

HOLOGRAPHIC WILSON LOOPS IN THE CONFINING BACKGROUNDS AT ZERO TEMPERATURE

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Abstract. *We study Wilson loops in the exact renormalization group flow at zero temperature via the 5D holographic model that reproduces the behavior of the QCD running coupling. Calculating the time-like rectangular Wilson loop we show that the model includes a confining phase at $T = 0$ in the IR region.*

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I. INTRODUCTION

In the recent years the holographic gauge/gravity duality has been shown to be a very useful tool to get theoretical insight into systems in the strongly coupling regime [1]. Some of holographic models represent a class of useful toy models, because they can be studied analytically while having features resembling the real QCD.

The description of RG flow via holography corresponds to a gravitational domain-wall solution interpolating between boundaries. These boundaries of the solutions are supposed to be related to fixed points of a dual theory and should have particular properties. The holographic direction corresponds to the energy scale, while the dilaton is related to the running coupling. In [2] it was constructed the holographic exact RG flows for the 5D dilaton gravity. For a particular case, it was shown that the behaviour of the running coupling mimics the dependence of the QCD running coupling on the energy scale. Thus it is of interest to perform a check if the confinement takes place for the backgrounds found in [2].

The Wilson loop represents one of criteria for studies of the IR region and manifests an area law behaviour in the case of the confinement. Using the holographic prescription of Wilson loops [3] one can explore a holographic dual for possible phase transitions. The aim of this work is to calculate the expectation value of a rectangular temporal Wilson loop for the holographic RG flow constructed in [2].

Recently, Wilson loops with different orientations in holographic RG flows with anisotropy were studied in [5, 6]. Using a so-called effective potential [1] it was shown that the model has a confinement-deconfinement phase transition. In [7, 8] it was also explored holographic Wilson loops in RG flow backgrounds estimating a position of the turning point. In this work we will follow the method of the effective potential [1, 5, 6] and show that for the holographic RG flow [2] at $T = 0$ one can observe a confinement. The work is organized as follows. In section 2 we give a brief description of the holographic model and corresponding zero-temperature solutions. In section 3 we calculate the holographic temporal Wilson loop and discuss the related effective potential. In section 4 we conclude.

II. THE HOLOGRAPHIC MODEL

The 5-dimensional holographic model [2] is given by an action of the form

$$\mathcal{S} = \frac{1}{16\pi G_5} \int d^5x \sqrt{|g|} \left(R - \frac{4}{3} (\partial\phi)^2 - C_1 e^{2k_1\phi} - C_2 e^{2k_2\phi} \right) + G.H., \quad (1)$$

where C_i, k_i with $i = 1, 2$ are constants. We choose the constants as $C_1 < 0$, and $C_2 > 0$. We note that the choice of the dilaton potential is motivated by a relation to a bosonic sector of supergravity [4]. Taking the dilatonic constants as $k_1 = k, k_2 = \frac{16}{9k}$ with $0 < k < \frac{4}{3}$, one can reduce the model (1) to the Toda chains and integrate this [2]. The solutions for the metric and the dilaton at zero temperature for the model (1) are

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} (-dt^2 + d\vec{x}^2) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2, \quad (2)$$

where $\vec{x} = (x_1, x_2, x_3)$ and the corresponding dilaton

$$\phi = -\frac{9k}{9k^2-16} \log F_1 + \frac{9k}{9k^2-16} \log F_2. \quad (3)$$

The functions F_1 and F_2 in (2)-(3) are given by

$$F_1 = \sqrt{\left| \frac{C_1}{2E_1} \right|} \sinh(\mu u), \quad F_2 = \sqrt{\left| \frac{C_2}{2E_2} \right|} \sinh\left(\frac{4}{3k}\mu(u-u_0)\right), \quad \mu = \sqrt{\left| \frac{3E_1}{2} \left(k^2 - \frac{16}{9}\right) \right|}. \quad (4)$$

The parameters u_0, E_1 and E_2 are the constants of integration and one has the zero energy constraint $E_1 + E_2 = 0$, with $E_1 < 0, E_2 > 0$.

