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MOBILITY EDGES IN ONE-DIMENSIONAL DISORDERED AHARONOV-BOHM RINGS

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Abstract. *We study numerically the localization properties of the eigenstates of a tight-binding Hamiltonian model for noninteracting electrons moving in a one-dimensional disordered ring pierced by an Aharonov-Bohm flux. By analyzing the dependence of the inverse participation ratio on Aharonov-Bohm flux, energy, disorder strength and system size, we find that all states in the ring are delocalized in the weak disorder limit. The states lying deeply in the band tails will undergo a continuous delocalization-localization transition as the disorder strength in the ring sweeps from the weak to the strong disorder regime.*

Keywords: Anderson transition; delocalization-localization transition; Aharonov-Bohm flux; vector potential.

Classification numbers: 72.15.Rn; 71.23.An; 72.20.Ee; 73.23.-b; 03.65.Ta.

I. INTRODUCTION

Anderson localization of quantum particles and classical waves in a random potential has been studied extensively for a long time [1–5]. Despite of a huge amount of research into this field, there still remain many unsolved problems and suprising results are being reported [6–8]. It has been widely known that in one and two dimensions, noninteracting quantum particles and classical waves are usually localized even in the presence of arbitrarily weak randomness [3, 4]. Several exceptions to this have also been known for some time. For example, a transition from power-localized to delocalized states has been found for a one-dimensional (1D) random Kronig-Penney model in the presence of a constant electric field [9]. Other interesting examples include a significant delocalization of p-polarized electromagnetic waves propagating in a randomly layered structure at a certain angle called the generalized Brewster angle [10–12]. When a short-range or long-range correlation is introduced into the random potential in one dimension, some localized states are shown to be transformed into extended ones [13–15]. In addition, extended states can also arise in 1D and 2D disordered systems when non-interacting quantum particles are subjected to either a constant imaginary vector potential [16, 17] or a constant real vector potential [18] in a closed system.

In this present paper, we revisit the problem studied in [18], where it has been shown analytically that non-interacting electrons in a 1D disordered ring threaded by an Aharonov-Bohm (AB) flux, which are subjected to a constant real vector potential, are delocalized in the weak disorder limit [18]. In the opposite limit of strong disorder, by using the modified Dean's method, the localized or extended nature of the eigenstates of a quantum particle moving in a 1D disordered AB ring has been studied numerically as well [19]. Noninteracting electrons moving in a 1D AB ring are well-known to carry persistent currents (PCs) [20–24]. It has been predicted theoretically that the averaged current amplitude decreases as the disorder strength increases [21, 22, 25–27]. There has been some discrepancy between the calculated and measured PC amplitudes. In Ref. [28], Bleszynski-Jayich and co-workers have observed that both the total magnitude of the PC and its temperature dependence are fully consistent with calculations based on a model of noninteracting electrons [29]. Such a model of free electrons has been also employed to explain the measured PCs in semiconductor ballistic rings with a few transverse channels [30]. These may provide some justification for using noninteracting models to study localization phenomena in AB rings.

The main purposes of our work is to verify the statement about the absence of localization in the ring threaded by an AB flux for sufficiently weak disorder [18] and to study the possibility of existence of delocalization - localization transition as the disorder strength in the ring sweeps from the weak to the strong disorder regime. It is important to mention that PCs in quantum rings are closely associated with the localized or extended nature of the electron eigenstates. The existence of such PCs can find some potential applications in the field of quantum information processing and quantum computing [31]. For instance, in Ref. [32] the authors have shown that quantum tunneling between states with nearly equal energies and opposite PCs in conducting (metal or semiconductor) rings with a barrier can give rise formation a flux qubit at low temperatures. This idea of formation of the flux qubit on mesoscopic rings has been proved to be still valid for the weakly interacting electrons. There is some analogy between this kind of the flux qubit and that one was built on a superconducting ring with Josephson junctions [33].

