3	ϕ^+_4	ξ^0	$i=1,2,3$
L	-2	2	2	-1	1	1	-1	1	2	2	-2	-2	-2	-2	-2	-2	

III. THE SCALAR POTENTIAL

The renormalizable potential contains three parts [16]. The first part is given by

$$
\begin{aligned}
V_{LNC} &= \mu^2_\chi\chi^\dagger\chi+\mu^2_\rho\rho^\dagger\rho+\mu^2_\eta\eta^\dagger\eta+\sum_{i=1}^{4}\mu^2_{\phi^+_i}\phi^+_i\phi^-_i+\sum_{i=1}^{2}\mu^2_{\varphi_i}\varphi^0_i\varphi^{0*}_i+\mu^2_\xi\xi^{0*}\xi^0\\
&+ \chi^\dagger\chi(\lambda_{13}\chi^\dagger\chi+\lambda_{18}\rho^\dagger\rho+\lambda_5\eta^\dagger\eta)+\rho^\dagger\rho(\lambda_{14}\rho^\dagger\rho+\lambda_6\eta^\dagger\eta)+\lambda_{17}(\eta^\dagger\eta)^2\\
&+ \lambda_7(\chi^\dagger\rho)(\rho^\dagger\chi)+\lambda_8(\chi^\dagger\eta)(\eta^\dagger\chi)+\lambda_9(\rho^\dagger\eta)(\eta^\dagger\rho)\\
&+ \chi^\dagger\chi\left(\sum_{i=1}^{4}\lambda^{\chi\phi}_i\phi^+_i\phi^-_i+\sum_{i=1}^{2}\lambda^{\chi\varphi}_i\varphi^0_i\varphi^{0*}_i+\lambda_{\chi\xi}\xi^{0*}\xi^0\right)\\
&+ \rho^\dagger\rho\left(\sum_{i=1}^{4}\lambda^{\rho\phi}_i\phi^+_i\phi^-_i+\sum_{i=1}^{2}\lambda^{\rho\varphi}_i\varphi^0_i\varphi^{0*}_i+\lambda_{\rho\xi}\xi^{0*}\xi^0\right)\\
&+ \eta^\dagger\eta\left(\sum_{i=1}^{4}\lambda^{\eta\phi}_i\phi^+_i\phi^-_i+\sum_{i=1}^{2}\lambda^{\eta\varphi}_i\varphi^0_i\varphi^{0*}_i+\lambda_{\eta\xi}\xi^{0*}\xi^0\right)\\
&+ \sum_{i=1}^{4}\phi^+_i\phi^-_i\left(\sum_{j=1}^{4}\lambda^{\phi\phi}_{ij}\phi^+_j\phi^-_j+\sum_{j=1}^{2}\lambda^{\phi\varphi}_{ij}\varphi^0_j\varphi^{0*}_j+\lambda^{\phi\xi}_i\xi^{0*}\xi^0\right)\\
&+ \sum_{i=1}^{2}\varphi^0_i\varphi^{0*}_i\left(\sum_{j=1}^{2}\lambda^{\varphi\varphi}_{ij}\varphi^0_j\varphi^{0*}_j+\lambda^{\varphi\xi}_i\xi^{0*}\xi^0\right)+\lambda_\xi(\xi^{0*}\xi^0)^2\\
&+ \Big\{\lambda_{10}\left(\phi^+_2\right)^2\left(\phi^-_3\right)^2+\lambda_{11}\left(\phi^+_2\right)^2\left(\phi^-_4\right)^2+\lambda_{12}\left(\phi^+_3\right)^2\left(\phi^-_4\right)^2\\
&+ w_1\left(\varphi^0_2\right)^2\varphi^0_1+w_2\chi^\dagger\rho\phi^-_3+w_3\eta^\dagger\chi\xi^0+w_4\left(\varphi^0_2\right)^2\varphi^{0*}_1+w_5\phi^+_3\phi^-_4\varphi^0_1+w_6\phi^+_3\phi^-_4\varphi^{0*}_1\\
&+ \chi\rho\eta(\lambda_1\varphi^0_1+\lambda_2\varphi^{0*}_1)+\chi^\dagger\rho\phi^-_4\left(\lambda_{15}\varphi^0_1+\lambda_{16}\varphi^{0*}_1\right)+\lambda_3\eta^\dagger\rho\phi^-_3\xi^0+\lambda_4\phi^+_1\phi^-_2\varphi^0_2\xi^0\\
&+ \left(\lambda_{19}\phi^-_3\phi^+_4+\lambda_{20}\phi^+_3\phi^-_4\right)\left(\varphi^0_2\right)^2+\lambda_{21}\left(\varphi^0_1\right)^3\varphi^{0*}_1\\
&+ \Bigg(\lambda_{22}\chi^\dagger\chi+\lambda_{23}\rho^\dagger\rho+\lambda_{24}\eta^\dagger\eta+\sum_{i=1}^{4}\lambda_{61i}\phi^+_i\phi^-_i+\sum_{i=1}^{2}\lambda_{62i}\varphi^0_i\varphi^{0*}_i\\
&+ \lambda_{25}\xi^{0*}\xi^0\Big)(\varphi^0_1)^2+h.c.\Big\}
\end{aligned}
\tag{13}
$$

