

## ABSENCE OF SINGULARITY IN SCHWARZSCHILD METRIC IN THE VECTOR MODEL FOR GRAVITATIONAL FIELD

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**Abstract.** *In this paper, based on the vector model for gravitational field we deduce an equation to determinate the metric of space-time. This equation is similar to equation of Einstein. The metric of space-time outside a static spherically symmetric body is also determined. It gives a small supplementation to the Schwarzschild metric in General theory of relativity but the singularity does not exist. Especially, this model predicts the existence of a new universal body after a black hole.*

### I. INTRODUCTION

From the assumption of the Lorentz invariance of gravitational mass, we have used the vector model to describe gravitational field [1]. From this model, we have obtained densities of Universe energy and vacuum energy equal to observed densities[2]. we have also deduced a united description to dark matter and dark energy[3].

In this paper, we deduce an equation to describe the relation between gravitational field, a vector field, with the metric of space-time. This equation is similar to the equation of Einstein. We say it as the equation of Einstein in the Vector model for gravitational field.

This equation is deduced from a Lagrangian which is similar to the Lagrangians in the vector-tensor models for gravitational field [4,5,6,7]. Nevertheless in those models the vector field takes only a supplemental role beside the gravitational field which is a tensor field. The tensor field is just the metric tensor of space- time. In this model the gravitational field is the vector field and its resource is gravitational mass of bodies. This vector field and the energy- momentum tensor of gravitational matter determine the metric of space-time. The second part is an essential idea of Einstein and it is required so that this model has the classical limit.

In this paper, we also deduce a solution of this equation for a static spherically symmetric body. The obtained metric is different to the Schwarzschild metric with a small supplementation. The especial feature of this metric is that black hole exits but has not singularity.

### II. LAGRANGIAN AND FIELD EQUATION

We choose the following action

$$S = S_{H-E} + S_{Mg} + S_g \quad (1)$$

with

$$S_{H-E} = \int \sqrt{-g}(R + \Lambda)d^4x$$

is the classical Hilbert-Einstein action,  $S_{Mg}$  is the gravitational matter action,

$$S_g = \frac{c^2}{16G\pi}\omega \int \sqrt{-g}(E_{g\mu\nu}E_g^{\mu\nu})d^4x$$

is the gravitational action. Where  $E_{g\mu\nu}$  is tensor of strength of gravitational field,  $\omega$  is a parameter in this model.

Variation of the action (1) with respect to the metric tensor leads to the following modified equation of Einstein

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - g_{\mu\nu}\Lambda = -\frac{8G\pi}{c^4}T_{Mg,\mu\nu} + \omega T_{g,\mu\nu} \quad (2)$$

Note that

- Variation of the Hilbert–Einstein action leads to the left–hand side of equation (2) as in General theory of relativity.
- Variation of the gravitational matter action  $S_{Mg}$  leads to the energy- momentum tensor of the gravitational matter

$$T_{Mg,\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_{Mg}}{\delta g^{\mu\nu}}$$

- Variation of the gravitational action  $S_g$  leads to the energy- momentum tensor of gravitational field

$$T_{g,\mu\nu} = \frac{-2}{\omega\sqrt{-g}} \frac{\delta S_g}{\delta g^{\mu\nu}}$$

Let us discuss more to two tensors in the right-hand side of equation (2). We recall that the original equation of Einstein is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - g_{\mu\nu}\Lambda = -\frac{8G\pi}{c^4}T_{\mu\nu}, \quad (3)$$

where  $T_{\mu\nu}$  is the energy- momentum tensor of the matter. For example, for a fluid matter of non–interacting particles with a proper inertial mass density  $\rho(x)$ , with a field of 4–velocity  $u^\mu(x)$  and a field of pressure  $p(x)$ , the energy-momentum tensor of the matter is [8, 9]

$$T^{\mu\nu} = \rho_0 c^2 u^\mu u^\nu + p(u^\mu u^\nu - g^{\mu\nu}) \quad (4)$$

If we say  $\rho_{g0}$  as the gravitational mass density of this fluid matter, the energy–momentum tensor of the gravitational matter is

$$T^{\mu\nu} = \rho_{g0} c^2 u^\mu u^\nu + p(u^\mu u^\nu - g^{\mu\nu}) \quad (5)$$

For a fluid matter of electrically charged particles with the gravitational mass  $\rho_{g0}$ , a field of 4–velocity  $u^\mu(x)$ , and a the electrical charge density  $\sigma_0(x)$ , the energy-momentum tensor of the gravitational matter is

$$T_{Mg}^{\mu\nu} = \rho_{g0} c^2 u^\mu u^\nu + \frac{1}{4\pi} \left( -F_\alpha^\mu F^{\alpha\nu} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)_g \quad (6)$$

The word "g" in the second term group indicates that we choose the density of gravitational mass which is equivalent to the energy density of the electromagnetic field. Where  $F_{\alpha\beta}$  is the electromagnetic field tensor.