II. THEORETICAL MODEL AND FORMALISM

II.1. Model

The disordered 1D AB ring system with no electron-electron interaction may be represented by the following model Hamiltonian:

$$H = -V\sum_j (e^{i\theta} a_{j+1}^\dagger a_j + e^{-i\theta} a_j^\dagger a_{j+1}) + \sum_j \varepsilon_j a_j^+ a_j, \tag{1}$$

where ε_j is a random site energy distributed uniformly between $-W/2$ and $W/2$ and V measures the strength of hopping. The phase θ is given by

$$\theta = \frac{2\pi a}{L}\frac{\phi}{\phi_0} = \frac{2\pi}{N}\frac{\phi}{\phi_0}, \tag{2}$$

where L is the circumference of the ring and a is the lattice constant. N is the number of sites in the ring and ϕ is the total magnetic flux through the ring. ϕ_0 $(= hc/e)$ is the flux quantum with h the Planck constant, c the speed of the light and $-e$ the electron charge. $a_j^\dagger$ (a_j) is the fermionic creation (annihilation) operator at site j.

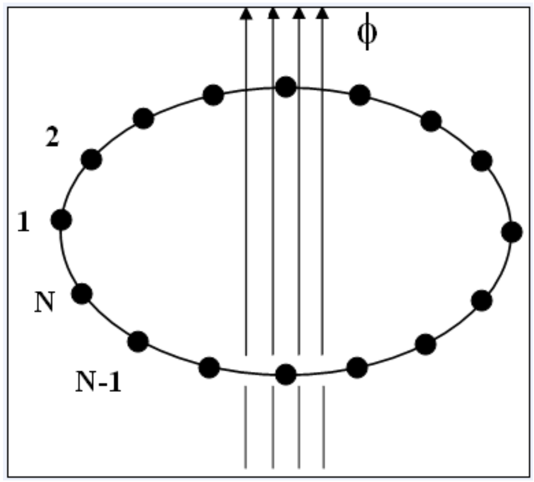

Fig. 1. One-dimensional ring of circumference L pierced by a magnetic flux ϕ.

The time-independent Schrödinger equation for noninteracting electrons follows from Eq. (1) and has the form [18]

$$\begin{aligned} -e^{i\frac{2\pi}{N}\frac{\phi}{\phi_0}}\varphi_{n+1} - e^{-i\frac{2\pi}{N}\frac{\phi}{\phi_0}}\varphi_{n-1} + \varepsilon_n\varphi_n = E\varphi_n, \\ if\ n = 2,3,...,N-1, \\ -e^{i\frac{2\pi}{N}\frac{\phi}{\phi_0}}\varphi_2 - e^{-i\frac{2\pi}{N}\frac{\phi}{\phi_0}}\varphi_N + \varepsilon_1\varphi_1 = E\varphi_1, \\ -e^{i\frac{2\pi}{N}\frac{\phi}{\phi_0}}\varphi_1 - e^{-i\frac{2\pi}{N}\frac{\phi}{\phi_0}}\varphi_{N-1} + \varepsilon_N\varphi_N = E\varphi_N, \end{aligned} \tag{3}$$

where φ_n is the amplitude of an eigenstate wave function at site n and E is the energy eigenvalue. Our system is schematically illustrated in Fig. 1.

II.2. Inverse participation ratio

Eq. (3) defines an eigenvalue problem of the form

$$A\psi = E\psi, \tag{4}$$

where $\psi = (\varphi_1,\ \varphi_2, \cdots,\ \varphi_N)^{\mathrm{T}}$ and A is a sparse $N\times N$ constructed from the coefficients of φ_n's on the left-hand side of Eq. (3). We solve Eq. (4) numerically and obtain the N eigenvalues and the corresponding eigenfunctions.

In order to study the localized or extended nature of the eigenstates, we calculate a quantity called the inverse participation ratio (IPR). For the k-th eigenfunction $(\varphi_1^{(k)},\ \varphi_2^{(k)}, \cdots,\ \varphi_N^{(k)})^{\mathrm{T}}$ with the eigenvalue E_k, the IPR is defined by [34]

$$IPR(E_k) = \frac{\sum_{n=1}^{N} |\varphi_n^{(k)}|^4}{(\sum_{n=1}^{N} |\varphi_n^{(k)}|^2)^2}, \tag{5}$$

which estimates the degree of spatial extension or localization of eigenstates. For an infinite system, the IPR varies from 0 for extremely delocalized states to 1 for extremely localized ones. For a finite system, it behaves like $1/N$ as N increases for delocalized states spreading uniformly over the entire system. On the other hand, localized states exhibit much higher values, which do not vary much as N is increased. Therefore, the finite-size scaling analysis of the IPR gives a very useful information about the localized or extended nature of the eigenstates.