The second part is a lepton number violating one (the subgroup $U(1)_{L_g}$ is violated) and the third breaking softly $Z_4\times Z_2$ are given in Ref. [16].

Expanding the Higgs potential around VEVs, ones get the constraint conditions at the **tree** levels as follows

$$w_3 = 0, \tag{14}$$

$$\begin{aligned}
-\mu_\chi^2 &= v_\chi^2\lambda_{13} + \frac{1}{2}v_\eta^2\lambda_5 + \frac{1}{2}\lambda_{\chi\xi}v_\xi^2\,,\\
-\mu_\eta^2 &= v_\eta^2\lambda_{17} + \frac{1}{2}v_\chi^2\lambda_5 + \frac{1}{2}\lambda_{\eta\xi}v_\xi^2\,,\\
-\mu_\xi^2 &= \frac{1}{2}\lambda_{\chi\xi}v_\chi^2 + \frac{1}{2}\lambda_{\eta\xi}v_\eta^2\,.
\end{aligned} \tag{15}$$

Applying the constraint conditions in (14), the charged scalar sector contains two massless fields: η_2^+ and χ_2^+ which are Goldstone bosons eaten by the W^+ and Y^+ gauge bosons, respectively. The other massive fields are ϕ_1^+, ϕ_2^+ and ϕ_4^+ with respective masses

$$\begin{aligned}
m^2_{\phi_1^+} &= \mu^2_{\phi_1^+} + \frac{1}{2}\left[v_\chi^2\lambda_1^{\chi\phi} + v_\eta^2\lambda_1^{\eta\phi} + v_\xi^2\lambda_1^{\phi\xi}\right],\\
m^2_{\phi_2^+} &= \mu^2_{\phi_2^+} + \frac{1}{2}\left[v_\chi^2\lambda_2^{\chi\phi} + v_\eta^2\lambda_2^{\eta\phi} + v_\xi^2\lambda_2^{\phi\xi}\right],\\
m^2_{\phi_4^+} &= \mu^2_{\phi_4^+} + \frac{1}{2}\left[v_\chi^2\lambda_4^{\chi\phi} + v_\eta^2\lambda_4^{\eta\phi} + v_\xi^2\lambda_4^{\phi\xi}\right].
\end{aligned} \tag{16}$$

In addition, in the basis (ρ_1^+ , ρ_3^+ , ϕ_3^+), there is the mass mixing matrix

$$M^2_{charged} = \begin{pmatrix} A + \frac{1}{2}v_\eta^2\,(\lambda_6+\lambda_9) & 0 & \frac{1}{2}v_\eta v_\xi\lambda_3\\ 0 & A + \frac{1}{2}\left(v_\chi^2\lambda_7 + v_\eta^2\lambda_6\right) & \frac{1}{\sqrt{2}}v_\chi w_2\\ \frac{1}{2}v_\eta v_\xi\lambda_3 & \frac{1}{\sqrt{2}}v_\chi w_2 & \mu^2_{\phi_3^+} + B_3 \end{pmatrix}, \tag{17}$$

where we have used the following notations

$$\begin{aligned}
A &\equiv \mu_\rho^2 + \frac{1}{2}\left[v_\chi^2\lambda_{18} + \lambda_{\rho\xi}v_\xi^2\right],\\
B_i &\equiv \frac{1}{2}\left(v_\chi^2\lambda_i^{\chi\phi} + v_\eta^2\lambda_i^{\eta\phi} + v_\xi^2\lambda_i^{\phi\xi}\right), \quad i = 1,2,3,4\,.
\end{aligned} \tag{18}$$

