

# Unparticle effects on dark matter production from photon and axion-like

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**Abstract.** *In this work, we study the effects of vector unparticle on the  $a\gamma \rightarrow \chi\bar{\chi}$  process, where  $\chi$  is a dark matter fermion. Our numerical results show that the cross-section takes values from  $8.23 \times 10^{-15}$  barn to  $5.12 \times 10^{-14}$  barn. Consequently, the cross-section with unparticle effect should be about  $10^{26}$ – $10^{28}$  times larger than obtained via pure photon exchange. This enhancement could be important for searching for dark matter, as well as unparticle.*

Keywords: unparticle; axion-like; heavy axion-like particles.

Classification numbers: 95.35.+d; 12.60.-i; 14.80.-j.

## 1. Introduction

As is well known, Dark Matter (DM) makes up most of the mass of galaxies and galaxy clusters, and is responsible for the way galaxies are organized on grand scales. Fermion dark matter is a type of DM candidate made of spin-1/2 particles which are neutral under gauge interactions and possess gravitational effects but interact weakly with Standard Model particles. Therefore, it becomes very important to consider the interactions between DM and ordinary particles.

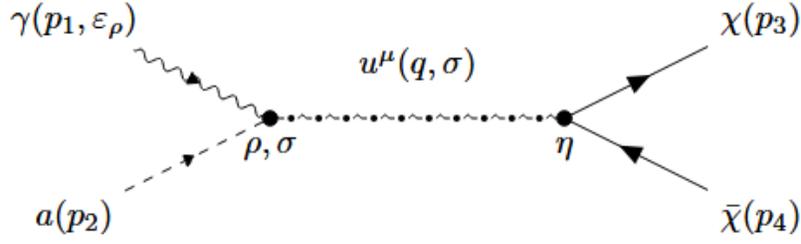
Noteworthy, unparticle physics were proposed by Howard Georgi [1] as a way to describe a scale invariant “hidden sector” that interacts with the Standard Model. Unlike ordinary particles, unparticles have a continuous mass spectrum rather than a discrete mass. They cannot be observed directly but must be inferred from their effects on Standard Model particles.

In the last few years, we have investigated unparticle effects on Bhabha scattering [2] and on axion-like particles production in  $e^+e^-$  collisions [3].

In this paper, we consider dark matter production from photon and axion-like via unparticle exchange.

## 2. The cross sections and the creations of dark matter from photon and axion-like via unparticle and photon exchange

The above-mentioned process is described by the Feynman diagram presented in Fig. 1.



**Fig. 1.** The Feynman diagram for  $a\gamma \rightarrow \chi\bar{\chi}$  process via radion.

The effective Lagrangian of describing photon ( $\gamma$ ), axion-like ( $a$ ) and dark matter fermion ( $m_\chi$ ) interaction is given by [4]

$$\mathcal{L}_{\text{int}} = \frac{C_3}{\Lambda_U^{d_u}} \phi_a F_{\mu\nu} \partial^\mu O_u^\nu + \frac{C_4}{\Lambda_U^{d_u}} \phi_a \tilde{F}_{\mu\nu} \partial^\mu O_u^\nu. \quad (1)$$

By setting

$$A = \frac{C_3}{\Lambda_U^{d_u}} \quad B = \frac{C_4}{\Lambda_U^{d_u}}, \quad (2)$$

we get the interaction vertex of  $\gamma(p_1, \rho) - a(p_2) - u(q, \sigma)$  as follows

$$A [-(p_1 \cdot q) \delta_\sigma^\rho + p_{1\sigma} q^\rho] + B \epsilon_{\sigma\mu\alpha\beta} p_1^\alpha g^{\rho\beta} q^\mu. \quad (3)$$

The amplitude for this process is

$$iM = \epsilon_\rho \left\{ A [-(p_1 \cdot q) \delta_\sigma^\rho + p_{1\sigma} q^\rho] + B \epsilon_{\sigma\mu\alpha\beta} p_1^\alpha g^{\rho\beta} q^\mu \right\} \frac{iA_{d_u}}{2 \sin(d_u \pi)} (-g^{\sigma\eta}) \times (-q^2 - i\epsilon)^{d_u-2} \bar{u}(p_3) (C_1 \gamma_\eta + C_2 \gamma_\eta \gamma_5) \frac{1}{\Lambda_u^{d_u-1}} v(p_4). \quad (4)$$