III. NUMERICAL RESULTS

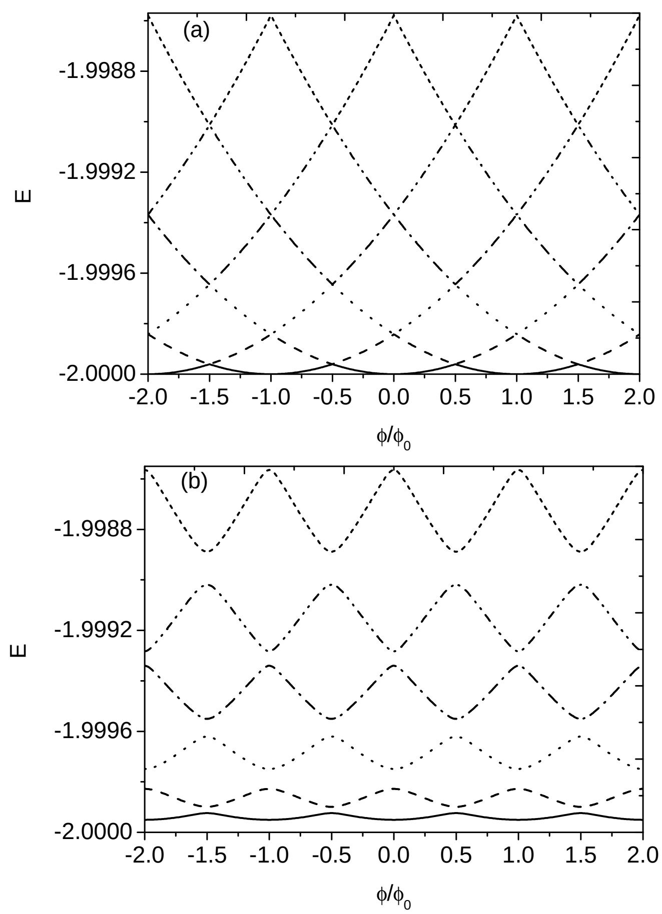

Fig. 2. Electron energy E versus normalized magnetic flux ϕ/ϕ_0 for the ground and five lowest excited states when (a) $W = 0$ and (b) $W = 0.005$.

It has been widely accepted [21, 22, 25–27] that the disorder suppresses the PC strongly in metallic and semiconducting mesoscopic rings. This can be explained qualitatively based on the energy spectrum of electrons within the nearest-neighbor tight-binding model. In Fig. 2, we reproduce the well-known results [21] that the energy eigenvalue is a periodic function of the magnetic flux for both clean and disordered cases. In the clean case with $W = 0$, the energy level curves form intersecting parabolas as shown in Fig. 2(a). When the disorder is introduced into the ring, the band gaps open precisely at the intersection points $\phi/\phi_0 = 0, \pm 0.5, \pm 1, \dots$ as in Fig. 2(b). As W increases, the level repulsion is enhanced and the band gaps are broadened gradually, which makes the energy level curves smoother. It is known that the PC is proportional to the slope of the energy vs. magnetic flux curve [16], therefore the PC will be substantially reduced with increasing the disorder strength. We notice that the PC is zero at integer and half-integer values of ϕ/ϕ_0.