The solutions possess the Poincare invariance. It's convenient to come to the domain wall coordinates, thus the metric takes the form

$$ds^2 = e^{2\mathcal{A}(w)} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + dw^2. \quad (5)$$

We note that the change of the coordinates is given by $dw = F_1^{\frac{16}{9k^2-16}} F_2^{\frac{9k^2}{16-9k^2}} du$, so the scale factor is

$$A = \frac{4}{9k^2-16} \log F_1 + \frac{9k^2}{4(16-9k^2)} \log F_2. \quad (6)$$

The quantity $e^{\mathcal{A}}$ is supposed to be associated with the energy scale of the dual theory, so it should be monotonic. The constant of integration u_0 in (4) splits our solution into the following branches: **a)** $u < u_0$; **b)** $u_0 < u < 0$, **c)** $u > 0$, **d)** $u_0 = 0$ ¹. In [2, 9] it was discussed in details the features of this solutions. It was shown that the solution **a)** has a non-monotonic behaviour of the scale factor that precludes to give an interpretation of this solutions as a holographic RG flow. The solution **b)** obeys the condition of the monotonicity of the scale factor and moreover corresponds to a confining theory in the IR limit and a free theory in the UV limit. The solutions **c)** and **d)** have monotonic scale factors but describe theories that are free both in the UV and IR limits. In what follows we will focus on the RG flow related to the solution **a)** and omit the discussion of others solution.

We note that for the holographic RG flow corresponding to the solution **b)** the boundary with $u \sim 0$ corresponds to the UV limit and the boundary near u_0 point is related to the IR limit. Both boundaries are spacetimes with a hyperscaling violation.

III. THE RECTANGULAR TIME-LIKE WILSON LOOP

In the holographic prescription the expectation value of the rectangular temporal Wilson loop of size $T \times \ell$ can be defined through the Nambu-Goto action S_{NG} of a string stretched between two quarks in a considered background

$$\langle W \rangle \sim e^{-S_{NG}}. \quad (7)$$

In this section we discuss the Nambu-Goto action for the general solution (2) in the domain wall coordinates (5). This corresponds to consideration of Wilson loops in the holographic RG. For the string configuration we choose the following gauge [3]

$$\sigma^0 = t, \quad \sigma^1 = x_1, \quad w = w(x_1) \quad (8)$$

with $w(0) = w(\ell) = 0$, $w(\frac{\ell}{2}) = w_{min}$.

Then the induced metric has the following form

$$ds^2 = -e^{2\mathcal{A}} dt^2 + (e^{2\mathcal{A}} + w'^2) dx_1^2. \quad (9)$$

The Nambu-Goto action in the string frame can be presented in the following way

$$S_{NG} = \frac{1}{2\pi\alpha'} \int dx_1 dt e^{\frac{2\phi}{3}} \sqrt{|g|} = T \int dx_1 \left(\frac{F_2}{F_1} \right)^{\frac{6k}{9k^2-16}} \sqrt{e^{2\mathcal{A}} (e^{2\mathcal{A}} + w'^2)}. \quad (10)$$

Coming to the u -variable the distance between quarks is

$$\frac{\ell}{2} = \int_{u_*}^0 du \frac{ce^{3\mathcal{A}}}{\sqrt{e^{4\mathcal{A} + \frac{4}{3}\phi} - c^2}}. \quad (11)$$

¹In what follows we suppose that $u_0 < 0$.

