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TWO ELECTRO-WEAK PHASES IN THE $SU(2)_1 \otimes SU(2)_2 \otimes U(1)_Y$ MODEL

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Abstract. *Our analysis shows that SM-like electroweak phase transition (EWPT) in the* $SU(2)_1 \otimes SU(2)_2 \otimes U(1)_Y$ *(2-2-1) model is a first-order phase transition at the* 200 *GeV scale, enough for baryogenesis. This first order EWPT is described by a non-smooth correlation length function. The second VEV is larger than* 1.1 *TeV in a two-stage EWPT scenario.*

Keywords: Baryogenesis, 2-2-1 model.

Classification numbers: 12.15.Lk, 11.30.Qc.

I. INTRODUCTION

The Baryogenesis, a solution for the matter-antimatter Asymmetry of the Universe, has been seen in the Sakharov condition [1]. The most important is a first-order EWPT because that not only leads to a thermal imbalance [2, 3] but also makes a connection between the B and CP violations via non-equilibrium physics [4].

The EWPT has been studied in the Standard Model (SM) [2, 3, 5–9] as well as beyond SM [10–36]. The EWPT strength is larger than one at the 200 GeV scale in SM, but the Higgs boson mass must be less than 120 GeV [2, 3, 5–9]. Beyond SM there are various sources for the first-order EWPT, for instance heavy bosons, dark matter candidates [13–21, 26–32, 37–45] or composite Higgs. Another pretty important point is that there are proofs that EWPT or effective potential does not depend on the gauge. This allows us to calculate EWPT in the Landau gauge as simplest and also physically adequate gauge [33–36, 41, 46]. In models with more doubly charged particles or bosons, the strength will be larger [45].

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The $SU(2)_1 \otimes SU(2)_2 \otimes U(1)_Y$ Model (2-2-1 model) is a model beyond SM, which has a simple group structure. However, there are three coupling constants, three vacuum expectation values (VEVs); two exotic quarks which are in a doublet of $SU(2)_2$ group; one new charged and one new neutral gauge boson which are larger than 1.7 TeV [47]. This model has two new gauge bosons which can play an important role in the early universe. These particles and the frame of Higgs potential can be a reason for one first-order EWPT.

This article is organized as follows. In Sect.II, a short review of the 2-2-1 model and the corresponding Higgs potential will be presented. The electroweak phase transition structure will be driven in Sect.III. The first-order phase transition condition will be analyzed by the strength and correlation length in Sect.IV. Finally, in Sect.V we shall summarize and describe outlooks for this work.

II. REVIEW ON 2-2-1 MODEL

In this model, the gauge symmetry is $SU(2)_1 \otimes SU(2)_2 \otimes U(1)_Y$. It has the following gauge bosons: two massive bosons as SM $W^\pm$ boson and Z boson, one new charged boson $W'^\pm$, one new heavy neutral boson Z'. In particular, the model has two Higgs doublets H_1 and H_2, where the first is the SM-like Higgs doublet of $SU(2)_1$ and the second is the heavy Higgs doublet of $SU(2)_2$. Besides, in order to minimize the number of the particles and increase the decay width of the heavy scalar boson of H_2, a quark doublet $Q'^T = (U', D')$ is introduced [47].

II.1. Higgs potential

The Higgs potential with two doublets and one singlet is given by

$$\begin{aligned}
V(H_1, H_2, S') = \sum_{i=1,2} [\mu_1^2 H_i^\dagger H_i + \lambda_i (H_i^\dagger H_i)^2] + \mu_s^2 S'^2 + \lambda_S S'^4 \\
+ \mu_3 S'^3 + S'(\mu_{1S} H_1^\dagger H_1 + \mu_{2S} H_2^\dagger H_2) \\
+ \lambda_{12} H_1^\dagger H_1 H_2^\dagger H_2 + \lambda_{1S} S'^2 H_1^\dagger H_1 \\
+ \lambda_{2S} S'^2 H_2^\dagger H_2, \qquad (1)
\end{aligned}$$

where the scalar fields can be expressed as

$$H_i = \begin{pmatrix} G_i^\dagger \\ (v_i + h_i + iG_i^0)/\sqrt{2} \end{pmatrix}, \qquad (2)$$

$$S' = (v_S + S)/\sqrt{2}. \qquad (3)$$

In Eqs. (2) and (3), G_i^+, G_i^0 are the Nambu-Goldstone bosons. $h_{1,2}$ and S are the scalar bosons. $v_{1,2,S}$ are VEVs. By using the minimal conditions, $\frac{\partial V(H_1, H_2, S')}{\partial v_i} = 0$, we obtain

$$\begin{aligned}
&\mu_1^2 + \lambda_1 v_1^2 + \frac{1}{2}(\lambda_{12} v_2^2 + \lambda_{1S} v_S^2) + \frac{1}{\sqrt{2}\mu_{1S} v_S} = 0, \\
&\mu_2^2 + \lambda_2 v_2^2 + \frac{1}{2}(\lambda_{12} v_1^2 + \lambda_{2S} v_S^2) + \frac{1}{\sqrt{2}\mu_{2S} v_S} = 0, \\
&\mu_S^2 v_S + \lambda_S v_S^3 + \frac{3\mu_S}{2\sqrt{2}} v_S^2 + \frac{1}{2\sqrt{2}}(\mu_{1S} v_1^2 + \mu_{2S} v_2^2) + \frac{1}{2}(\lambda_{1S} v_1^2 + \lambda_{2S} v_2^2) v_S = 0. \qquad (4)
\end{aligned}$$

