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EVALUATION OF AVERAGE DIRECTIONAL EFFECTIVE EMISSIVITIES OF ISOTHERMAL CYLINDRICAL-INNER-CONE CAVITIES USING MONTE CARLO METHOD

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Abstract. *We have used the Monte Carlo method based on successive reflections and ray tracing to calculate the average normal directional effective emissivities of isothermal cylindrical-inner-cone cavities for various geometrical parameters. A simplified specular-directional diffuse reflection model was applied in our calculations for cavities working in the infrared spectral range. Our results are in good agreement comparing with what obtained by other authors. The algorithm developed by us has an advantage in simplicity and time saving of calculations. It can be used in blackbody cavity design considerations, especially in the cylindrical-inner-cone cases.*

Keywords: blackbody, isothermal cavity, effective emissivity, specular-diffuse reflection model, Monte Carlo method, pseudo-random number, ray tracing.

Classification numbers: 44.40.+a; 07.57.-c; 42.15.-i.

I. INTRODUCTION

The blackbody cavities as artificial sources of radiation are often used for the calibration of radiation thermometers, thermal imagers and radiometers. In case of on-field operating instruments, limited size and weight of cavities are important requirements of a radiation source [1]. Cavities having cylindrical - inner - cone geometry are usually considered in practice. They provide moderately collimated beam with high and uniform effective emissivities with compact geometrical dimensions [1, 2].

The directional spectral effective emissivities are the primary radiation characteristics of a blackbody cavity. Commonly, for a non-isothermal cavity with reference temperature T_0, its local directional effective emissivity can be represented in the form [3]:

$$\varepsilon_e(\overrightarrow{\xi},\overrightarrow{\omega},\lambda,T_\xi,T_0) = \varepsilon_e(\overrightarrow{\xi},\overrightarrow{\omega},\lambda) + \Delta\varepsilon_e(\overrightarrow{\xi},\overrightarrow{\omega},\lambda,T_\xi,T_0) \tag{1}$$

where $\overrightarrow{\xi}$ is position vector of an unit area of cavity wall having local temperature T_ξ, $\overrightarrow{\omega}$ is its radiation direction at wavelength λ , and T_0 is reference temperature of a blackbody determined by Plancks law. In Eq. (1), the term $\varepsilon_e(\overrightarrow{\xi},\overrightarrow{\omega},\lambda)$ is the amount of effective emissivity for isothermal condition which does not depend on cavity temperature, and the term $\Delta\varepsilon_e(\overrightarrow{\xi},\overrightarrow{\omega},\lambda,T_\xi,T_0)$ is the non-isothermal addition in the total value of local directional effective emissivity [3, 4].

Calculation methods of radiation characteristics are mostly used to investgate blackbody cavities. As shown in Eq. (1), to determine effective emissivities of cavities, the calculations of effective emissivities for isothermal conditions, $\varepsilon_e(\overrightarrow{\xi},\overrightarrow{\omega},\lambda)$, are an indispensable step.

The effective emissivity depends on the cavity geometry and its intrinsic surface emissivity [3–5]. The quantitative relationship between the cavity effective emissivity and its structure should be investigated at design stage to evaluate the quality of the blackbody being designed. For this purpose, calculated effective emissivity values of isothermal cavities can be used. The theoretical methods for calculating effective emissivities involve the resolution of complex integral equations describing radiative exchange process in blackbody cavities [2, 5]. For cavities having non-standard geometrical configuration, these methods are difficult to apply.

Nowadays, the Monte Carlo simulation method is most flexible that can be applied to calculations of effective emissivity for blackbody cavities. Based on probabilistic approach to the radiative phenomena and the law of large number, the Monte Carlo method allows to define the parameters of a stochastic model of interested system. This method offers the possibilities to investigate the radiation characteristics of cavities with any geometry [4–6].

In this paper, we have proposed an algorithm of Monte Carlo simulation method for calculation of directional, including normal, effective emissivities of isothermal cylindrical-inner-cone cavities. The method is based on the successive reflection ray-tracing using statistical weight approach. A simplified specular-directional diffuse reflectance model of cavity surfaces was employed during simulation. The relationship between directional effective emissivities and various cavity parameters, together with different values of optical property of cavity walls, has been studied. Our results are compared with the results obtained by other authors.