Note that because of the close equality between the inertial mass and the gravitational mass, the tensor  $T_{\mu\nu}$  is closely equivalent to the tensor  $T_{Mg,\mu\nu}$ . The only distinct character is that the inertial mass depends on inertial frame of reference while the gravitational mass does not depend one. However the value of  $\rho_0$  in the equation (4) is just the proper density of inertial mass, therefore it also does not depend on inertial frame of reference. Thus, the modified equation of Einstein(2) is principally different with the original equation of Einstein(3) in the present of the gravitational energy- momentum tensor in the right-hand side.

From the above gravitational action, the gravitational energy-momentum tensor is

$$T_{g,\mu\nu} = \frac{-2}{\omega\sqrt{-g}} \frac{\delta S_g}{\delta g^{\mu\nu}} = \frac{c^2}{4G\pi} \left( E_{g,\mu}^\alpha E_{g,\nu\alpha} - \frac{1}{4} g_{\mu\nu} E_g^{\alpha\beta} E_{g,\alpha\beta} \right) \quad (7)$$

Where  $E_{g,\alpha\beta}$  is the tensor of strength of gravitational field [1]. The expression of (7) is obtained in the same way with the energy- momentum tensor of electromagnetic field.

Let us now consider the equation (2) for the space-time outside a body with the gravitational mass  $M_g$  (this case is similar to the case of the original equation of Einstein for the empty space). However in this case, the space is not empty although it is outside the field resource, the gravitational field exists everywhere. We always have the present of the gravitational energy-momentum tensor in the right-hand side of the equation (2). When we reject the cosmological constant  $\Lambda$ , the equation (2) leads to the following form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \omega T_{g,\mu\nu} \quad (8)$$

or

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{c^2\omega}{4G\pi} \left( E_{g,\mu}^\alpha E_{g,\alpha\nu} - \frac{1}{4} g_{\mu\nu} E_g^{\alpha\beta} E_{g,\alpha\beta} \right) \quad (9)$$

### III. THE EQUATIONS OF GRAVITATIONAL FIELD IN CURVATURE SPACE-TIME

We have known the equations of gravitational field in flat space-time [1]

$$\partial_k E_{g,mn} + \partial_m E_{g,nk} + \partial_n E_{g,km} = 0 \quad (10)$$

and

$$\partial_i D_g^{ik} = J_g^k \quad (11)$$

The metric tensor is flat in these equations.

When the gravitational field exists, because of its influence to the metric tensor of space-time, we replace the ordinary derivative by the covariant derivative. The above equations become

$$E_{g,mn;k} + E_{g,nk;m} + E_{g,km;n} = 0 \quad (12)$$

and

$$\frac{1}{\sqrt{-g}} \partial_i \left( \sqrt{-g} D_g^{ik} \right) = J_g^k \quad (13)$$

#### IV. MakeUppercaseThe Metric Tensor of Space-Time outside A Static Spherically Symmetrical Body

We resolve the equations (9,12,13) outside a resource to find the metric tensor of space– time. Thus we have the following equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{c^2\omega}{4G\pi} \left( E_{g,\mu}^\alpha E_{g,\alpha\nu} - \frac{1}{4}g_{\mu\nu}E_g^{\alpha\beta} E_{g,\alpha\beta} \right) \quad (14)$$

$$E_{g,mn;k} + E_{g,nk;m} + E_{g,km;n} = 0 \quad (15)$$

and

$$\partial_i \left( \sqrt{-g} E_g^{ik} \right) = 0 \quad (16)$$

Because the resource is static spherically symmetrical body, we also have the metric tensor in the Schwarzschild form as follows [8]

$$g_{\mu\alpha} = \begin{pmatrix} e^\nu & 0 & 0 & 0 \\ 0 & -e^\lambda & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2 \theta \end{pmatrix} \quad (17)$$

and

$$g^{\mu\alpha} = \begin{pmatrix} e^{-\nu} & 0 & 0 & 0 \\ 0 & -e^{-\lambda} & 0 & 0 \\ 0 & 0 & -r^{-2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2 \sin^2 \theta} \end{pmatrix} \quad (18)$$