For the Nambu-Goto action we have

$$\frac{S_{NG}}{2} = \frac{T}{2\pi\alpha'} \int_{u_*}^0 du \frac{e^{7\mathcal{A} + \frac{4\phi}{3}}}{\sqrt{e^{4\mathcal{A} + \frac{4\phi}{3}} - c^2}}. \quad (12)$$

As expected the Nambu-Goto action is divergent near 0 (the UV boundary). Doing the renormalization one can write

$$\frac{S_{NG}}{2} = \frac{T}{2\pi\alpha'} \int_{u_*}^0 du \left(\frac{e^{7\mathcal{A} + \frac{4\phi}{3}}}{\sqrt{e^{4\mathcal{A} + \frac{4\phi}{3}} - c^2}} - e^{5\mathcal{A} + \frac{2\phi}{3}} \right) + \int_{u_*}^{u(\Lambda)} du e^{5\mathcal{A} + \frac{2\phi}{3}}, \quad (13)$$

where we introduced the cutoff parameter $u(\Lambda)$.

One can define the so-called effective potential with $u' = 0$ as

$$V_{eff} = e^{2\mathcal{A} + \frac{2\phi}{3}} = F_1^{\frac{8-6k}{9k^2-16}} F_2^{\frac{9k^2-12k}{2(16-9k^2)}}. \quad (14)$$

Then in terms of V_{eff} the distance between two endpoints of the string can be represented as

$$\frac{\ell}{2} = \int_{u_*}^0 du e^{-\phi} \frac{V_{eff}(u) \sqrt{V_{eff}(u)}}{\sqrt{\frac{V_{eff}^2(u)}{V_{eff}^2(u_*)} - 1}} \quad (15)$$

and the Nambu-Goto action is

$$\frac{S_{NG}}{2} = \frac{T}{2\pi\alpha'} \int_{u_*}^{\epsilon} du \frac{e^{-\phi} V_{eff}^3(u) \sqrt{V_{eff}(u)}}{\sqrt{V_{eff}^2(u) - V_{eff}^2(u_*)}}, \quad (16)$$

where ϵ is a cutoff parameter. One can see that (15)-(16) make sense if V_{eff} is a decreasing function for the interval $0 < u < u_*$.

Let us suppose that

$$V'_{eff}(u)|_{u=u_{min}} = 0, \quad V''_{eff}(u_{min}) > 0. \quad (17)$$

Near the point $u = u_{min}$ we have $V''_{eff}(u_{min}) > 0$ and

$$\sqrt{\frac{V_{eff}^2(u)}{V_{eff}^2(u_{min})} - 1} = \sqrt{\frac{V''_{eff}(u_{min})}{V_{eff}(u_{min})}} (u - u_{min}) + \mathcal{O}(u - u_{min})^2. \quad (18)$$

Inserting (18) into (15)-(16) one gets for the distance between quarks

$$\begin{aligned} \frac{\ell}{2} &= \int_{u_{min}}^0 du e^{-\phi} \frac{V_{eff}^{\frac{5}{4}}(u_{min}) \left(V_{eff}(u_{min}) + V_{eff}''(u_{min})(u - u_{min})^2 \right)^{\frac{3}{4}}}{\sqrt{V_{eff}''(u_{min})(u - u_{min})}} \\ &= \frac{2V_{eff}^{5/4}(u_{min})e^{-\phi(u_{min})}}{3V_{eff}''^{3/2}(u_{min})} \left(V_{eff}(u_{min}) + V_{eff}''(u_{min})(u - u_{min})^2 \right)^{\frac{3}{4}} \\ &\times \left(V_{eff}''(u_{min}) - \frac{3V_{eff}(u_{min})}{(u - u_{min})^2} {}_2F_1\left(1, 1, \frac{5}{4}, -\frac{V_{eff}(u_{min})}{V_{eff}''(u_{min})(u - u_{min})^2}\right) \right) \end{aligned} \tag{19}$$