II. METHODOLOGY

II.1. Background

Inner walls of blackbody cavities are opaque by nature, and according to the generalized Kirchhoff's law and the reciprocity theorem, the average directional effective emissivity of a cavity in isothermal conditions can be expressed as follows [3, 4, 7]:

$$\varepsilon_e(\overrightarrow{\omega},\lambda) = 1-\rho_e(\overrightarrow{\omega},\lambda) = \alpha_e(-\overrightarrow{\omega},\lambda) \tag{2}$$

In Eq. (2), $\rho_e(\overrightarrow{\omega},\lambda)$ is the directional spectral effective reflectivity, $\overrightarrow{\omega}$ is the direction of observation, which coincides with the direction of radiation, and $\alpha_e(-\overrightarrow{\omega},\lambda)$ is directional effective absorptivity of cavity. The later can be evaluated by the number of successive multiple reflections of irradiation inside cavities. The lager the maximum number of successive reflection of incident radiation inside the cavity, the higher its effective absorptivity can be achieved.

In real situations, an intermediate surface of an opaque cavity may interact with the radiation resulting in partial reflection and absorption by that surface. According to the conservation laws of energy, after k times of successively reflections from surfaces of a cavity, the energy of reflected rays can be expressed as:

$$E_r(k) = E\times\rho^k \tag{3}$$

where $E_r(k)$ is the energy of the reflected ray after k times of reflections within the cavity (k =1,2,3,...), E is the initial energy of the incident ray, and ρ is the cavity surface reflectivity. If k is large enough, $E_r(k)\to 0$, or the initial ray is considered to be totally absorbed within the cavity.

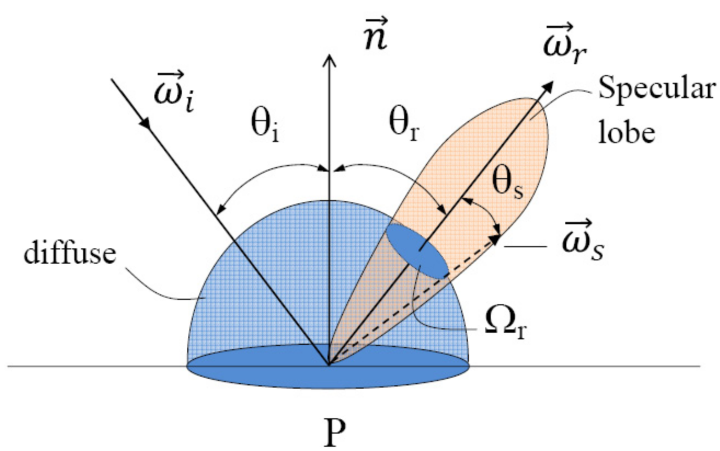

Fig. 1. A simplified directional diffuse reflection model.

The Monte Carlo simulation method based on probabilistic approach can be used to calculate radiation characteristics of an isothermal cavity [3, 7]. Such method includes tracing random propagation of rays within a cavity to estimate the probability of certain events (e.g., absorption of irradiated ray inside cavity or its escape from the cavity after a certain combination of successive reflections). In general, ray tracing between opaque surfaces can be reduced to a consecutive search of intersection points of the ray being traced with those surfaces. It is easily implemented by solving the following system of equations [3]:

$$\begin{cases} \overrightarrow{\xi} & = \overrightarrow{\xi_0} + \overrightarrow{\omega} t, \\ \Phi(\overrightarrow{\xi}) & = 0 \end{cases} \tag{4}$$

where t is parameter, $\overrightarrow{\xi_0}$ is position vector of starting point of a ray, $\overrightarrow{\omega}$ is unit direction vector of the ray, $\overrightarrow{\xi}$ is position vector of intersection point, and $\Phi(\overrightarrow{\xi}) = 0$ is equation describing a surface.

In Monte Carlo simulation method, instead of energy, a statistical weight ϖ is assigned to each initially generated ray. If such ray entered cavity through aperture, it suffered multiple reflection during interaction with cavity inner surfaces. After k times of reflections, the statistical weight of that ray will be as $\varpi(k) = \varpi.\rho^k$. The trajectory of a traced ray is ended in two cases: i) If the statistical weight $\varpi(k)$ of a traced ray became negligible (smaller than the pre-specified uncertainty of calculations) and such ray was referred to totally absorbed by cavity [3], and ii) if the traced ray escaped from the cavity after reflections. The statistical weight of each ray after its travel inside the cavity with k-time reflections is lost in an amount of [1-$\varpi(k)$].