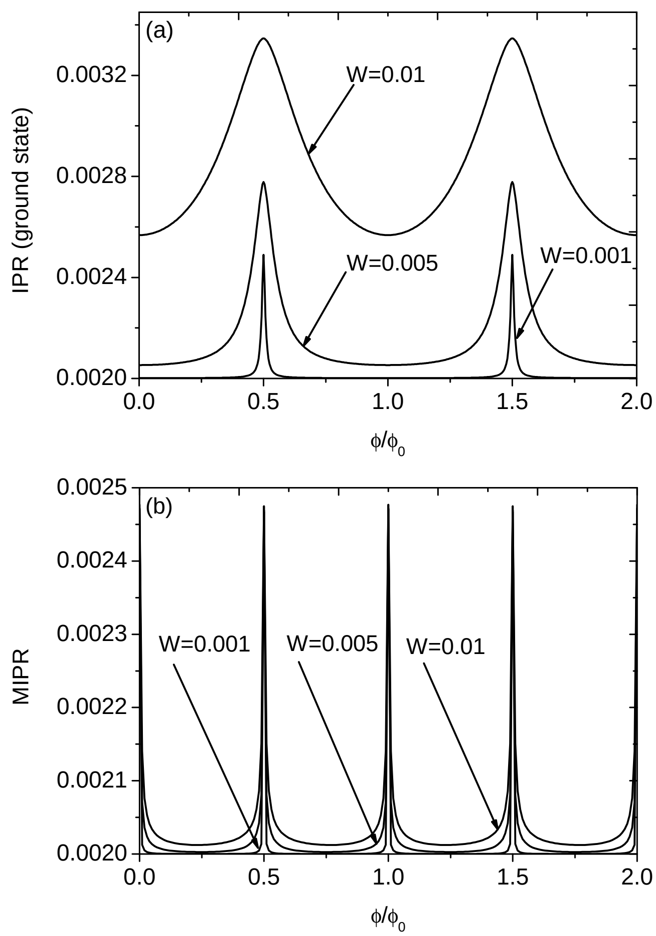

Fig. 3. (a) IPR of the ground state and (b) the mean IPR for a typical disorder realization versus normalized magnetic flux ϕ/ϕ_0 when $N = 500$ and $W = 0.001,\ 0.005,\ 0.01$.

In Fig. 3, we plot the IPR of the ground state and the mean IPR, which is obtained by averaging the IPR over all states, as a function of the magnetic flux ϕ/ϕ_0 when $N = 500$ and

$W = 0.001,\ 0.005,\ 0.01$. The results support the statement in Refs. [35, 36] that due to the AB effect, all physical properties of a mesoscopic ring are periodic in ϕ with period $\phi_0 = hc/e$. We see from Fig. 3(a) that the IPR of the ground state takes maxima at $\phi/\phi_0 = 1/2,\ 3/2, \cdots$ and minima at $\phi/\phi_0 = 0,\ 1,\ 2, \cdots$. In Fig. 3(b), we find that the mean IPR shows maxima at all values of ϕ where the PC is zero.