The conditions in Eqs. (4 - 7) yield

$$A + \frac{1}{2}v_\eta^2\,(\lambda_6+\lambda_9) > 0\,, A + \frac{1}{2}\left(v_\chi^2\lambda_7 + v_\eta^2\lambda_6\right) > 0\,, \mu^2_{\phi_3^+} + B_3 > 0\,, \tag{19}$$

$$\begin{aligned}
&\sqrt{\left(A + \frac{1}{2}v_\eta^2\,(\lambda_6+\lambda_9)\right)\left(A + \frac{1}{2}\left(v_\chi^2\lambda_7 + v_\eta^2\lambda_6\right)\right)} > 0\,,\\
&\frac{1}{2}v_\eta v_\xi\lambda_3 + \sqrt{\left(A + \frac{1}{2}v_\eta^2\,(\lambda_6+\lambda_9)\right)\left(\mu^2_{\phi_3^+} + B_3\right)} > 0\,,\\
&\frac{1}{\sqrt{2}}v_\chi w_2 + \sqrt{\left(A + \frac{1}{2}\left(v_\chi^2\lambda_7 + v_\eta^2\lambda_6\right)\right)\left(\mu^2_{\phi_3^+} + B_3\right)} > 0\,.
\end{aligned} \tag{20}$$

Note that in this case the constraints in (19) are just enough or other word speaking, if the conditions of semi-definition for *diagonal elements* are fulfilled then other ones are automatically satisfied.

Now we turn into CP-odd Higgs sector. There are three massless fields: I_χ ,I_η and I_{ξ^0}. The field I_{φ_2} has the following squared mass

$$m^2_{I_{\varphi_2}} = \mu^2_{\varphi_2} + B'_2\,, \tag{21}$$

where

$$B'_n \equiv \frac{1}{2}\left(v^2_\chi\lambda^{\chi\varphi}_n + v^2_\eta\lambda^{\eta\varphi}_2 + v^2_\xi\lambda^{\varphi\xi}_n\right), \quad n = 1,2\,. \tag{22}$$

There are other two mass matrices as follows: Firstly, in the basis $(I_{\chi^0_1}, I_{\eta^0_3})$, the matrix is

$$m^2_{CPodd1} = \frac{\lambda_8}{2}\begin{pmatrix} v^2_\eta & -v_\chi v_\eta \\ -v_\chi v_\eta & v^2_\chi \end{pmatrix}. \tag{23}$$

The matrix in (23) provides two physical states

$$\begin{aligned} G_1 &= \cos\theta_a I_{\chi^0_1} + \sin\theta_a I_{\eta^0_3}\,, \\ A_1 &= -\sin\theta_a I_{\chi^0_1} + \cos\theta_a I_{\eta^0_3}\,, \end{aligned} \tag{24}$$

where

$$\tan\theta_a = \frac{v_\eta}{v_\chi}\,. \tag{25}$$

The field G_1 is massless while the field A_1 has mass as follows

$$m^2_{A_1} = \frac{\lambda_8 v^2_\chi}{2\cos^2\theta_a}\,. \tag{26}$$

Secondly, in the basis (I_{φ_1}, I_ρ), the matrix is

$$m^2_{CPodd2} = \begin{pmatrix} \mu^2_{\varphi_1} - C + B_1 & \frac{1}{2}v_\chi v_\eta(\lambda_1 - \lambda_2) \\ \frac{1}{2}v_\chi v_\eta(\lambda_1 - \lambda_2) & A + \frac{\lambda_6}{2}v^2_\eta \end{pmatrix}, \tag{27}$$

where we have denoted

$$C \equiv v^2_\chi\lambda_{22} + v^2_\eta\lambda_{24} + v^2_\xi\lambda_{25} \tag{28}$$

The conditions in (4) yield

$$\mu^2_{\varphi_1} - C + B_1 > 0\,, \quad A + \frac{\lambda_6}{2}v^2_\eta > 0\,, \tag{29}$$

$$\frac{1}{2}v_\chi v_\eta(\lambda_1 - \lambda_2) + \sqrt{\left(\mu^2_{\varphi_1} - C + B_1\right)\left(A + \frac{\lambda_6}{2}v^2_\eta\right)} > 0\,. \tag{30}$$

The above conditions provide the following constraints:

i) If $\lambda_1 < \lambda_2$, then

$$\left(\mu^2_{\varphi_1} - C + B_1\right)\left(A + \frac{\lambda_6}{2}v^2_\eta\right) > \frac{v^2_\chi v^2_\eta}{4}(\lambda_1 - \lambda_2)^2\,.$$

ii) If $\lambda_1 > \lambda_2$, there are only conditions given in (30).