Suppose that a radiation in direction $\overrightarrow{\omega}$ consisting of N rays entered a cavity. The effective absorptivity of that cavity can be expressed by the total loss of statistical weights of all rays as follows:

$$\alpha_e(\overrightarrow{\omega}, \lambda) = \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{m} [1 - \varpi_i(k)], \tag{5}$$

where $i = 1, 2, \ldots, N$ is index of the i-th initial ray; $k = 1, 2, \ldots, m$ is the number of reflections the i-th ray undergone during its life. If the conditions of Eq. (2) were held, one can get the effective emissivity of a cavity by the relations: $\alpha_e(-\overrightarrow{\omega}, \lambda) = \varepsilon_e(\overrightarrow{\omega}, \lambda)$.

The reflection from real-world surfaces is specular-diffuse resulting in arbitrary angular distribution of reflected radiation rays. This distribution is often characterized with the Bi-directional Distribution Function (BRDF), describing the dependence of radiation scattering on irradiation and reflection directions of radiation. The reflection models used for Monte Carlo radiation calculation usually consists of linear combination of specular and diffuse components [3, 6, 8].

$$f_r(\overrightarrow{\omega}_i, \overrightarrow{\omega}_r, \lambda) = k_{r,d} . f_{r,d}(\overrightarrow{\omega}_i, \overrightarrow{\omega}_r, \lambda) + k_{r.s} . f_{r,s}(\overrightarrow{\omega}_i, \overrightarrow{\omega}_r, \lambda), \tag{6}$$

where f_r is the BRDF of a surface; $f_{r,d}; f_{r,s}$ are BRDF of diffuse and specular reflection components, accordingly; $k_{r,d}; k_{r,s}$ are non-negative coefficients, $k_{r,d} + k_{r,s}$= 1,; $\overrightarrow{\omega}_i, \overrightarrow{\omega}_r$ are incident, perfect reflection radiation directions, and surface normal vectors, respectively (see Fig. 1). The BRDF should satisfy the reciprocal principle and energy conservation law in order to closely approximate the physical nature of radiation reflection [6], i.e.

$$f_r(\overrightarrow{\omega}_i, \overrightarrow{\omega}_r, \lambda) = f_r(\overrightarrow{\omega}_r, \overrightarrow{\omega}_i, \lambda); \int_{\Omega_r} f_r(\overrightarrow{\omega}_i, \overrightarrow{\omega}_r, \lambda)(\overrightarrow{\omega}_r, \overrightarrow{n}) d\overrightarrow{\omega}_r \leq 1, \forall \overrightarrow{\omega}_i, \tag{7}$$

In Eq. (7), $\rho(\overrightarrow{\omega}_i, \lambda) = \int_{\Omega_r} f_r(\overrightarrow{\omega}_i, \overrightarrow{\omega}_r, \lambda)(\overrightarrow{\omega}_r, \overrightarrow{n}) d\overrightarrow{\omega}_r$ is the total hemispherical reflectivity of a surface.

It is known that the angular distribution of the reflected radiation depends on the surface roughness, and for long wavelengths, surfaces become more specular.In this case, the surface reflection of radiation can be described as specular - directional diffuse, i.e. the reflected rays tend to be distributed around the perfect specular direction $\overrightarrow{\omega}_r$ within solid angles,Ω_r, forming a certain symmetric lobe (Fig. 1) [6, 8, 9].

II.2. Simulation algorithm development

We have considered a cylindrical-inner-cone cavity geometry with length L, radius R, half-angle of inner cone φ. A diaphragm with radius r is placed at the aperture that helps to form a moderately collimated radiation beam along axial direction (with some angular divergence β). Suppose that the cavities satisfied the conditions: i) The cavities were designed to work in the infrared spectral range and were in the isothermal condition, and ii) The inner surfaces of cavities were opaque, their optical characteristics were uniform everywhere in the cavities and the surface emissivity were given as ε_w.