The mass-squared matrix for the scalar bosons has the form [47]:

$$M^2 = \begin{pmatrix} m^2_{h_1} & m^2_{h_1 h_2} & m^2_{h_1 S} \\ m^2_{h_2 h_1} & m^2_{h_2} & m^2_{h_2 S} \\ m^2_{S h_1} & m^2_{S h_2} & m^2_{S} \end{pmatrix}$$
$$= \begin{pmatrix} 2\lambda_1 v_1^2 & \lambda_{12} v_1 v_2 & \frac{\mu_{1S} v_1}{\sqrt{2}} + \lambda_{1S} v_S v_1 \\ \lambda_{12} v_1 v_2 & 2\lambda_2 v_2^2 & \frac{\mu_{2S} v_2}{\sqrt{2}} + \lambda_{2S} v_S v_2 \\ \frac{\mu_{1S} v_1}{\sqrt{2}} + \lambda_{1S} v_S v_1 & \frac{\mu_{2S} v_2}{\sqrt{2}} + \lambda_{2S} v_S v_2 & 2\lambda_S v_S^2 + \frac{3\mu_S}{2\sqrt{2}} v_S - \frac{1}{2\sqrt{2}} \frac{\mu_{1S} v_1^2 + \mu_{2S} v_2^2}{v_S} \end{pmatrix}, \quad (5)$$

where the masses of Higgs bosons are

$$\begin{aligned} m_h^2 &= m_{h_1}^2 = 2\lambda_1 v^2, \\ m_{h_2}^2 &= 2\lambda_2 v_2^2, \\ m_S^2 &= 2\lambda_S v_S^2 + \frac{3\mu_S v_S}{2\sqrt{2}} - \frac{\mu_{1S} v^2 + \mu_{2S} v_2^2}{2\sqrt{2} v_S}. \end{aligned} \quad (6)$$

In Eqs.(6), h_1 is considered as the SM-like Higgs h so we use h instead of h_1 from now on. h_2 and S are not yet physical particles because they are mixed together as in Eq.(5).

We can diagonalize the matrix in Eq.(5) and obtain the masses of two physical particles [47]:

$$m^2_{H/H_S} = \frac{m_S^2 + m_{h_2}^2}{2} \pm \frac{1}{2}\sqrt{(m_S^2 - m_{h_2}^2)^2 + 4m_{23}^4}, \quad (7)$$

where $m_{23}^2 = \lambda_{2S} v_2 v_S + v_2 \mu_{2S}/\sqrt{2}$. However, we approximate that μ_{2S} and λ_{2S} are very small (see Ref. [47]), so that $m_{23} \sim 0$ and we neglect this mixing so $m_{H_S} = m_S, m_{h_2} = m_H$. In our analysis, we use H and H_S instead of h_2 and S.

II.2. Gauge boson sector

The masses of the gauge bosons can be found in the kinetic part of the Lagrangian

$$\mathscr{L} = (D_\mu H_1)^\dagger (D_\mu H_1) + (D_\mu H_2)^\dagger (D_\mu H_2) + (D_\mu S')^\dagger (D_\mu S'). \quad (8)$$

We can find the masses of gauge bosons by writing the covariant deravative as:

$$D_\mu = (\partial_\mu - i g_i T_a^{(i)} A^a_{i\mu} - i g_Y Y B_\mu), \quad (9)$$

where g_i and $A^a_{i\mu}$ $(a = 1,2,3)$ are the gauge coupling parameters and gauge fields of $SU(2)_i$, g_Y and B_μ are the gauge coupling and gauge field of $U(1)_Y$, $T_a^{(i)} = \sigma_a/2$, where σ_a are the Pauli matrices and Y is the hypercharge of a particle. The covariant derivative of H_1 and H_2 can be rewritten as:

$$D_\mu H_i \supset \begin{pmatrix} g_i A^3_{i\mu}/2 + g_Y B_\mu/2 & g_i W^+_{i\mu}/\sqrt{2} \\ g_i W - +_{i\mu}/\sqrt{2} & -g_i A^3_{i\mu}/2 + g_Y B_\mu/2 \end{pmatrix} \begin{pmatrix} 0 \\ (v_i + h_i)/\sqrt{2} \end{pmatrix}, \quad (10)$$