Generally, physical states of matrix (27) are

$$\begin{pmatrix} A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} \cos\theta_\rho & \sin\theta_\rho \\ -\sin\theta_\rho & \cos\theta_\rho \end{pmatrix} \begin{pmatrix} I_{\varphi_1} \\ I_\rho \end{pmatrix}, \tag{31}$$

where the mixing angle is given by

$$\tan 2\theta_\rho = \frac{v_\chi v_\eta (\lambda_1 - \lambda_2)}{\left(\mu^2_{\varphi_1} - C + B_1 - A - \frac{\lambda_6}{2} v^2_\eta\right)}, \tag{32}$$

and their squared masses as follows

$$\begin{aligned} m^2_{A_2} &= \frac{1}{2}\left\{A + D_1 - \sqrt{(A-D_1)^2 + v^2_\eta\left[2(A-D_1)\lambda_6 + v^2_\eta \lambda^2_6 + v^2_\chi(\lambda_{13}-\lambda_{14})^2\right]}\right\}, \\ m^2_{A_3} &= \frac{1}{2}\left\{A + D_1 + \sqrt{(A-D_1)^2 + v^2_\eta\left[2(A-D_1)\lambda_6 + v^2_\eta \lambda^2_6 + v^2_\chi(\lambda_{13}-\lambda_{14})^2\right]}\right\}, \end{aligned} \tag{33}$$

where

$$D_1 = \mu^2_{\varphi_1} + B_1 - C + \frac{1}{2} v^2_\eta \lambda_6 . \tag{34}$$

Next, the CP-even scalar sector is our task. Ones have one massive field, namely R_{φ_2} with mass

$$\begin{aligned} m^2_{R_{\varphi_2}} = m^2_{I_{\varphi_2}} &= \mu^2_{\varphi_2} + B'_2 \\ &= \mu^2_{\varphi_2} + \frac{1}{2}\left(v^2_\chi \lambda^{\chi\varphi}_2 + v^2_\eta \lambda^{\eta\varphi}_2 + v^2_\xi \lambda^{\varphi\xi}_2\right). \end{aligned} \tag{35}$$

As mentioned in Ref. [15], the lightest scalar φ^0_2 is possible DM candidate. Therefore from (35), the following condition is reasonable

$$\mu^2_{\varphi_2} = -\frac{1}{2}\left(v^2_\chi \lambda^{\chi\varphi}_2 + v^2_\xi \lambda^{\varphi\xi}_2\right). \tag{36}$$

In this case, the model contains the complex scalar DM φ^0_2 with mass

$$m^2_{R_{\varphi_2}} = m^2_{I_{\varphi_2}} = \frac{1}{2} v^2_\eta \lambda^{\eta\varphi}_2 . \tag{37}$$

Other three mass matrices are

iii) In the basis $(R_{\chi^0_1}, R_{\eta^0_3})$, the matrix is

$$m^2_{CPeven1} = \frac{\lambda_8}{2}\begin{pmatrix} v^2_\eta & v_\chi v_\eta \\ v_\chi v_\eta & v^2_\chi \end{pmatrix}. \tag{38}$$

This matrix is completely similar to that in (23). Thus, two physical states are

$$\begin{aligned} R_{G_1} &= \cos\theta_a R_{\chi^0_1} + \sin\theta_a R_{\eta^0_3}, \\ H_1 &= -\sin\theta_a R_{\chi^0_1} + \cos\theta_a R_{\eta^0_3}, \end{aligned} \tag{39}$$

where R_{G_1} is massless while the field H_2 has mass as follows

$$m^2_{H_1} = m^2_{A_1} = \frac{\lambda_8 v^2_\chi}{2\cos^2\theta_a}. \tag{40}$$

iV) In the basis $(R_\rho\,, R_{\varphi_1})$, the matrix is

$$m^2_{CPeven2} = \begin{pmatrix} A+\frac{\lambda_6}{2}v_\eta^2 & -\frac{1}{2}v_\chi v_\eta(\lambda_1+\lambda_2) \\ -\frac{1}{2}v_\chi v_\eta(\lambda_1+\lambda_2) & \mu^2_{\varphi_1}+C+B_1 \end{pmatrix}. \tag{41}$$