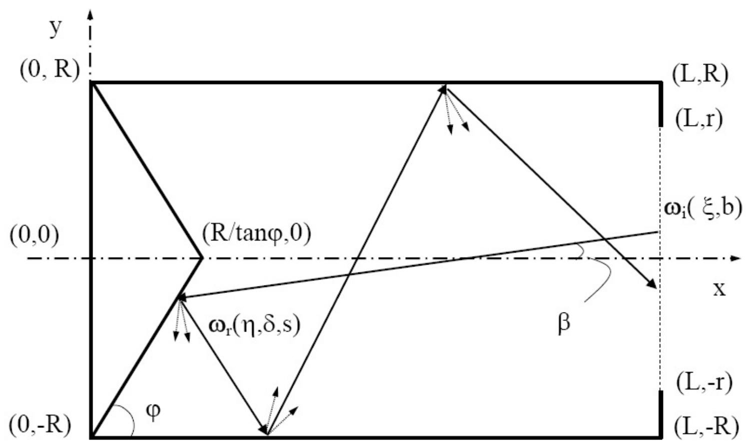

Fig. 2. A cylindrical-inner-cone geometry.

A simulation algorithm based on Monte Carlo method was developed to compute the isothermal normal effective emissivity of such cavities, based on the following considerations:

- The total cavity surface reflectivity, $\rho_w = 1 - \varepsilon_w$, is represented as a linear combination of specular and diffuse components, and the angular distributions of reflected radiations can be characterized by Eq. (6).

- The effective emissivity for investigated cavities can be determined as in Eq. (2)

As the cylindrical-inner-cone cavity is rotationally symmetrical, we can compute its directional effective emissivity in the plane consisting of initial ray (Fig. 2). All calculation results obtained in this plane are being true for the rest ones of cavity. In this case, the surface equations $\Phi(\overrightarrow{\xi}) = 0$ in Eq. (4) become the equations of lines on the plane of interest. Consequently, the BRDF of cavity surfaces depends on incident and reflection angles, θ_i and θ_r (Fig. 1), on investigated plane only, instead of solid ones. Since we are interested in calculation of cavity effective radiation characteristics in the normal direction within the small range of wavelengths, the cavity surfaces can be considered to be grey in the spectral range of interest. This 2-dimensional model leads to simplicity in calculation and computational saving.

Based on the above assumptions, we employed a simplified version of Phong's reflection model [9] to describe the radiation reflection behavior of the cavity surfaces. The BRDF of such model is described by Eq. (6). Then, the diffuse BRDF is independent of incident angles and is defined as

$$f_{r,d} = \frac{1}{\pi} \tag{8}$$

And the specular-like BRDF is expressed as:

$$f_{r,s}(\overrightarrow{\omega}_i, \overrightarrow{\omega}_r) = \frac{\upsilon+2}{2\pi} \cdot \frac{(\overrightarrow{\omega}_r . \overrightarrow{\omega}_s)^{\upsilon}}{(\overrightarrow{\omega}_i . \overrightarrow{n})} \tag{9}$$

or

$$f_{r,s}(\theta_i, \theta_r) = \frac{\upsilon+2}{2\pi} \cdot \frac{\cos^{\upsilon}(\theta_s)}{\cos\theta_i} \tag{10}$$

where θ_i and θ_r are the incident and the perfect specular reflection angles, accordingly; θ_s is the angle between the perfect specular and specular-like reflection directions, $\overrightarrow{\omega}_r$ and $\overrightarrow{\omega}_s$, respectively; $\overrightarrow{n}$ is the normal vector of the surface at the point of interaction; the power υ characterizes the shape of reflection lobe on the investigated plane (the larger υ, the sharper the reflection lobe) (Fig. 1).

The probabilistic approach in Monte Carlo method leads to the need of describing the distributions of radiation by some probability density functions (PDF) for the appropriate random variables. In the case of hemispherical reflectance, the integrand function in Eq. (7) is replaced by a PDF, $p(\theta_i, \theta_r) = f_r(\theta_i, \theta r)\cos\theta_r$. And using the reciprocal principle, we can also write as $p(\theta_i, \theta_r) = f_r(\theta_i, \theta r)\cos\theta_i$. Modeling an arbitrary PDF is usually based on generation of pseudo-random series of floating-point numbers, uniformly distributed on the [0,1] interval. There are five such numbers; ξ, b, n, δ and s that are used for our simulation. Their meanings are described later in this paper.

In Fig. 2, we suppose that $\overrightarrow{\omega}$ is the viewing direction through the aperture of the cavity and it is nearly parallel to the cavity longitudinal axis. The initial radiation in the direction $\overrightarrow{\omega}$ is generated at the aperture section. This simulated radiation is considered to consist of a rather large number, N, of rays. To ensure a statistical error less than 10^{-4}, all simulations should be performed with $N \geq 10^6$ [3,6].