Hence, we obtain the total squared amplitude

$$|M|^2 = -D^2 \left\{ 4 \left[ |A|^2 (2(|C_1|^2 + |C_2|^2)) + 2|B|^2 (|C_1|^2 - |C_2|^2) \right] [(q \cdot p_1)(p_1 \cdot p_3)(q \cdot p_4) + (q \cdot p_1)(p_1 \cdot p_4)(q \cdot p_3)] - 12|A|^2 |C_2|^2 (q \cdot p_1)^2 (p_3 \cdot p_4) - 8 \left[ |A|^2 (|C_1|^2 + |C_2|^2) + |B|^2 (|C_1|^2 - |C_2|^2) \right] s(p_1 \cdot p_3)(p_1 \cdot p_4) \right\}, \quad (5)$$

here

$$D = \frac{-A_{d_u}}{2 \sin(d_u \pi)} (-1) (-q^2 - i\epsilon)^{d_u-2} \frac{1}{\Lambda_u^{d_u-1}}, \quad (6)$$

and

$$A_{d_u} = \frac{16 \pi^2 \sqrt{\pi}}{(2\pi)^{2d_u}} \frac{\Gamma(d_u + \frac{1}{2})}{\Gamma(d_u - 1) \Gamma(2d_u)}. \quad (7)$$

In the center of mass frame, four-momenta of particles are defined

$$p_1 = (E_1, \vec{p}), \quad p_2 = (E_2, -\vec{p}), \quad p_3 = (E, \vec{k}), \quad p_4 = (E, -\vec{k}), \quad (8)$$

$$E_1 \simeq E, \quad E_2 \simeq E, \quad (9)$$

and

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 = q^2 = 4E^2, \quad (10)$$

$$|\vec{p}| = \frac{\sqrt{s}}{2} \sqrt{1 - \frac{4m_a^2}{s}}, \quad |\vec{k}| = \frac{\sqrt{s}}{2} \sqrt{1 - \frac{4m_\chi^2}{s}}, \quad (11)$$

where  $s$  is the center of mass energy,  $m_a$  and  $m_\chi$  are axion-like mass and DM fermion mass, respectively.

From these, we have

$$p_1 \cdot p_2 = \frac{s}{2} \left( 1 - \frac{2m_a^2}{s} \right), \quad (12)$$

$$p_3 \cdot p_4 = \frac{s}{2} \left( 1 - \frac{2m_\chi^2}{s} \right), \quad (13)$$

$$p_1 \cdot p_3 = p_2 \cdot p_4 = \frac{s}{4} \left[ 1 - \sqrt{\left( 1 - \frac{4m_a^2}{s} \right) \left( 1 - \frac{4m_\chi^2}{s} \right) \cos \theta} \right], \quad (14)$$

$$p_1 \cdot p_4 = p_2 \cdot p_3 = \frac{s}{4} \left[ 1 + \sqrt{\left( 1 - \frac{4m_a^2}{s} \right) \left( 1 - \frac{4m_\chi^2}{s} \right) \cos \theta} \right], \quad (15)$$

$$q \cdot p_1 = \frac{s}{2} \left( 1 - \frac{2m_a^2}{s} \right), \quad q \cdot p_3 = q \cdot p_4 = \frac{s}{2}, \quad (16)$$

with  $\theta$  is the angle between  $\vec{p}$  and  $\vec{k}$ .

Finally, we get

$$\begin{aligned}
|M|^2 = -D^2 \frac{s^3}{2} & \left\{ 2(|A|^2 + |B|^2)(|C_1|^2 - |C_2|^2) \left(1 - \frac{2m_a^2}{s}\right) + 6|A|^2|C_2|^2 \frac{m_\chi^2}{s} \right. \\
& - [ |A|^2(|C_1|^2 + |C_2|^2) + |B|^2(|C_1|^2 - |C_2|^2) ] \\
& \left. \times \left[ 1 - \left(1 - \frac{4m_a^2}{s}\right) \left(1 - \frac{4m_\chi^2}{s}\right) \cos^2 \theta \right] \right\}. \tag{17}
\end{aligned}$$

From (17) we find the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{k}|}{|\vec{p}|} \overline{|M|^2} = \frac{1}{192\pi^2 s} \frac{\sqrt{1 - \frac{4m_\chi^2}{s}}}{\sqrt{1 - \frac{4m_a^2}{s}}} |M|^2, \tag{18}$$