where $W_i^{\pm} = (A_i^1 \mp iA_i^2)/\sqrt{2}$ are the charged gauge fields. Since they are not mixed with each other, we can easily obtain the masses of SM-like and the new charged gauge boson as:

$$m_W = \frac{gv}{2} \qquad \text{and} \qquad m_{W'} = \frac{g_2 v_2}{2}.$$

The mass matrix of the neutral gauge-boson sector is given by:

$$\mathscr{L}_M = \frac{1}{8}\begin{pmatrix} A^3_{2\mu} \\ A^3_{1\mu} \\ B_\mu \end{pmatrix}^T \begin{pmatrix} v_2^2 g_2^2 & 0 & -v_2^2 g_2 g_Y \\ 0 & v_1^2 g^2 & -v_1^2 g g_Y \\ -v_2^2 g_2 g_Y & -v_1^2 g g_Y & (v_1^2+v_2^2) g_Y^2 \end{pmatrix} \begin{pmatrix} A_2^{3\mu} \\ A_1^{3\mu} \\ B^\mu \end{pmatrix}. \tag{11}$$

We can easily find the massless photon field A_μ and two massive neutral gauge bosons $Z_{1\mu}$ and $Z_{2\mu}$,

$$\begin{pmatrix} A^3_{2\mu} \\ A^3_{1\mu} \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_\theta & 0 & -s_\theta \\ 0 & 1 & 0 \\ s_\theta & 0 & c_\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_W & s_W \\ 0 & -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_{2\mu} \\ Z_{1\mu} \\ A_\mu \end{pmatrix}, \tag{12}$$

where

$$s_\theta = \sin\theta = \frac{g_Y}{\sqrt{g_2^2+g_Y^2}}, \qquad c_\theta = \cos\theta = \frac{g_2}{\sqrt{g_2^2+g_Y^2}}, \qquad g' = g_Y c_\theta,$$

$$s_W = \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}, \qquad c_W = \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}},$$

θ_W is the Weiberg angle in the SM.

The mass-squared matrix for the two new bosons Z_1 and Z_2 is given by:

$$M^2_{Z_1Z_2} = \begin{pmatrix} m^2_{Z_1} & m^2_{Z_1Z_2} \\ m^2_{Z_1Z_2} & m^2_{Z_2} \end{pmatrix}, \tag{13}$$

with

$$m^2_{Z_1} = \frac{v^2}{4}(g^2+g'^2), \qquad m^2_{Z_2} = \frac{v_2^2 g_2^4 + v^2 g'^4}{4(g_2^2-g'^2)},$$

$$m^2_{Z_1Z_2} = \frac{v^2 g'^2}{4}\sqrt{\frac{g^2+g'^2}{g_2^2-g'^2}}.$$

After diagonalizing the mass matrix, we receive the mass eigenstates Z and Z':

$$Z = Z_1\cos\theta_Z - Z_2\sin\theta_Z, \qquad Z' = Z_1\sin\theta_Z + Z_2\cos\theta_Z, \tag{14}$$

where their mixing angle θ_Z is

$$\sin 2\theta_Z = \frac{2m^2_{Z_1Z_2}}{m^2_{Z'} - m^2_Z}. \tag{15}$$

The physical masses of the two neutral gauge bosons Z and Z' are:

$$m_Z^2 = \frac{m_{Z_1}^2 + m_{Z_2}^2}{2} + \frac{1}{2}\sqrt{(m_{Z_2}^2 - m_{Z_1}^2)^2 + 4m_{Z_1Z_2}^4},$$
$$m_{Z'}^2 = \frac{m_{Z_1}^2 + m_{Z_2}^2}{2} - \frac{1}{2}\sqrt{(m_{Z_2}^2 - m_{Z_1}^2)^2 + 4m_{Z_1Z_2}^4}.$$

Finally, the Yukawa sector can be expressed as follows:

$$-\mathscr{L} = y_F \bar{Q}_L' Q_R' S' + y_b \bar{Q}_L' H_2 b_R + y_t \bar{Q}_L' \tilde{H}_2 t_R + m_\psi \bar{Q}_L' Q_R' + H.c \tag{16}$$

III. ELECTROWEAK PHASE TRANSITION STRUCTURE IN THE 2-2-1 MODEL

The purpose of this section is to find the effective potential of 2-2-1 model. The process will be similar to the one of SM. Higgs components and gauge bosons are the main contributors to EWPT, so determining the mass of these particles can affect the phase separation.