Simulation process includes a set of iterations of rays and the total number of iterations is N. The initial ray equation is defined by using the two pseudo-random numbers ξ [0,1] and b[0,1], where ξ is assigned to the position of the starting point, y_0, of a ray on the aperture section, its coordinate is sampled within the range of $[-R, R]$; b is assigned to the divergence angle of initially entering ray, that is sampled within the range of $[-\beta, \beta]$. The equation of an initial ray is defined as:

$$ax + by + c = 0; (a^2 + b^2 \neq 0) \tag{11}$$

with $a = \tan\beta; b = -1; c = [y_0 - L\tan\beta]$ in our case.

Initially, a statistical weight $\varpi_0 = 1$ is assigned for each simulated ray. Next, this ray is traced by searching its intersection point with any surface of cavity. To do this we have to solve the Eq. (4) with $\Phi(\overrightarrow{\xi}) = 0$ is being the equation of intersected line between a plane consisting initial ray and a surface of cavity. This equation of line is easily found by 2 given points lying on it. The coefficients in this equation depend on cavity geometrical parameters such as L, R, φ. In our case, there are 5 such equations of cavity border sides that must be found.

Two straight lines described by equations $ax + by + c = 0$ and $Ax + By + C = 0, (aB - Ab \neq 0)$, will intersect each other at the unique point having a co-ordinate

$$x = \frac{bC - cB}{aB - Ab}; y = \frac{cA - Ca}{aB - Ab} \tag{12}$$

A line that passes through 2 points $((x_1, y_1)$ and (x_2, y_2) will be intersected with a line described by general equation Eq. (11) if the following condition is held:

$$(ax_1 + by_1 + c).(ax_2 + by_2 + c) < 0 \tag{13}$$

These conditions should be evaluated to determine exactly which side of cavity will interact with the traced ray.

Since the interaction points of a ray and a certain cavity surface, including diaphragm, are found, the ray tracing is continued after the direction of reflected ray was sampled by appropriate PDFs. In this 2-D simulation model, the reflection is defined as follows:

- Reflected ray is in the same plane of incident one as shown on Fig. 2. The incident angles, θ_i, are determined by solving line equations (Eqs. (4), (11), (12)); $\theta_r = \theta_i$, and θ_s are chosen empirically (Fig. 1).

- The type of reflection (diffuse or specular) is determined by a pseudo-random number $\eta[0,1]$: in the case of $0 \leq \eta < k_{r,d}$, the reflection is chosen as diffuse, otherwise, when $k_{r,d} \leq \eta < k_{r,d} + k_{r,s} = 1$, it is specular.

- A direction of diffusely reflected ray is determined by using a pseudo-random number σ[0,1], that is assigned to hemisphere angle, [0,π], and is sampled according to the diffuse PDF, $p_d = \frac{1}{\pi}.\cos\theta_i$. Similarly, a direction of specularly - like reflected ray within the reflection lobe is found by applying a pseudo-random number s [0,1], that is assigned to the lobe angle, $[-\theta_s, \theta_s]$, which is sampled by specular PDF, $p_s = \frac{\upsilon+2}{2\pi}.\cos^{\upsilon}(\theta_s)$, υ also is chosen empirically. After that, the equations of reflected rays are determined to continue their tracing inside cavity until rays stopped.

- The statistical weight of traced ray after k times of reflections is reduced to

$$\varpi(k) = \varpi_0.\rho_w^k \prod_{i,j} p_d(i) p_s(j) \tag{14}$$

with $i = 0, \ldots, m$; $j = 0, \ldots, l$; $m + l = k$; and k=1,2,...

The iteration of a ray (or the trajectory of a traced ray) is ended if the reflected ray intersected with aperture to escape the cavity or its statistical weight $\varpi(k)$ became less than a negligible pre-defined value, τ, or traced ray has been considered to be totally absorbed within cavity after k times of reflections. After termination of each ray, its statistical weight is tallied. Summing up all the losses of statistical weights of rays after their existence inside the cavity, we can compute the normal effective absorptivity, $\alpha_e(-\overrightarrow{\omega})$, of cavities by using Eq. (5). Taking into account that all the conditions for Eq. (2) are held, finally we get the normal effective emissivities of isothermal cylindrical-inner-cone cavities by relation $\alpha_e(-\overrightarrow{\omega}) = \varepsilon_e(\overrightarrow{\omega})$.