The left–hand side of (14) is the tensor of Einstein, it has only the non–zero components as follows [8, 9, 10]

$$R_{00} - \frac{1}{2}g_{00}R = e^{\nu-\lambda} \left( -\frac{\lambda'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} e^\nu \quad (19)$$

$$R_{11} - \frac{1}{2}g_{11}R = -\frac{\nu'}{r} - \frac{1}{r^2} + \frac{1}{r^2} e^\lambda \quad (20)$$

$$R_{22} - \frac{1}{2}g_{22}R = e^{-\lambda} \left[ \frac{r^2}{4} \nu' \lambda' - \frac{r^2}{4} (\nu')^2 - \frac{r^2}{2} \nu'' - \frac{r}{2} (\nu' - \lambda') \right] \quad (21)$$

$$R_{33} - \frac{1}{2}g_{33}R = \left( R_{22} - \frac{1}{2}g_{22}R \right) \sin^2 \theta \quad (22)$$

$$R_{\mu\nu} = 0, g^{\mu\nu} = 0 \quad \text{with } \mu \neq \nu$$

The tensor of strength of gravitational field  $E_{g,\mu\nu}$  when it is corrected the metric tensor needs corresponding to a static spherically symmetrical gravitational  $E_g(r)$  field. From

the form of  $E_{g,\mu\nu}$  in flat space–time [1]

$$E_{g,\mu\nu} = \begin{pmatrix} 0 & -\frac{E_{gx}}{c} & -\frac{E_{gy}}{c} & -\frac{E_{gz}}{c} \\ \frac{E_{gx}}{c} & 0 & H_{gz} & -H_{gy} \\ \frac{E_{gy}}{c} & -H_{gz} & 0 & H_{gx} \\ \frac{E_{gz}}{c} & H_{gy} & -H_{gx} & 0 \end{pmatrix} \quad (23)$$

For static spherically symmetrical gravitational field, the magneto-gravitational components  $H_g = 0$ . We consider only in the  $x$ - direction, therefore the components  $E_{gy}, E_{gz} = 0$ . We find a solution of  $E_{g,\mu\nu}$  in the following form

$$E_{g,\mu\nu} = \frac{1}{c} E_g(r) \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (24)$$

Note that because  $E_{g,\mu\nu}$  is a function of only  $r$ , it satisfies the equation (15) regardless of function  $E_g(r)$ . The function is found at the same time with  $\mu$  and  $\nu$  from the equations (14) and (16). Raising indices in (24) with  $g^{\alpha\beta}$  in (18), we obtain

$$E_g^{\mu\alpha} = \frac{1}{c} e^{-(\nu+\lambda)} E_g(r) \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (25)$$

and

$$\sqrt{-g} E_g^{\mu\alpha} = \frac{1}{c} e^{-\frac{1}{2}(\nu+\lambda)} E_g(r) r^2 \sin \theta \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (26)$$

Substituting (26) into (16), we obtain an only nontrivial equation

$$\left[ e^{-\frac{1}{2}(\nu+\lambda)} E_g(r) r^2 \sin \theta \right]' = 0 \quad (27)$$

We obtain a solution of (27)

$$e^{-\frac{1}{2}(\nu+\lambda)} E_g(r) r^2 \sin \theta = \text{constant}$$

or

$$E_g(r) = e^{\frac{1}{2}(\nu+\lambda)} \frac{\text{constant}}{r^2} \quad (28)$$

We require that space–time is Euclidian one at infinity, it leads to that both  $\nu \rightarrow 0$  and  $\lambda \rightarrow 0$  when  $r \rightarrow \infty$ , therefore the solution (28) has the normal classical form when  $r$  is large, i.e.