First, we have the Higgs Lagrangian of 2-2-1 model, which contains the kinetic energy and potential parts as:

$$\mathscr{L}_{\text{Higgs}} = (D_\mu H_1)^\dagger (D_\mu H_1) + (D_\mu H_2)^\dagger (D_\mu H_2) + (D_\mu S')^\dagger (D_\mu S') + V(H_1, H_2, S'). \tag{17}$$

After averaging over all space, we get:

$$\langle H_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i \end{pmatrix}, \qquad i = 1,2 \tag{18}$$
$$\langle S' \rangle = \frac{1}{\sqrt{2}} v_S. \tag{19}$$

Lagrangian is rewritten as below since we can consider v, v_2 and v_S as variables from now on.

$$\mathscr{L}_{Higgs} = \frac{1}{2}\partial^\mu v \partial_\mu v + \frac{1}{2}\partial^\mu v_2 \partial_\mu v_2 + \frac{1}{2}\partial^\mu v_S \partial_\mu v_S + V_0(v, v, v_S) + \sum_{i=\text{boson}} m_i^2(v, v_2, v_S) W^\mu W_\mu \tag{20}$$

in which W runs over all gauge and Higgs fields.

Since each symmetry breaking only generates masses for the parts which depend on its VEV, we can split the masses of particles into 3 parts as:

$$m^2(v_S, v_2, v) = m^2(v_S) + m^2(v_2) + m^2(v). \tag{21}$$

Table 1 contains the masses of particles in this model [47], which depend on the VEVs; n is the degree of freedom; $g = 0.654, g' = 0.407$; g_2 is unknown and it should be larger than 2 [47].

Table 1. Masses of bosons and fermions in the 2-2-1 model.

Particles	$m^2(v)$	$m^2(v_2)$	$m^2(v_S)$	n
$m^2_{W^\pm}$	$\frac{g^2v^2}{4}$	0	0	6
$m^2_{W'^\pm}$	0	$\frac{g_2^2v_2^2}{4}$	0	6
$m^2_{Z_1}\sim m^2_Z$	$(g^2+g'^2)\frac{v^2}{4}$	0	0	3
$m^2_{Z_2}\sim m^2_{Z'}$	$\frac{1}{4}\frac{g'^4v^2}{g_2^2-g'^2}$	$\frac{1}{4}\frac{g_2^4v_2^2}{g_2^2-g'^2}$	0	3
$m^2_h=m^2_{h_1}$	$2\lambda_1v^2$	0	0	1
$m^2_H=m^2_{h_2}$	0	$2\lambda_2v_2^2$	0	1
$m^2_{H_S}=m^2_S$	$-\frac{\mu_{1S}v^2}{2\sqrt{2}v_S}$	$-\frac{\mu_{2S}v_2^2}{2\sqrt{2}v_S}$	$2\lambda_Sv_S^2+\frac{3\mu_Sv_S}{2\sqrt{2}}$	1
m^2_t	$f_t^2v^2$	0	0	-12
$m^2_T\sim m^2_{U'}=m^2_Q$	0	0	$(m_\psi+\frac{y_F}{\sqrt{2}}v_S)^2$	-12
$m^2_B\sim m^2_{D'}=m^2_Q$	0	0	$(m_\psi+\frac{y_F}{\sqrt{2}}v_S)^2$	-12

The tree potential V_0 has the form:

$$V_0(v,v_2,v_S)=V(\langle H_1\rangle,\langle H_2\rangle,\langle S'\rangle) \tag{22}$$

$$\begin{aligned}&=\sum_{i=1,2}[\mu_1^2\langle H_i\rangle^\dagger\langle H_i\rangle+\lambda_i(\langle H_i\rangle^\dagger\langle H_i\rangle)^2]+\mu_s^2\langle S'\rangle^2+\lambda_S\langle S'\rangle^4\\&\quad+\mu_3\langle S'\rangle^3+\langle S'\rangle(\mu_{1S}\langle H_1\rangle^\dagger\langle H_1\rangle+\mu_{2S}\langle H_2\rangle^\dagger\langle H_2\rangle)\\&\quad+\lambda_{12}\langle H_1\rangle^\dagger\langle H_1\rangle\langle H_2\rangle^\dagger\langle H_2\rangle+\lambda_{1S}\langle S'\rangle^2\langle H_1\rangle^\dagger\langle H_1\rangle\\&\quad+\lambda_{2S}\langle S'\rangle^2\langle H_2\rangle^\dagger\langle H_2\rangle\end{aligned} \tag{23}$$

$$\begin{aligned}&=\frac{\mu_1^2}{2}v^2+\frac{\mu_2^2}{2}v_2^2+\frac{\lambda_1}{4}v^4+\frac{\lambda_2}{4}v_2^4+\frac{\mu_S^2}{2}v_S^2+\frac{\lambda_S}{4}v_S^4\\&\quad+\frac{\mu_3}{2\sqrt{2}}v_S^3+\frac{1}{2\sqrt{2}}v_S(\mu_{1S}v^2+\mu_{2S}v_2^2)+\lambda_{12}v^2v_2^2+\lambda_{1S}v_S^2v^2+\lambda_{2S}v_S^2v_2^2\\&=V_0(v)+V_0(v_2)+V_0(v_S).\end{aligned} \tag{24}$$

A scenario is to have 2 phase transitions, where v_S and v_2 are at the same scale. The first symmetry breaking $SU(2)_1\otimes SU(2)_2\otimes U(1)_Y\to SU(2)_L\otimes U(1)_Y$ is directly turned on, without the mediate stage which generates the masses for exotic quarks. This phase transition generates mass for all the new particles through $v_2=v_S$. The electroweak phase transition is like the one in SM.