As before, ones get

$$A+\frac{\lambda_6}{2}v_\eta^2 > 0\,, \quad \mu^2_{\varphi_1}+C+B_1 > 0\,, \tag{42}$$

$$-\frac{1}{2}v_\chi v_\eta(\lambda_1+\lambda_2)+\sqrt{\left(A+\frac{\lambda_6}{2}v_\eta^2\right)\left(\mu^2_{\varphi_1}+C+B_1\right)} > 0\,. \tag{43}$$

Thus, if $\lambda_1+\lambda_2 > 0$, then

$$\left(A+\frac{\lambda_6}{2}v_\eta^2\right)\left(\mu^2_{\varphi_1}+C+B_1\right) > \frac{v_\chi^2 v_\eta^2}{4}(\lambda_6+\lambda_2)^2\,. \tag{44}$$

If $\lambda_1+\lambda_2 \leq 0$, there are only conditions in (42).

The physical states of matrix (41) are

$$\begin{pmatrix} H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{pmatrix}\begin{pmatrix} R_\rho \\ R_{\varphi_1} \end{pmatrix}, \tag{45}$$

where the mixing angle is given by

$$\tan 2\theta_r = \frac{v_\chi v_\eta(\lambda_1+\lambda_2)}{\left(\mu^2_{\varphi_1}+C+B_1-A-\frac{\lambda_6}{2}v_\eta^2\right)}, \tag{46}$$

and their squared masses are identified by

$$\begin{aligned} m^2_{H_2} &= \frac{1}{2}\left\{A+D_2-\sqrt{(A-D_2)^2+v_\eta^2\left[2(A-D_2)\lambda_6+v_\eta^2\lambda_6^2+v_\chi^2(\lambda_{13}+\lambda_{14})^2\right]}\right\}, \\ m^2_{H_3} &= \frac{1}{2}\left\{A+D_2+\sqrt{(A-D_2)^2+v_\eta^2\left[2(A-D_2)\lambda_6+v_\eta^2\lambda_6^2+v_\chi^2(\lambda_{13}+\lambda_{14})^2\right]}\right\}, \end{aligned} \tag{47}$$

where

$$D_2 = \mu^2_{\varphi_1}+B_1+C+\frac{1}{2}v_\eta^2\lambda_6\,. \tag{48}$$

v) In the basis $(R_\chi\,, R_\eta\,, R_{\xi^0})$, the matrix is

$$m^2_{CPeven3} = \begin{pmatrix} 2v_\chi^2\lambda_{13} & v_\chi v_\eta\lambda_5 & \lambda_{\chi\xi}v_\chi v_\xi \\ v_\chi v_\eta\lambda_5 & 2v_\eta^2\lambda_{17} & \lambda_{\eta\xi}v_\eta v_\xi \\ \lambda_{\chi\xi}v_\chi v_\xi & \lambda_{\eta\xi}v_\eta v_\xi & 2\lambda_\xi v_\xi^2 \end{pmatrix}. \tag{49}$$

Again, in this case the constraints in Eqs (4 - 9) are given by

$$\lambda_{13} > 0\,, \lambda_{17} > 0\,, \lambda_{\xi} > 0\,, \tag{50}$$

$$v_\chi v_\eta \lambda_5 + \sqrt{\left(2v_\chi^2\lambda_{13}\right)\left(2v_\eta^2\lambda_{17}\right)} > 0 \Rightarrow \lambda_5 > -2\sqrt{(\lambda_{13}\lambda_{17})}\,, \tag{51}$$

$$\lambda_{\chi\xi} v_\chi v_\xi + \sqrt{\left(2v_\chi^2\lambda_{13}\right)\left(2\lambda_\xi v_\xi^2\right)} > 0 \Rightarrow \lambda_{\chi\xi} > -2\sqrt{(\lambda_{13}\lambda_\xi)}\,, \tag{52}$$

$$\lambda_{\eta\xi} v_\eta v_\xi + \sqrt{\left(2v_\eta^2\lambda_{17}\right)\left(2\lambda_\xi v_\xi^2\right)} > 0 \Rightarrow \lambda_{\eta\xi} > -2\sqrt{(\lambda_{17}\lambda_\xi)}\,, \tag{53}$$