$$E_g(r) \rightarrow -\frac{GM_g}{r^2}$$

Therefore

$$constant = -GM_g \quad (29)$$

To solve the equation (14), we have to calculate the energy–momentum tensor in the right–hand side of it. We use (28) to rewrite the tensor of strength of gravitational field in three forms as follows

$$E_{g,\mu\alpha} = \frac{1}{c} e^{\frac{1}{2}(\nu+\lambda)} \left( -\frac{GM_g}{r^2} \right) \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (30)$$

and

$$E_g^{\mu\alpha} = \frac{1}{c} e^{-\frac{1}{2}(\nu+\lambda)} \left( -\frac{GM_g}{r^2} \right) \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (31)$$

and

$$E_{g\mu}^{\alpha} = \frac{1}{c} \left( -\frac{GM_g}{r^2} \right) \begin{pmatrix} 0 & e^{\frac{1}{2}(\nu-\lambda)} & 0 & 0 \\ e^{\frac{1}{2}(\lambda-\nu)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (32)$$

we obtain the following result

$$\begin{aligned} T_{g,\mu\alpha} &= \frac{c^2}{4G\pi} \left[ E_{g,\mu\beta} E_{g,\alpha}^{\beta} - \frac{1}{4} g_{\mu\alpha} E_{g,kl} E_g^{kl} \right] \\ &= -\frac{GM_g^2}{8\pi r^4} \begin{pmatrix} e^{\nu} & 0 & 0 & 0 \\ 0 & -e^{\lambda} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \end{aligned} \quad (33)$$

From the equations(14),(19,20,21,22) and(33), we have the following equations

$$e^{\nu-\lambda} \left( \frac{\lambda'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = \omega \frac{GM_g^2}{8\pi r^4} e^{\nu} \quad (34)$$

$$-\frac{\nu'}{r} - \frac{1}{r^2} + \frac{1}{r^2} e^{\lambda} = -\omega \frac{GM_g^2}{8\pi r^4} e^{\lambda} \quad (35)$$

$$e^{-\lambda} \left[ \frac{r^2}{4} \nu' \lambda' - \frac{r^2}{4} (\nu')^2 - \frac{r^2}{2} \nu'' - \frac{r}{2} (\nu' - \lambda') \right] = \omega \frac{GM_g^2}{8\pi r^4} r^2 \quad (36)$$

Multiplying two members of (34) with  $e^{-(\nu-\lambda)}$  then add it with (35), we obtain

$$\nu' + \lambda' = 0 \implies \nu + \lambda = constant \quad (37)$$

Because both  $\nu$  and  $\lambda$  lead to zero at infinity, the constant in (37) has to be zero. Therefore, we have

$$\nu = -\lambda \quad (38)$$

Using (37), we rewrite (36) as follows

$$e^\nu \left[ -\frac{r^2}{4} \nu'^2 - \frac{r^2}{4} (\nu')^2 - \frac{r^2}{2} (\nu'') - \frac{r}{2} (\nu' + \nu') \right] = \omega \frac{GM_g^2}{8\pi r^4} r^2$$

or

$$e^\nu \left[ (\nu')^2 + \nu'' + \frac{2}{r} \nu' \right] = -\omega \frac{GM_g^2}{8\pi r^4} \quad (39)$$

$$e^\nu \left[ (\nu')^2 + \nu'' \right] + \frac{2}{r} \nu' e^\nu = -\omega \frac{GM_g^2}{8\pi r^4} \quad (40)$$

We rewrite (40) in the following form

$$(e^\nu \nu')' + \frac{2}{r} (\nu') e^\nu = -\omega \frac{GM_g^2}{8\pi r^4} \quad (41)$$

Putting  $y = e^\nu \nu'$ , (41) becomes

$$y' + \frac{2}{r} y = -\omega \frac{GM_g^2}{8\pi c^2 r^4} \quad (42)$$

The differential equation (42) has the standard form as follows

$$y' + p(r)y = q(r) \quad (43)$$

The solution  $y(r)$  is as follows [10]. Putting

$$\eta(r) = e^{\int p(r) dr} = e^{\int \frac{2}{r} dr} = e^{2 \ln(r)} = r^2 \quad (44)$$

We have

$$\begin{aligned} y(r) &= \frac{1}{\eta(r)} \left( \int q(r)\eta(r) dr + A \right) dr \\ &= \frac{1}{r^2} \left[ \int \left( -\omega \frac{GM_g^2}{4\pi r^4} \right) r^2 dr + A \right] \\ &= \frac{1}{r^2} \left[ \omega \frac{GM_g^2}{4\pi r} + A \right] \\ &= \omega \frac{GM_g^2}{4\pi r^3} + \frac{A}{r^2}, \end{aligned} \quad (45)$$

where  $A$  is an integral constant.