Multi-stage EWPT has been considered in many beyond SM models. Separation into several phases of EWPT is due to the square of particle mass without the mixing of VEVs (except H_s). This problem may be well addressed in [40].

$$\begin{aligned}\text{2-2-1 model: } & SU(2)_1\otimes SU(2)_2\otimes U(1)_Y\\ & \Downarrow\\ \text{SM model: } & SU(2)_L\otimes U(1)_Y\\ & \Downarrow\\ \text{QED: } & U(1)_Q\end{aligned}$$

The mass of H_S has a mixing of VEVs because the Higgs potential has the interaction among S', H_1 and H_2, $S'(\mu_{1S}H_1^\dagger H_1+\mu_{2S}H_2^\dagger H_2)$. This will lead to a difficulty in phase separation. This interaction makes complex in the mass generation to H_s and the Higgs potential has auto-CP violation. In the next section we will approximate the mass of H_s, it can participate in one or two phases.

IV. TWO PHASE TRANSITIONS

When v_S is at the same scale with v_2, we set $v_{S0}\approx v_{20}$ then

$$\begin{aligned}m_{H_S}(v_{20}) =& 2\lambda_S v_{20}^2+\frac{1}{2\sqrt{2}}(3\mu_S-\mu_{2S})v_{20},\\ m_Q(v_{20}) =& m_\psi+\frac{y_F}{\sqrt{2}}v_{20},\end{aligned} \tag{25}$$

where Q is the exotic quark T and B.

IV.1. The first phase transition $SU(2)_1\otimes SU(2)_2\otimes U(1)_Y\to SU(2)_L\otimes U(1)_Y$

There are all of the new bosons and fermions in this phase transition, such as W',Z',H,H_s,T,B. The effective potential of $SU(2)_1\otimes SU(2)_2\otimes U(1)_Y\to SU(2)_L\otimes U(1)_Y$ phase transition is

$$\begin{aligned}V_{eff}(v_2) =& V_0(v_2)\\ &+\frac{1}{64\pi^2}\left[6m_{W'}^4(v)\ln\frac{m_{W'}^2(v)}{Q^2}+3m_{Z'}^4(v)\ln\frac{m_{Z'}^2(v)}{Q^2}\right.\\ &\qquad+m_H^4(v)\ln\frac{m_H^2(v)}{Q^2}+m_{H_S}^4(v)\ln\frac{m_{H_S}^2(v)}{Q^2}\\ &\qquad\left.-24m_Q^4(v)\ln\frac{m_Q^2(v)}{Q^2}\right]\\ &+\frac{T^4}{4\pi^2}\left[6F_-\left(\frac{m_{W'}(v)}{T}\right)+3F_-\left(\frac{m_{Z'}(v)}{T}\right)\right.\\ &\qquad+F_-\left(\frac{m_H(v)}{T}\right)+F_-\left(\frac{m_{H_S}(v)}{T}\right)\\ &\qquad\left.+24F_+\left(\frac{m_Q(v)}{T}\right)\right],\end{aligned}$$

where

$$F_{\pm}\left(\frac{m_\phi}{T}\right) = \int_0^{\frac{m_\phi}{T}} \alpha J_{\pm}^{(1)}(\alpha,0)d\alpha, \tag{26}$$

$$J_{\pm}^{(1)}(\alpha,0) = 2\int_\alpha^\infty \frac{(x^2-\alpha^2)^{\nu/2}}{e^x \pm 1}dx, \tag{27}$$

$$\Rightarrow \begin{cases} J_{-}^{(1)}(\alpha,0) = \dfrac{\pi^2}{3} - \pi\alpha - \dfrac{\alpha^2}{2}(\ln\dfrac{\alpha}{4\pi} + C - \dfrac{1}{2}) + \alpha^2\mathscr{O} \\ J_{+}^{(1)}(\alpha,0) = \dfrac{\pi^2}{6} - \dfrac{\alpha^2}{2}(\ln\dfrac{\alpha}{\pi} + C - \dfrac{1}{2}) + \alpha^2\mathscr{O}. \end{cases} \tag{28}$$

v_{20} is the symmetry breaking scale of this phase transition, then we can write the effective potential as:

$$V_{eff}(v_2) = \frac{\lambda_T}{4}v_2^4 - \frac{\theta}{3}Tv_2^3 + \frac{\gamma(T^2-T_0^2)}{2}v_2^2, \tag{29}$$