$$\begin{aligned} &\sqrt{\left(2v_\chi^2\lambda_{13}\right)\left(2v_\eta^2\lambda_{17}\right)\left(2\lambda_\xi v_\xi^2\right)} + v_\chi v_\eta \lambda_5 \sqrt{\left(2\lambda_\xi v_\xi^2\right)} + \lambda_{\chi\xi} v_\chi v_\xi \sqrt{\left(2v_\eta^2\lambda_{17}\right)} \\ &+\lambda_{\eta\xi} v_\eta v_\xi \sqrt{\left(2v_\chi^2\lambda_{13}\right)} > 0 \\ &\Rightarrow 2\sqrt{\lambda_{13}\lambda_{17}\lambda_\xi} + \lambda_5\sqrt{\lambda_\xi} + \lambda_{\chi\xi}\sqrt{\lambda_{17}} + \lambda_{\eta\xi}\sqrt{\lambda_{13}} > 0\,, \end{aligned} \tag{54}$$

$$\begin{aligned} &\left(2v_\chi^2\lambda_{13}\right)\left(2v_\eta^2\lambda_{17}\right)\left(2\lambda_\xi v_\xi^2\right) - [(v_\chi v_\eta\lambda_5)^2\left(2\lambda_\xi v_\xi^2\right) + (\lambda_{\chi\xi} v_\chi v_\xi)^2\left(2v_\eta^2\lambda_{17}\right) \\ &+ \left(2v_\chi^2\lambda_{13}\right)(\lambda_{\eta\xi} v_\eta v_\xi)^2] + 2v_\chi v_\eta\lambda_5.\lambda_{\chi\xi} v_\chi v_\xi.\lambda_{\eta\xi} v_\eta v_\xi > 0 \\ &\Rightarrow 4\lambda_{13}\lambda_{17}\lambda_\xi - [(\lambda_5)^2\lambda_\xi + (\lambda_{\chi\xi})^2\lambda_{17} + (\lambda_{\eta\xi})^2\lambda_{13}] + \lambda_5.\lambda_{\chi\xi}.\lambda_{\eta\xi} > 0\,. \end{aligned} \tag{55}$$

III.1. Special cases

To find solutions in Higgs sector, we should make some simplifications.

III.1.1. *The SM-like Higgs boson*

We consider now the matrix (49): with the basis $(R_\chi\,, R_\eta\,, R_{\xi^0})$

$$m^2_{CPeven3} = \begin{pmatrix} 2v_\chi^2\lambda_{13} & v_\chi v_\eta\lambda_5 & \lambda_{\chi\xi} v_\chi v_\xi \\ v_\chi v_\eta\lambda_5 & 2v_\eta^2\lambda_{17} & \lambda_{\eta\xi} v_\eta v_\xi \\ \lambda_{\chi\xi} v_\chi v_\xi & \lambda_{\eta\xi} v_\eta v_\xi & 2\lambda_\xi v_\xi^2 \end{pmatrix}. \tag{56}$$

Let us assume a simplified worth to be considered scenario which is characterized by the following relations:

$$\lambda_5 = \lambda_{13} = \lambda_{17} = \lambda_\xi = \lambda_{\chi\xi} = \lambda_{\eta\xi} = \lambda\,, \qquad v_\xi = v_\chi. \tag{57}$$

The system of Eqs.(48 - 53) leads to another constraint, namely

$$\sqrt{\lambda} > 0\,. \tag{58}$$

In this scenario, the squared matrix (49) for the electrically neutral CP even scalars in the basis $(R_\eta\,, R_\chi\,, R_{\xi^0})$ takes the simple form:

$$m^2_{CPeven3} = \lambda \begin{pmatrix} 2x^2 & x & x \\ x & 2 & 1 \\ x & 1 & 2 \end{pmatrix} v_\chi^2\,, \qquad x = \frac{v_\eta}{v_\chi}\,. \tag{59}$$

In this scenario, we find the that the physical scalars included in the matrix $m^2_{CPeven3}$ are:

$$\begin{pmatrix} h \\ H_4 \\ H_5 \end{pmatrix} \simeq \begin{pmatrix} -1+\frac{x^2}{9} & \frac{x}{3} & \frac{x}{3} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \frac{\sqrt{2}}{3}x & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} R_\eta \\ R_\chi \\ R_{\xi^0} \end{pmatrix}, \tag{60}$$

where h is the 126 GeV SM like Higgs boson. Thus, we find that the SM-like Higgs boson h has couplings very close to SM expectation with small deviations of the order of $\frac{v_\eta^2}{v_\chi^2}$. In addition, the squared masses of the physical scalars included in the matrix $m^2_{CPeven3}$ take the form:

$$m_h^2 \simeq \frac{4}{3}\lambda v_\eta^2, \qquad m_{H_4}^2 \simeq \lambda v_\chi^2, \qquad m_{H_5}^2 \simeq 3\lambda v_\chi^2. \tag{61}$$