Substituting  $y = e^\nu \nu'$ , we have

$$e^\nu \nu' = (e^\nu)' = \omega \frac{GM_g^2}{4\pi r^3} + \frac{A}{r^2} \quad (46)$$

or

$$\begin{aligned} e^\nu &= \int \left( \omega \frac{GM_g^2}{4\pi r^3} + \frac{A}{r^2} \right) dr \\ &= -\omega \frac{GM_g^2}{8\pi r^2} - \frac{A}{r} + B \end{aligned} \quad (47)$$

where  $B$  is a new integral constant.

We shall determine the constants  $A, B$  from the non-relativistic limit. We know that the Lagrangian describing the motion of a particle in gravitational field with the potential  $\varphi_g$  has the form [11]

$$L = -mc^2 + \frac{mv^2}{2} - m\varphi_g \quad (48)$$

The corresponding action is

$$S = \int Ldt = -mc \int \left( c - \frac{v^2}{2c} + \frac{\varphi_g}{c} \right) dt = -mc \int ds \quad (49)$$

we have

$$ds = \left( c - \frac{v^2}{2c} + \frac{\varphi_g}{c} \right) dt \quad (50)$$

that is

$$\begin{aligned} ds^2 &= \left( c^2 + \frac{v^4}{4c^2} + \frac{\varphi_g}{c^2} - v^2 + 2\varphi_g - \frac{v^2\varphi_g}{c^2} \right) dt^2 \\ &= \left( c^2 + 2\varphi_g \right) dt^2 - v^2 dt^2 + \dots \\ &= c^2 \left( 1 + 2\frac{\varphi_g}{c^2} \right) dt^2 - dr^2 + \dots \end{aligned} \quad (51)$$

Where we reject the terms which lead to zero when  $c$  approaches to infinity. Comparing (51) with the our line element (we reject the terms in the coefficient of  $dr^2$ )

$$ds^2 = e^\nu c^2 dt^2 - dr^2 \quad (52)$$

we get

$$\begin{aligned} -\frac{A}{r} + B &\equiv 2\frac{\varphi_g}{c^2} + 1 \\ &\equiv -2\frac{GM_g}{c^2 r} + 1 \end{aligned} \quad (53)$$

From (53) we have

$$A = 2\frac{GM_g}{c^2}, \quad B = 1 \quad (54)$$

The constant  $\omega$  does not obtain in the non relativistic limit, we shall determine it later. Thus, we get the following line element

$$ds^2 = c^2 \left( 1 - 2\frac{GM_g}{c^2 r} - \omega \frac{GM_g^2}{8\pi r^2} \right) dt^2 - \left( 1 - 2\frac{GM_g}{c^2 r} - \omega \frac{GM_g^2}{8\pi r^2} \right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (55)$$

We put  $\frac{\omega}{8\pi} = \frac{G\omega'}{c^4}$  and rewrite the line element (55)

$$ds^2 = c^2 \left( 1 - 2\frac{GM_g}{c^2 r} - \omega' \frac{G^2 M_g^2}{c^4 r^2} \right) dt^2 - \left( 1 - 2\frac{GM_g}{c^2 r} - \omega' \frac{G^2 M_g^2}{c^4 r^2} \right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (56)$$



We determine the parameter  $\omega'$  from the experiments in the Solar system. We use the Robertson - Eddington expansion [9] for the metric tensor in the following form

$$ds^2 = c^2 \left( 1 - 2\alpha \frac{GM_g}{c^2 r} - 2(\beta - \alpha\gamma) \frac{G^2 M_g^2}{c^4 r^2} + \dots \right) dt^2 - (1 - 2\gamma \frac{GM_g}{c^2 r} + \dots) dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (57)$$