III. RESULTS AND DISCUSSION

The developed Monte Carlo simulation algorithm is implemented in LabView environment. This software tool has been applied to cylindrical-inner-cone cavities designing works with the purpose of their use as radiation sources in the infrared imaging calibration. The effective directional emissivities have been calculated for isothermal cavities with specular-diffuse walls in our work.

In order to validate our model, we have compared our results with those of J.Wang *et.al* [10]. In their work, the radiation characteristics of diffuse isothermal cavities have been calculated, using the STEEP3 blackbody emissivity modeling program. By analyzing the history of inverse emitting rays and using uniform specular-diffuse (USD) reflection model in the Monte Carlo simulation algorithm, the authors have evaluated the spectral and directional effective emissivities, including normal average ones, of a cylindrical-inner-cone cavity. This cavity had the parameters: $\varphi = 60^o$, L/R = 6 and its wall emissivities,ε_w, was set to 0.7,0.8 and 0.9 , respectively, in their calculations. Note that, the USD reflection model is expressed as the sum of perfect specular and perfect diffuse (Lambertian) components.

The comparison of results is shown in Table 1. Our results were obtained for cavity with geometrical parameters similar to [9] and without diaphragm ($r = R$), using a simplified 2-dimensional, specular directional-diffuse reflection model. The model parameters were chosen empirically ($\beta = 1.5^o, k_{r,d} = 0.3, \theta_s = 10.0^o, \tau = 0.001, \upsilon = 1$). In our calculation, the normal effective emissivity of such cavity was calculated with the same set of wall emissivity values, as in [**?**]. The total number of generated rays in each simulation, N, is 10^6, in our calculations.

Table 1. Average normal effective emissivities comparison for a cylindrical-inner-cone cavity with $\varphi = 60^o$, L/R = 6, and r/R =1.

Wall emissivities, (ε_w)	Average normal effective emissivities, ($\varepsilon_{e,n}$)	
	J.Wangs results (2013)	Our results
0.7	0.99125	0.991084 (σ = 2.62E-05)
0.8	0.99475	0.994903 (σ = 1.79E-05)
0.9	0.99757	0.997723 (σ = 1.44E-05)

As shown in Table 1, our results have a good convergence with the standard deviations of 10^{-5} (with the number of simulations equal to 20), and they are in good agreement with those of J.Wang *et.al*, with the uncertainty in the range of 0.01% in average. The differences between our and J.Wangs results may be explained by the fact that instead of USD reflection model as used in [10], the 2-dimension directional diffuse one was applied in our calculations. In contrast, our method has an advantage: use of such model leads to considerable simplicity and time saving in our calculations.

We have used our self-programmed LabView code to evaluate the relationship of various parameters of cavity and its average normal effective emissivities. The restriction $L \geq 2R/\tan\varphi$ [3], was applied to cavity geometry to ensure that a viewing solid angle from any point on the aperture could entirely enclose the conical base of the cavity.

Fig. 3 demonstrates relations $\varepsilon_{e,nof}(\varphi)$ for a cavity with the same set of cavity parameters (L/R=6, $r/R = 1$) and three values of wall emissivity, $\varepsilon_w = 0.7$, 0.8 and 0.9, respectively.