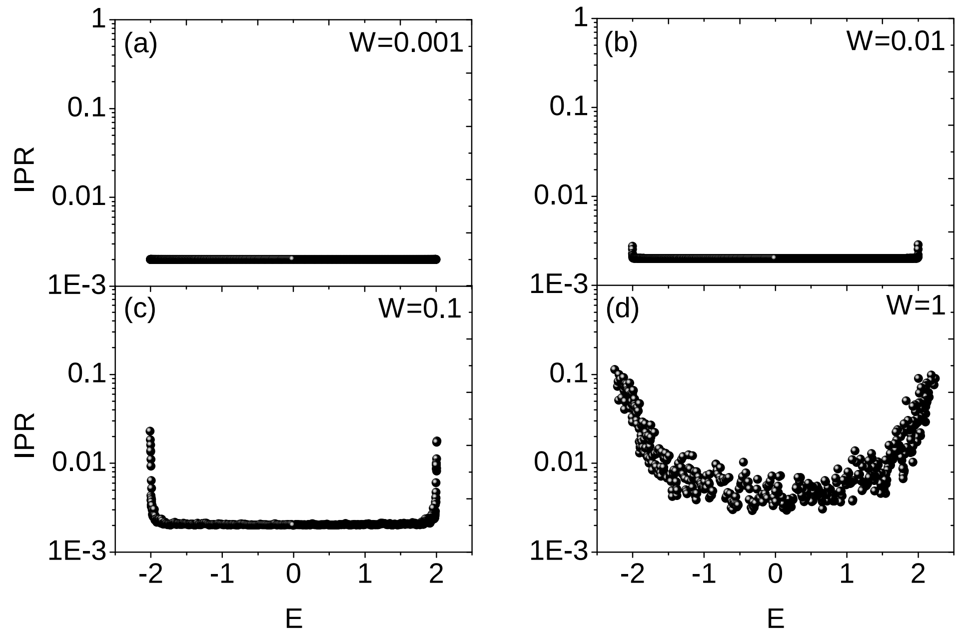

Fig. 4. IPR versus energy E for a typical disorder realization, when $\phi/\phi_0 = 0.25$, $N = 500$ and $W = 0.001,\ 0.01,\ 0.1,\ 1$.

Our main aim in this work is to verify the statement about the absence of localization in the ring threaded by an AB flux for sufficiently weak disorder [18] and to investigate the nature of the low-lying states in the strong disorder regime. In Fig. 4, we plot the IPR versus energy for a typical disorder realization, when $\phi/\phi_0 = 0.25$, $N = 500$ and $W = 0.001,\ 0.01,\ 0.1,\ 1$. The delocalized states are characterized by the criterion IPR $\approx N^{-1}$. From this, we see clearly that all eigenstates in the ring are delocalized in the weak disorder limit. Our numerical result is fully consistent with the analytical calculation reported in [18]. As the disorder parameter W increases, however, the IPRs of the states deep in the band tails begin to increase rapidly, implying that these states become localized. There appears a pair of mobility edges separating the localized states near the band edges from the extended states around the band center. In the strong disorder limit, all states become localized.

Next, we focus on the ground state. In Fig. 5, we show the IPR of the ground state as a function of the disorder strength W at $\phi/\phi_0 = 0.25$ and 0.5 for a fixed system size $N = 500$. This result is obtained by averaging over 100 independent random configurations of ε_j. We find that the ground state is delocalized at small values of W, where IPR $\approx N^{-1}$. As W increases, however, the IPR increases to values much larger than N^{-1}, clearly indicating the occurrence

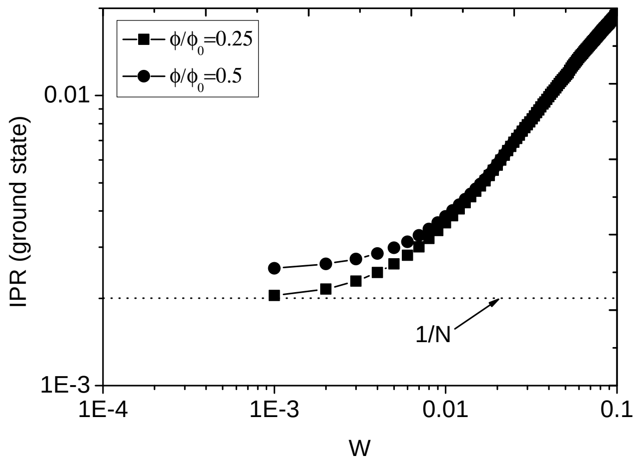

Fig. 5. IPR of the ground state as a function of the disorder strength W when $N = 500$ and $\phi/\phi_0 = 0.25,\ 0.5$. This result is obtained by averaging over 100 realizations of disorder.

of a delocalization-localization transition. Although this effect may suppress the PC in the ring significantly, the current still exists as long as the disorder strength is not too large, which is due to the fact that the PC in an isolated ring is obtained by adding contributions from many different energy levels.