where

$$\begin{aligned}
\lambda_T = \frac{m_H^2(v_{20}) + m_{H_s}^2(v_{20})}{2v_{20}^2}\Bigg\{1 + \frac{1}{8\pi^2 v_{20}^2(m_H^2(v_{20})+m_{H_s}^2(v_{20}))}\Bigg[&6m_{W'^\pm}^4(v_{20})\ln\frac{bT^2}{m_{W'^\pm}^2(v_{20})} \\
&+ 3m_{Z'}^4(v_{20})\ln\frac{bT^2}{m_{Z'}^2(v_{20})} + m_H^4(v_{20})\ln\frac{bT^2}{m_H^2(v_{20})} \\
&+ m_{H_s}^4(v_{20})\ln\frac{bT^2}{m_{H_s}^2(v_{20})} - 24m_Q^4(v_{20})\ln\frac{b_F T^2}{m_Q^2(v_{20})}\Bigg]\Bigg\}
\end{aligned}$$

$$\theta = \frac{1}{4\pi v_{20}^3}\left[6m_{W'^\pm}^3(v_{20}) + 3m_{Z'}^3(v_{20}) + m_H^3(v_{20}) + m_{H_s}^3(v_{20})\right]$$

$$\gamma = \frac{1}{12v_{20}^2}\left[6m_{W'^\pm}^2(v_{20}) + 3m_{Z'}^2(v_2) + m_H^2(v_{20}) + m_{H_s}^2(v_{20}) + 12m_Q^2(v_{20})\right]$$

$$\begin{aligned}
T_0^2 = \frac{1}{2\gamma}\Bigg\{&m_H^2(v_{20}) + m_{H_s}^2(v_{20}) \\
&- \frac{1}{8\pi^2 v_{20}^2}\left[6m_{W'^\pm}^4(v_{20}) + 3m_{Z'}^4(v_{20}) + m_H^4(v_{20}) + m_{H_s}^4(v_{20}) - 24m_Q^4(v_{20})\right]\Bigg\}.
\end{aligned}$$

There are five variables, which are the masses at 0K of W', Z', H, H_S bosons and two exotic quarks. With $b = 49.5, b_F = 3.67$, we set $m_H(v_{20}) = m_{H_S}(v_{20}) = Y$ and $m_{Z'}(v_{20}) = m_{W'}(v_{20}) = m_Q(v_{20}) = X$, then we will have two variables running. After that, we choose an arbitrary value of the symmetry breaking scale of this phase transition and plot Y as a function of X with the condition $S \geq 1$ to get the upper limit of variable X then change the scale and continue plotting until getting the value 1.7 TeV as a bounder of X. Then we find $v_{20} = 1110$ GeV the at-least value that fits the $\rho-$ parameter condition and the range of the transition strength is $1 \leq S < 8$.

We can see the range of unknown masses from the Fig. 1 and the new coupling constant of $SU(2)_1$ can be found as:

$$0 < g_2 < 3.06. \tag{30}$$

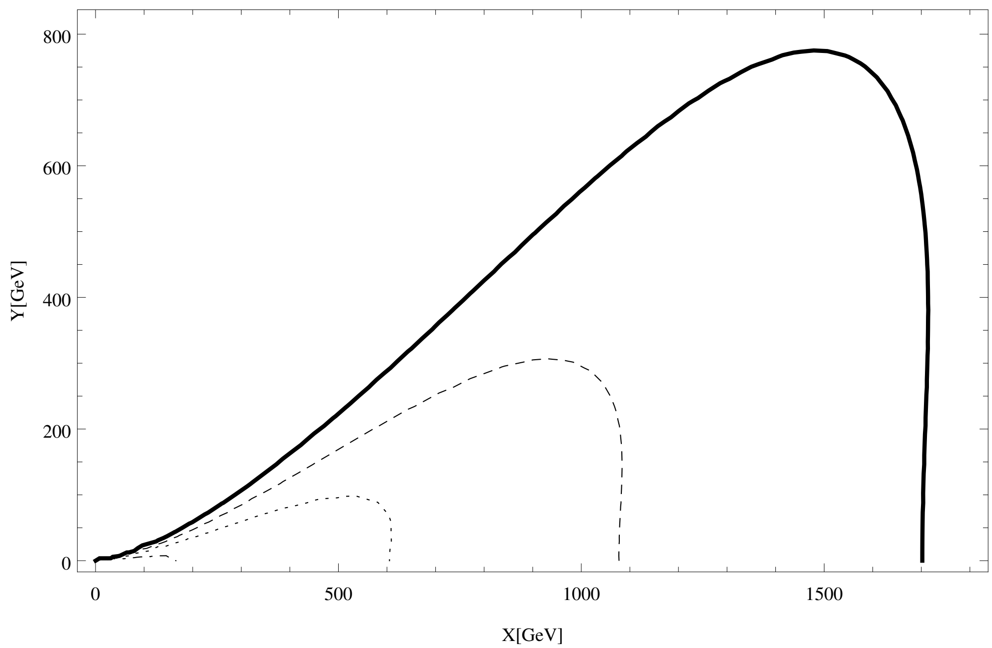

Fig. 1. The symmetry breaking scale $v_{20} = 1110$ GeV. Thick contour $S = 1$, dashed contour $S = 1.5$, dotted contour $S = 2.5$, dashed-dotted contour $S_{max} = 8$.