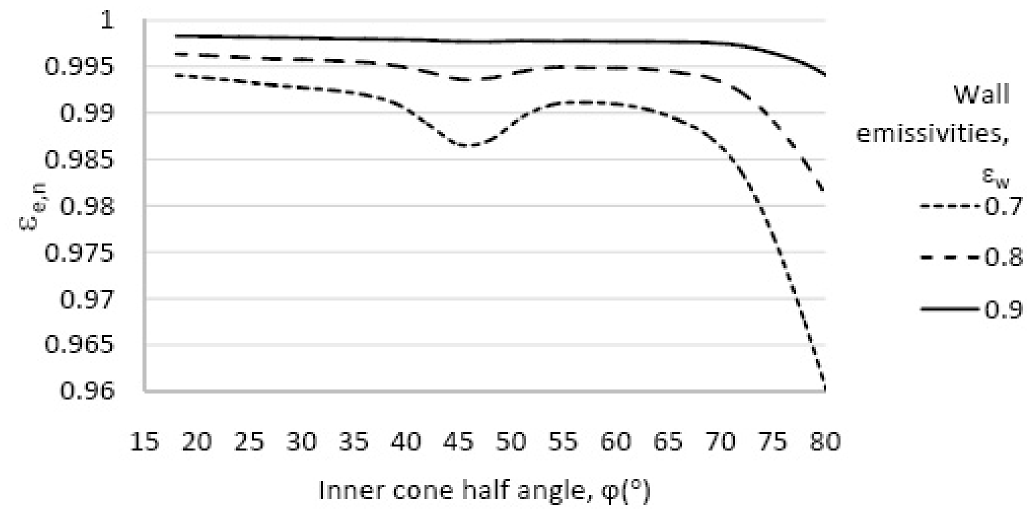

Fig. 3. Normal effective emissivity, $\varepsilon_{e,n}$, as a function of inner cone half angle,φ, for various values of wall emissivity, ε_w (in the case of isothermal cavity with parameters L/R=6, r/R=1).

With the defined set of geometrical parametersWith the defined set of geometrical parameters, the highest values of the normal effective emissivity of cavity, $\varepsilon_{e,n}$, are obtained in the ranges of $20^{\circ} < \varphi < 40^{\circ}$ and $55^{\circ} < \varphi < 70^{\circ}$ for all ε_w. For the lower and higher values of φ, the values of effective emissivities $\varepsilon_{e,n}$ strongly depend on the wall emissivity: For the same half angle φ, the higher value of $\varepsilon_w$is the higher value of $\varepsilon_{e,n}$ can be obtained. In particular, for ε_w=0.9, the values of $\varepsilon_{e,n}$ are relatively highest among three cases and seem to be independent of the cone angles, φ, in the range of interest.

In Fig. 4, the curves demonstrating relations between $\varepsilon_{e,n}$ and ratio L/R with given $\varepsilon_w = 0.7$ for various half angles, $\varphi = 30^{\circ}, 45^{\circ}$ and 60° were plotted. With $\varphi = 60^{\circ}$, the ratio $L/R > 6$ should be chosen to have $\varepsilon_{e,n} > 0.99$. In the case of $\varphi = 30^{\circ}$, cavities can be designed with a reasonable short overall length ($L/R \geq 3$) ensuring high effective emissivity. The angle $\varphi = 45^{o}$ may be not a good choice for cavity construction design with lower values of wall emissivity: if $\varepsilon_w = 0.7$, the highest value of $\varepsilon_{e,n}$ does not exceed 0.99 even though the values of L/R are high. The rest curves in Fig.4 show the relations between $\varepsilon_{e,n}$ and ratio L/R with given φ = 60 $^{\circ}$ for ε_w=0.7, 0.8 and 0.9, respectively. The ratio L/R can be reduced if the cavity walls have higher intrinsic emissivity. A cavity could be designed with L$/R \geq$2 if the wall emissivity, ε_w, is equal to 0.9 (φ = 60 $^{\circ}$). In contrast, the ratio L/R greater than 6 must be chosen, if ε_w = 0.7 for the same φ = 60 $^{\circ}$. There are critical values of L/R for each set of φ and ε_w. The values of L/R larger than those critical ones may not lead to improvement of expected effective emissivities of cavities in question.

The relation between ratio R/r and calculated values of $\varepsilon_{e,n}$ in the case of a cavity with L/R =6, $\varphi = 60^{o}$ for ε_w=0.7, 0.8, and 0.9, was demonstrated in Fig. 5, respectively. The directional effective emissivities becomes higher with increasing R/r. In other words, cavities having smaller diaphragm radius, , give higher effective emissivity, $\varepsilon_{e,n}$, with the same wall emissivity.