Taking into account the fact that mass of the SM Higgs boson is equal to 126 GeV, from (61) we obtain

$$\lambda \approx 0.187\,. \tag{62}$$

Combining with the limit from the rho parameter in Ref. [16]

$$3.57\,\mathrm{TeV} \leq v_\chi \leq 6.9\,\mathrm{TeV}$$

yields

$$\begin{aligned} 1.5\,\mathrm{TeV} &\leq m_{H_4} \leq 2.61\,\mathrm{TeV}\,, \\ 2.6\,\mathrm{TeV} &\leq m_{H_5} \leq 4.5\,\mathrm{TeV}\,. \end{aligned} \tag{63}$$

III.1.2. *The charged Higgs bosons*

The charged scalar sector contains two massless fields: G_{W^+} and G_{Y^+} which are Goldstone bosons eaten by the longitudinal components of the W^+ and Y^+ gauge bosons, respectively. The other massive fields are ϕ_1^+, ϕ_2^+ and ϕ_4^+ with respective masses given in (18).

In the basis $(\rho_1^+, \rho_3^+, \phi_3^+)$, the squared mass matrix is given in (17). Let us make effort to simplify this matrix. Note that μ_χ^2, μ_η^2, and μ_ξ^2 can be derived using relations (14) and (57). In addition, it is reasonable to assume

$$\mu_\rho^2 = -\frac{v_\chi^2}{2}(\lambda_{18}+\lambda_{\rho\xi}) \approx \mu_\eta^2\,, \quad \mu^2_{\phi_3^+} = -\frac{v_\chi^2}{2}(\lambda_2^{\chi\phi}+\lambda_2^{\phi\xi})\,, \tag{64}$$

we obtain the simple form of the squared mass matrix of the charged Higgs bosons,

$$M^2_{chargeds} = \begin{pmatrix} A+\frac{1}{2}v_\eta^2(\lambda_6+\lambda_9) & 0 & \frac{\lambda_3}{2}v_\eta v_\chi \\ 0 & \frac{1}{2}\left(v_\chi^2\lambda_7+\lambda_6 v_\eta^2\right) & \frac{1}{\sqrt{2}}v_\chi w_2 \\ \frac{\lambda_3}{2}v_\eta v_\chi & \frac{1}{\sqrt{2}}v_\chi w_2 & \frac{1}{2}v_\eta^2\lambda_2^{\eta\phi} \end{pmatrix}. \tag{65}$$

The matrix (65) predicts that there may exist two light charged Higgs bosons $H_{1,2}^+$ with masses at the electroweak scale and the mass of H_3^+ which is mainly composed of ρ_3^+ is around 3.5 TeV. In addition, the Higgs boson H_1^+ almost does not carry lepton number, whereas the others two do.

Generally, the Higgs potential always contains mass terms which mix VEVs. However, these terms must be small enough to avoid high order divergences (for examples, see Refs. [17,18]) and provide baryon asymmetry of Universe by the strong electroweak phase transition (EWPT).

Ignoring the mixing terms containing λ_3 in (65) does not affect other physical aspects, since the above mentioned terms just increase or decrease small amount of the charged Higgs bosons. Therefore, without lose of generality, neglecting the terms with λ_3 satisfies other aims such as EWPT.

Hence, in the matrix of (65), the coefficient λ_3 is reasonably assumed to be zero. Therefore we get immediately one physical field ρ_1^+ with mass given by

$$m^2_{\rho_1^+} = \frac{1}{2} v_\eta^2 (\lambda_6 + \lambda_9) . \tag{66}$$

The other fields mix by submatrix given at the bottom of (65). The limit $\rho_1^+ = H_1^+$ when $\lambda_3 = 0$ is very interesting for discussion of the Higgs contribution to the ρ parameter.