IV.2. The second Phase transition $SU(2)_L \otimes U(1)_Y \to U(1)_Q$

This phase transition involves a part of new Higgs bosons H_S, a part of new gauge boson Z', with the masses of them being functions of v as the 3rd column in Table 1. Importantly this phase involves the two SM particles $W^{\pm}$, Higgs h boson and top quark. This phase is SM-like but it has more new particles.

In Table 1, the mass of H_s in this phase is $-\dfrac{\mu_{1S}v^2}{2\sqrt{2}v_S}$ which depends on v, v_S. This means that H_s is involved in this phase. But we assume $v \ll v_S$ and in this phase the dynamic variable is v so we can approximate $-\dfrac{\mu_{1S}}{2\sqrt{2}v_S} \sim const$. Therefore, this approximation as considering the contribution of H_s is like "an effective mass" ($m^2_{H_s}(v) = const.v^2$).

The symmetry breaking scale is $v = 246$ GeV. The same as the 1st EWPT, the effective potential in this stage can be written as:

$$V_{eff}(v) = \frac{\lambda'_T}{4}v^4 - \theta' T v^3 + \gamma'(T^2 - T_0'^2)v^2, \tag{31}$$

where

$$\lambda'_T = \frac{m_h^2(v_0)}{2v_0^2}\left\{1 + \frac{1}{8\pi^2 v_0^2 m_h^2(v_0)}\left[6m^4_{W^\pm}(v_0)\ln\frac{bT^2}{m^2_{W^\pm}(v_0)} + 3m_Z^4(v_0)\ln\frac{bT^2}{m_Z^2(v_0)} + 3m^4_{Z'}(v_0)\ln\frac{bT^2}{m^2_{Z'}(v_0)} \right.\right.$$
$$\left.\left. + m_h^4(v_0)\ln\frac{bT^2}{m_h^2(v_0)} + m^4_{H_S}(v_0)\ln\frac{bT^2}{m^2_{H_S}(v_0)} - 12m_t^4(v_0)\ln\frac{b_F T^2}{m_t^2(v_0)}\right]\right\},$$

$$\theta' = \frac{1}{12\pi v_0^3}\left[6m_{W^\pm}^3(v_0)+3m_Z^3(v_0)+3m_{Z'}^3(v_0)+m_h^3(v_0)+m_{H_S}^3(v_0)\right],$$
$$\gamma' = \frac{1}{24v_0^2}\left[6m_{W^\pm}^2(v_0)+3m_Z^2(v_0)+3m_{Z'}^2(v_0)+m_h^2(v_0)+m_{H_S}^2(v_0)+6m_t^2(v_0)\right],$$
$$T_0'^2 = \frac{1}{4\gamma'}\left\{m_h^2(v_0)-\frac{1}{8\pi^2v_0^2}\left[6m_{W^\pm}^4(v_0)+3m_Z^4(v_0)+3m_{Z'}^4(v_0)+m_h^4(v_0)+m_{H_S}^4(v_0)-12m_t^4(v_0)\right]\right\}.$$

In this potential, we set the mass of SM-like Higgs boson $m_h(v_0) = 125$ GeV then there are two unknown masses $m_{Z'}(v_0)$ and $m_{H_S}(v_0)$. Here, θ' has more distributions of Z' and H_S which do not appear in SM. The larger θ' is, the larger the strength is. Therefore, the strength will be stronger than one and that of SM.

To illustrate more clearly the SM-like first-order EWPT, we compute correlated lengths, ξ, such as non-smooth functions under temperature as below:

$$\frac{\partial^2 V_{eff}(v)}{\partial v^2}(v_{eq},T)\bigg|_{v_{eq}} = \xi^{-2}, \tag{32}$$

$$\tag{33}$$

where

$$v_{eq} = \begin{cases} 0 & ,T > T_C \\ v_m = \dfrac{\theta T - \sqrt{(\theta T)^2 - 4\lambda_T\gamma(T^2-T_0^2)}}{2\lambda_T} & ,T < T_C \end{cases} \tag{34}$$

$$\Rightarrow \xi(T) = \frac{1}{\sqrt{3\Lambda_T v_{eq}^2 - 2\theta T v_{eq} + \gamma(T^2-T_0^2)}}. \tag{35}$$

The correlation length is a function which depends on temperature and VEV at the stable state v_{eq}. The equilibrium is also temperature depending, which equals to zero when temperature is below the critical value and to v_m when temperature is larger than T_C. The two parts of $\xi(T)$ graph represents for two different phases.