and for the string action

$$\begin{aligned} \frac{S}{2} &= \frac{T}{2\pi\alpha'} \int_{u_{min}}^\epsilon du e^{-\phi} \frac{V_{eff}^{\frac{3}{4}}(u_{min}) \left(V_{eff}(u_{min}) + V_{eff}''(u_{min})(u - u_{min})^2 \right)^{\frac{7}{4}}}{\sqrt{V_{eff}''(u_{min})(u - u_{min})}} \\ &= \frac{2V_{eff}^{3/4}(u_{min})e^{-\phi(u_{min})}}{7\sqrt{V_{eff}''(u_{min})}} \left(V_{eff}(u_{min}) + V_{eff}''(u_{min})(u - u_{min})^2 \right)^{\frac{3}{4}} \\ &\times \left((u - u_{min})^2 V_{eff}'' + \frac{10}{21} V_{eff}(u_{min}) \right. \\ &\left. - \frac{7V_{eff}^2(u_{min})}{(u - u_{min})^2 V_{eff}''(u_{min})} {}_2F_1\left(1, 1, \frac{5}{4}, -\frac{V_{eff}(u_{min})}{V_{eff}''(u_{min})(u - u_{min})^2}\right) \right), \end{aligned} \tag{20}$$

where ${}_2F_1$ is a Gauss hypergeometric function and $\ell \rightarrow +\infty$ as $u \rightarrow u_{min} - 0$ and $S \rightarrow \infty$ as $u \rightarrow u_{min} - 0$. Comparing (19)-(20) one can see the area law for the quark-antiquark potential. We note that the stationary point, $V_{eff}'|_{u=u_{min}} = 0$ is usually called a dynamical domain wall point and obeys the equation

$$V_{eff}' = 0. \tag{21}$$

In Fig. 1 we present the plot of V_{eff} (14) as a function of u for the holographic RG flow defined on the interval $(u_0; 0)$ with $u_0 = -1$. From

Fig. 1 we observe that V_{eff} has a minimum u_{min} in $(u_0; 0)$. It also confirms that we observe a confined phase in the background (2) corresponding to a theory with $T = 0$. We also see that for smaller values of k the minimum of the potential is closer to the IR boundary u_0 .

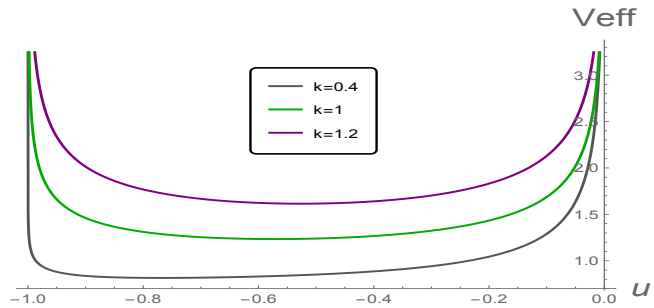


Fig. 1. The "potential" V_{eff} as a function of u for the holographic RG flows defined on $(u_0; 0)$ with $u_0 = -1$, $E_1 = E_2 = -1$, $C_1 = -C_2 = -1$, $k = 0.4, 1, 1.2$.

IV. DISCUSSION

In this paper we have discussed the 5D background of the holographic RG flow at $T = 0$ constructed in [2]. The model can resemble some QCD features, particularly, the dependence of the running coupling on the energy scale. Following the holographic prescription we have discussed a rectangular time-like Wilson loop in the RG flow background. For this purpose we have considered a string stretched between two quarks in the RG flow geometry. Knowing the action of the string we have calculated the effective potential and performed some analysis of the Nambu-Goto action and the distance between quarks near a supposed minimum of the effective potential. We have shown that the effective potential indeed has a point of minimum which tends to the IR boundary for small values of k , that defines the shape of the dilaton potential. Thus we have demonstrated that the holographic RG flow defined for the region $(u_0, 0)$ has a confinement phase.

A forthcoming problem is to study more carefully a string tension corresponding to the temporal Wilson loops. We are also going to explore the Wilson loops for holographic RG flows at finite temperature from [2] and consider a confinement-deconfinement phase transition as it was done in [6]. It would be interesting to study the Regge spectrum of mesons in these holographic models.

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