Analysis of electroweak phase transition shows that the term of VEV mixing at the top-right corner should be negligible [17, 18] or

$$\lambda_3 \simeq 0 . \tag{67}$$

Therefore, from (17), it follows that ρ_1^+ is physical field with mass

$$m^2_{\rho_1^+} = A + \frac{1}{2} v_\eta^2 \left(\lambda_6 + \lambda_9\right) , \tag{68}$$

and two massive *bilepton* scalars ρ_3^+ and ϕ_3^+ mix each other by matrix at the right-bottom corner. Taking into account the conditions in (4) yields

$$A + \frac{1}{2}\left(v_\chi^2 \lambda_7 + v_\eta^2 \lambda_6\right) > 0 , \mu^2_{\phi_3^+} + B_3 > 0 , \tag{69}$$

$$\frac{1}{\sqrt{2}} v_\chi w_2 + \sqrt{\left(A + \frac{1}{2}\left(v_\chi^2 \lambda_7 + v_\eta^2 \lambda_6\right)\right)\left(\mu^2_{\phi_3^+} + B_3\right)} > 0 . \tag{70}$$

From (70) it follows that if $w_2 < 0$, then

$$\left(A + \frac{1}{2}\left(v_\chi^2 \lambda_7 + v_\eta^2 \lambda_6\right)\right)\left(\mu^2_{\phi_3^+} + B_3\right) > \frac{v_\chi^2 w_2^2}{2} ,$$

but if $w_2 > 0$, *there are only conditions in* (69).

It is worth mentioning that the masses of three charged scalars $\phi_i^+, i = 1, 2, 4$ are still not fixed.

Let us deal with the charged Higgs boson sector by assuming

$$\lambda_6 = \lambda_7 = \lambda_9 = \lambda_{18} = \lambda_3^{\chi\phi} = \lambda_3^{\eta\phi} = \lambda_3^{\phi\xi} = \lambda' . \tag{71}$$

With this assumption, we have

$$\begin{aligned} \mu_\chi^2 &= -\frac{\lambda}{2}(3v_\chi^2 + v_\eta^2) \simeq -\frac{3}{2}\lambda v_\chi^2 , \\ \mu_\eta^2 &= -\lambda(v_\eta^2 + v_\chi^2) \simeq -\lambda v_\chi^2 , \\ \mu_\xi^2 &= -\frac{\lambda}{2}(v_\chi^2 + v_\eta^2) \simeq -\frac{1}{2}\lambda v_\chi^2 . \end{aligned} \tag{72}$$

In the basis $(\rho_1^+, \rho_3^+, \phi_3^+)$, the matrix in (17) becomes

$$M^2_{charged} = \begin{pmatrix} \mu_\rho^2+\lambda' v_\chi^2+\lambda' v_\eta^2 & 0 & \frac{\lambda'}{2}v_\eta v_\chi \\ 0 & \mu_\rho^2+\frac{\lambda'}{2}\left(3v_\chi^2+v_\eta^2\right) & \frac{1}{\sqrt{2}}v_\chi w_2 \\ \frac{\lambda'}{2}v_\eta v_\chi & \frac{1}{\sqrt{2}}v_\chi w_2 & \mu^2_{\phi_3^+}+\lambda'^2_\chi+\frac{1}{2}v_\eta^2) \end{pmatrix}. \quad (73)$$

Next, assuming

$$\mu_\rho^2 = \mu^2_{\phi_3^+} = \mu_\eta^2 = -\lambda' v_\chi^2\,, \quad (74)$$

we obtain

$$M^2_{newcharged} = \begin{pmatrix} \lambda'^2 & 0 & \frac{\lambda'}{2}x \\ 0 & \frac{\lambda'}{2}(1+x^2) & \frac{1}{\sqrt{2}}w_2 \\ \frac{\lambda'}{2}x & \frac{1}{\sqrt{2}}w_2 & \frac{\lambda'}{2}x^2 \end{pmatrix} v_\chi^2\,. \quad (75)$$

From (75), we get two charged Higgs bosons with masses at electoweak scale and one massive with mass around TeV ($\propto v_\chi$), in addition the Higgs boson composed mainly from ρ_1^+ does not carry lepton number, while the two others do.

IV. CONCLUSION

In this paper, we have applied the positivity of scalar mass spectra in the 3-3-1 model with CKS mechanism. We show that for the Higgs squared mass matrices, the conditions for positivity of the diagonal elements are most important since other constraints are followed from the first ones. In the model under consideration, the above conclusion is very helpful for the fixing parameters.

Since there are a lot of Higgs fields in this model, so the vacuum stability will be considered in the future study.

ACKNOWLEDGMENTS

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2017.356.

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