As we can see in those effective potential graphs, there are symmetry breaking processes from one minimum to two minima. When $T > T_C$, the effective potential has only one minimum at zero VEV. But when the temperature comes close to the critical value, there is a signal of another minimum. Finally, when the universe's temperature reaches T_C, the second minimum officially appears. After that, the universe continues to be cooled down leading to a new equilibrium. This is the process where the particles in our model turn from zero to finite masses.

Now we will draw the correlation length of $SU(2)_L \otimes U_Y(1) \to U_Q(1)$ phase transition by temperature with different values of unknown masses.

Correlation length is not a smooth function, which has a peak at the critical temperature. Since the peak is where two functions representing for two minima intersect, correlation length can describe a first-order phase transition. Besides, it does not have any rule except for the peak because the value of unknown masses are chosen randomly. The $SU(2)_L \otimes U_Y(1) \to U_Q(1)$ phase transition can be seen in Fig. 2.

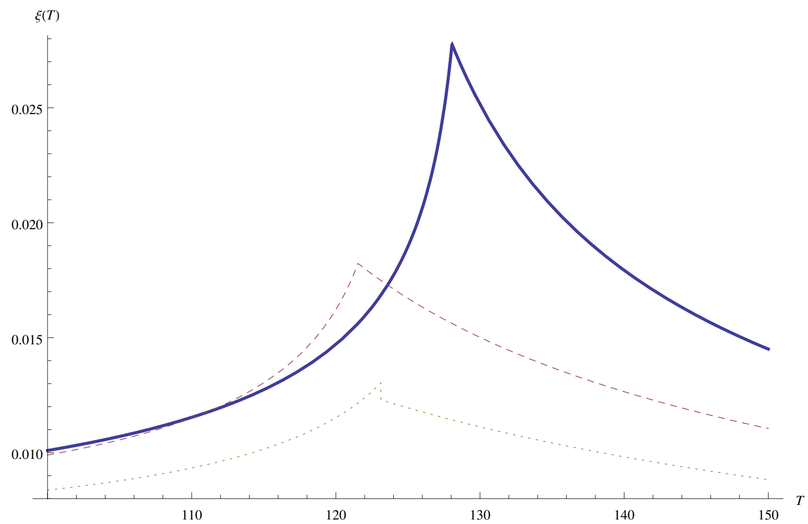

Fig. 2. This graph shows the correlation length of $SU(2)_L \otimes U_Y(1) \to U_Q(1)$ phase transition with different values of unknown masses. The dotted line: $m_{Z'}(v_0) = 252.2$ GeV, $m_{H_S}(v_0) = 404.7$ GeV for the transition strength $S = 2$, the critical temperature $T_C = 123.122$K. The dashed line: $m_{Z'}(v_0) = 163.4$ GeV, $m_{H_S}(v_0) = 381.5$ GeV for the transition strength $S = 1.5$, the critical temperature $T_C = 121.538$K. The thick line: $m_{Z'}(v_0) = 136.2$ GeV, $m_{H_S}(v_0) = 307.4$ GeV for the transition strength $S = 1$, the critical temperature $T_C = 128.054$K.

V. CONCLUSION AND OUTLOOKS

By using the high-temperature effective potential in the 2-2-1 model, the EWPT is strengthened by the new scalars to be the strongly first-order. Our results match the condition of $g_2 > 2$ in [47]. The EWPT can be calculated in a different way as in [33, 46]. The accuracy of a high-temperature expansion for the effective potential will be better than 5% if $\frac{m_{boson}}{T} < 2.2$, where m_{boson} is the relevant boson mass [48]. With our calculations, in the SM-like EWPT, the value of T_c is in the range [100,200] GeV so the maximum of m_{boson} is about 450 GeV. Therefore, our driving domain of boson mass is appropriate. The mass range of the bosons in other phase transitions also satisfies this condition.

In this model, H_s is a complex case, because its mass is intertwined between the VEVs. This complicates the separation of phase so in subsequent calculations, we will introduce a Higgs potential correction to determine clearly the mass of H_s. The tiny masses of neutrinos which can be explained in the see-saw mechanism [49], could be an extra reason for the matter-antimatter asymmetry and CP-violation. Therefore, in the next works, we can investigate again the EWPT by using neutrino data and the sphaleron rate.

Furthermore, the sphaleron is an important process in baryogenesis so we will continue to calculate and test the sphaleron solution in this model with the Cosmotransition code [50]. This code uses a Bessel function for $v(r)$ but it is not flexible in changing the value of wall.

This work could serve as the basis for the calculation of cross section of the decay Higgs to photons when connected to the data of LHC or Particle Data Group.

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