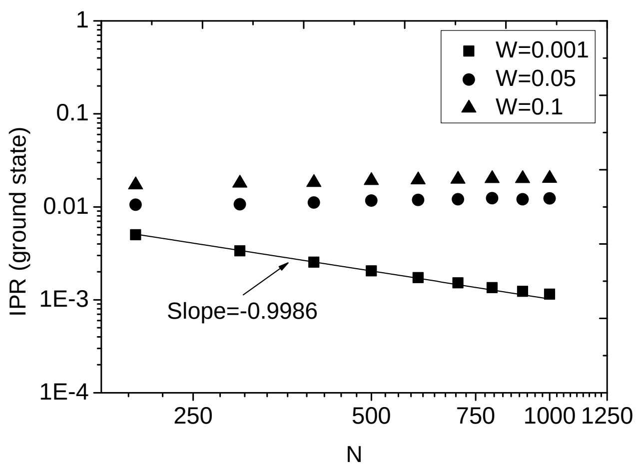

Fig. 6. Dependence of the IPR of the ground state obtained by averaging over 100 realizations of disorder on the system size N, when $\phi/\phi_0 = 0.25$ and $W = 0.001,\ 0.05,\ 0.1$. When $W = 0.001$, we obtain IPR $\approx 1/N$.

In order to establish the existence of the delocalization-localization transition, we plot the IPR of the ground state as a function of the system size N in Fig. 6, when $\phi/\phi_0 = 0.25$ and $W = 0.001,\ 0.05,\ 0.1$. The results are obtained by taking averages over 100 random configurations.

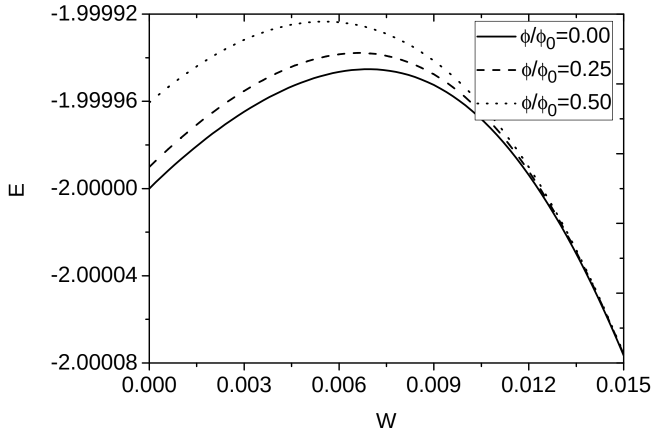

Fig. 7. A variation in the electron ground state energy with disorder strength at three different values of magnetic flux $\phi/\phi_0 = 0,\ 0.25$ and 0.5. The electron energy does not depend on the magnetic field when the disorder strength in the ring approaches a certain value.

We find that when W is 0.001, the IPR scales precisely as N^{-1}, confirming that the ground state is delocalized. When W is 0.05 and 0.1, however, the IPR becomes approximately independent of the system size, clearly indicating the localized nature of the ground state. In other words, the ground state undergoes an Anderson transition as the disorder strength increases. A similar transition occurs to the state at the upper band edge. The low-lying excited states also undergo an Anderson transition at higher critical values of W. The main mechanism driving the delocalization-localization transition is the weakening of the delocalizing effect of the vector potential in the strong disorder regime.

Before closing the present work, we investigate the influence of the magnetic field ϕ/ϕ_0 on the system under consideration. In Fig. 7, we show a variation in the electron ground state energy with disorder strength at three different values of $\phi/\phi_0 = 0,\ 0.25$ and 0.5. For W small where the magnetic flux plays a dominant role the ground state energy is gained towards the band center; hence, its localization length is enhanced on increasing the magnetic flux. When W is large enough, however, the ground state energy almost does not change with respect to the magnetic flux. This means that the considered system becomes like as a 1D disordered ring without the magnetic flux for which it is well known that all eigenstates are localized. This corroborates the above obtained behaviors. Note that due to the periodicity we have only considered the magnetic flux $\phi/\phi_0 \in [0, 1/2]$.

IV. CONCLUSION

We have studied the localization properties of the eigenstates of a tight-binding model for spinless noninteracting electrons moving in a 1D disordered ring threaded by an AB flux. By analyzing the dependence of the inverse participation ratio on magnetic flux, energy, disorder strength and system size, we have found that all states in the ring are delocalized in the weak disorder limit. Our numerical result is fully consistent with the analytical one presented in [18].

The states lying deeply in the band tails undergo a continuous Anderson transition as the disorder strength in the ring sweeps from the weak to the strong disorder regime. This results from the fact that the influence of the AB effect becomes weaker in the strong disorder regime.

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