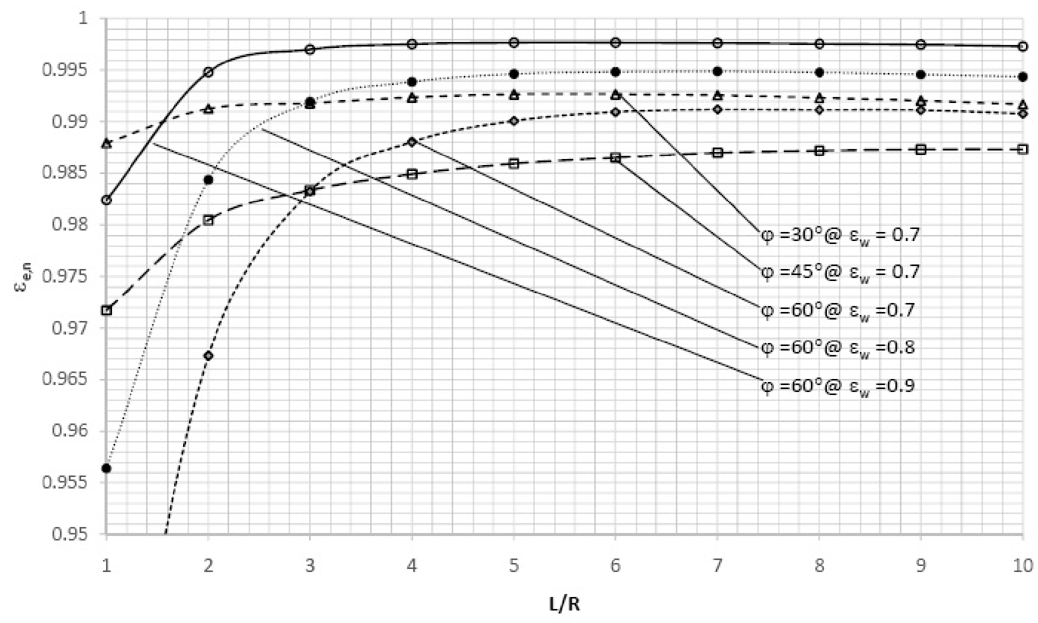

Fig. 4. Normal effective emissivity, $\varepsilon_{e,n}$, in relation with L/R ratio.

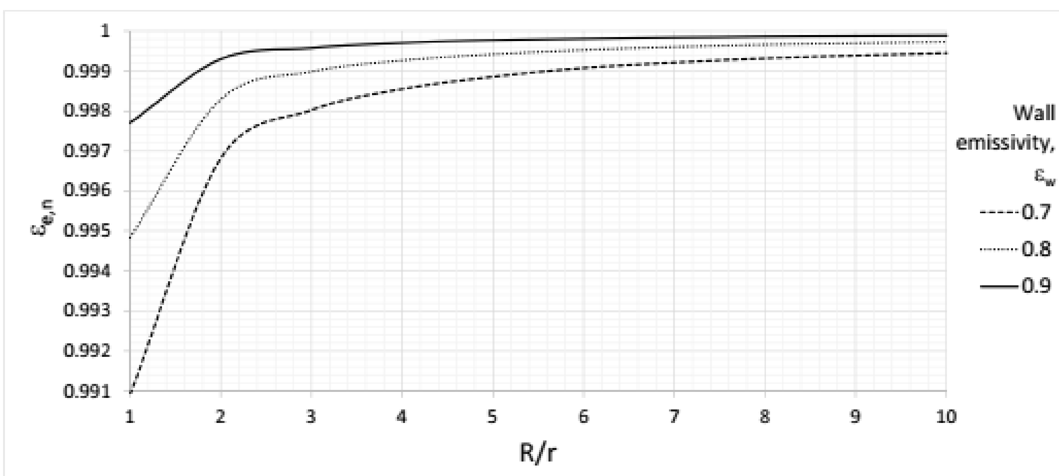

Fig. 5. Normal effective emissivities, ε_e , in relation with R/r ratio (in the case of $L/R = 6$, $\varphi = 60^\circ$).

It is easy to have the high effective emisivity for a certain set of cavity geometrical parameters, if high values of wall emissivities, ε_w, were chosen. The surface reflection property can be empirically tailored by setting appropriate θ_s and υ in Eq. (10). Taking that into account, our algorithm can be a helpful tool in engineering design of blackbody cavities.

IV. CONCLUSION

We have developed an algorithm based on Monte Carlo simulation and ray tracing methods to evaluate the average normal directional effective emissivity of cylindrical-inner-cone cavities, used as radiation sources for infrared imaging calibration. The simplified directional-diffuse reflection model was applied in our calculations for cavities working in the infrared spectral range under isothermal condition.

Our self-programmed LabView code has an advantage in simplicity and time saving of calculations. The relationships between normal effective emissivity, $\varepsilon_{e,n}$, and various cavity geometrical parameters for some set of wall emissivities values were carried out. Our results are in good agreement with those of other authors. It is evident that the software tool developed by us can be used in blackbody cavity design considerations, especially in the cylindrical-inner-cone cases.

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