When comparing (56) with (57), we have

$$\alpha = \gamma = 1 \quad (58)$$

and

$$\omega' = 2(1 - \beta) \quad (59)$$

The predictions of the Einstein field equations can be neatly summarized as

$$\alpha = \beta = \gamma = 1 \quad (60)$$

From the experimental data in the Solar system, people [9] obtained

$$\frac{2 - \beta + 2\gamma}{3} = 1.00 \pm 0.01 \quad (61)$$

With  $\gamma = 1$  in this model, we have

$$\omega' = 2(1 - \beta) = 0.00 \pm 0.06 \quad (62)$$

Thus  $|\omega'| \leq 0.006$  hence  $|\omega| \leq \frac{0.48G\pi}{c^4}$ . The line element (56) gives a very small supplementation to the Schwarzschild line element. We discuss more to this term  $\omega$ . We consider to the term  $e^\nu$ , it vanishes when

$$1 - 2\frac{GM_g}{c^2 r} - \omega' \frac{G^2 M_g^2}{c^4 r^2} = 0$$

or

$$c^4 r^2 - 2GM_g c^2 r - \omega' G^2 M_g^2 = 0 \quad (63)$$

If we choose  $\omega' < 0$ , equation (63) has two positive solutions

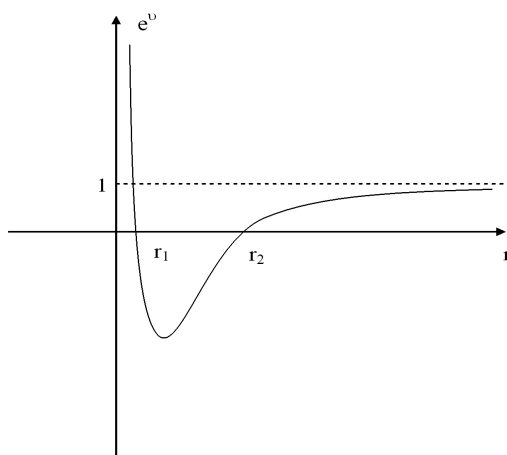
$$\begin{aligned} r_1 &= \frac{GM_g}{c^2} (1 - \sqrt{1 + \omega'}) \approx -\omega' \frac{GM_g}{2c^2} \\ r_2 &= \frac{GM_g}{c^2} (1 + \sqrt{1 + \omega'}) \approx \frac{2GM_g}{c^2} + \omega' \frac{GM_g}{2c^2} \end{aligned} \quad (64)$$

We calculate radii  $r_1, r_2$  for a body whose mass equals to Solar mass and for a galaxy whose mass equals to the mass of our galaxy with  $\omega' \approx -0.06$

- with  $M_g = 2 \times 10^{30} kg$ :  $r_1 \approx 30m$ ,  $r_2 \approx 3km$ .
- with  $M_g = 10^{11} \times 2 \times 10^{30} kg$ :  $r_1 \approx 3 \times 10^9 km$ ,  $r_2 \approx 3 \times 10^{11} km$ .

Thus, because of gravitational collapse, firstly at the radius  $r_2$  a body becomes a black hole but then at the radius  $r_1$  it becomes visible. Therefore, this model predicts the existence of a new universal body after a black hole.

The graph of  $e^\nu$  is showed in figure 1



**Fig. 1.** The graphic of function  $e^\nu$

## V. CONCLUSION

In conclusion, based on the vector model for gravitational field we have deduced a modified Einstein's equation. For a static spherically symmetric body, this equation gives a Schwarzschild metric with a black hole without singularity. Especially, this model predicts the existence of a new universal body after a black hole.

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## REFERENCES

- [1] Vo Van On, *Journal of Technology and Science Development*, Vietnam National University - Ho Chi Minh city, Vol.9(2006)5-11.
- [2] Vo Van On, *Communications in Physics*, **17**(2007)13-17.
- [3] Vo Van On, *Communications in Physics*, **17**, Supplement(2007)83-91.
- [4] R. Hellings and K. Nordvedt, *Phys. Rev. D* **7**, 35(1973)3593-3602.
- [5] K. Nordvedt, Jr and C.M. Will . *Astrophys J.* **177**(1972)775.
- [6] C. Eling and T. Jacobson and D. Mattingly. *arXiv: gr-qc / 0410001 v2 2005*
- [7] E . A. Lim . *arXiv: astro-phy/ 0407437 v2 2004*
- [8] R. Adler, M. Bazin , M. Schiffer, *Introduction To General Relativity*. McGraw-Hill Book Company(1965)
- [9] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of General Theory of Relativity*, Copyright 1972 by John Wiley and Sons, Inc
- [10] Bronstein I.N and Semendaev K.A, *Handbook of Mathematics for Engineers and Specialists*, M Nauka (in Russian), 1986
- [11] Nguyen Ngoc Giao, *Theory of gravitational field(General theory of relativity)*, Bookshefl of University of Natural Sciences,1999( in Vietnamese).

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