where

$$\overline{|M|^2} = \frac{1}{3} |M|^2. \tag{19}$$

Next, we obtain the total cross section

$$\begin{aligned}
\sigma = \frac{s^2}{384\pi} \frac{\sqrt{1 - \frac{4m_\chi^2}{s}}}{\sqrt{1 - \frac{4m_a^2}{s}}} \frac{A_{du}^2}{\sin^2(d_u \pi)} s^{2d_u - 4} \frac{1}{\Lambda_u^{4d_u - 2}} \\
\times \left\{ \left[ -|C_3|^2(2|C_1|^2 + |C_2|^2) + 2|C_4|^2(|C_1|^2 - |C_2|^2) \right] \left(1 - 2\frac{m_a^2}{s}\right) \right. \\
+ 3|C_3|^2|C_2|^2 \left(1 - 2\frac{m_a^2}{s}\right) \left(1 - 2\frac{m_\chi^2}{s}\right) \\
\left. + \left[ |C_3|^2(|C_1|^2 + |C_2|^2) + |C_4|^2(|C_1|^2 - |C_2|^2) \right] \left[ 1 - \frac{1}{3} \left(1 - 4\frac{m_a^2}{s}\right) \left(1 - 4\frac{m_\chi^2}{s}\right) \right] \right\}. \tag{20}
\end{aligned}$$

Taking  $C_1 = C_2 = 1/\sqrt{2}$ ,  $C_3 = C_4 = 1$  as input parameters [5–7], we have

$$\sigma = \frac{s^2}{384\pi} \frac{\sqrt{1 - \frac{4m_\chi^2}{s}}}{\sqrt{1 - \frac{4m_a^2}{s}}} \frac{A_{dU}^2}{\sin^2(d_U \pi)} s^{2d_U - 4} \frac{1}{\Lambda_U^{4d_U - 2}} \left( \frac{4m_a^2}{s} + \frac{m_\chi^2}{s} + 2\frac{m_a^2}{s} \frac{m_\chi^2}{s} \right). \tag{21}$$

In the same way above mentioned, we have determined the interaction vertex of  $\gamma(q_1, \rho) - \gamma(q_2, \sigma) - a(p)$  as follows

$$-\frac{1}{2} g_{a\gamma} \varepsilon_{\rho\sigma\mu\alpha} q_1^\mu q_2^\alpha. \tag{22}$$

Hence, we have found the total squared amplitude for the process  $a\gamma \rightarrow \chi\bar{\chi}$  via exchange of photon

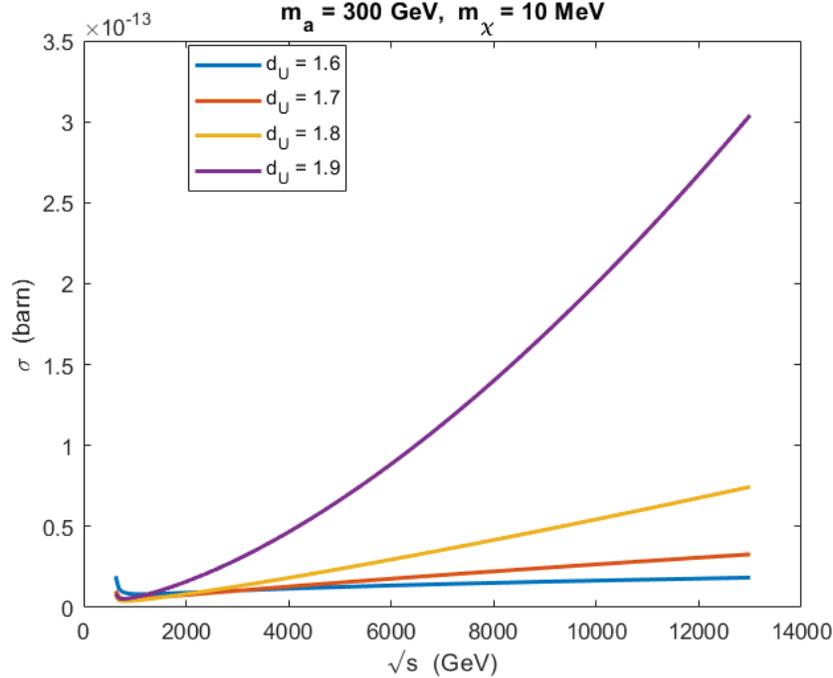
$$|M_\gamma|^2 = \frac{1}{64} g_{a\gamma}^2 \left[ \frac{s^2}{2} (\mu_\chi^2 - d_\chi^2) - \frac{s^2}{2} (\mu_\chi^2 - d_\chi^2) \left(1 - \frac{4m_\chi^2}{s}\right) \cos^2 \theta + 4s m_\chi^2 d_\chi^2 \right], \quad (23)$$

where,  $\mu_\chi$  and  $d_\chi$  correspond to the magnetic dipole moment and the electric dipole moment of the dark matter fermion  $\chi$ . The total cross section for this process is

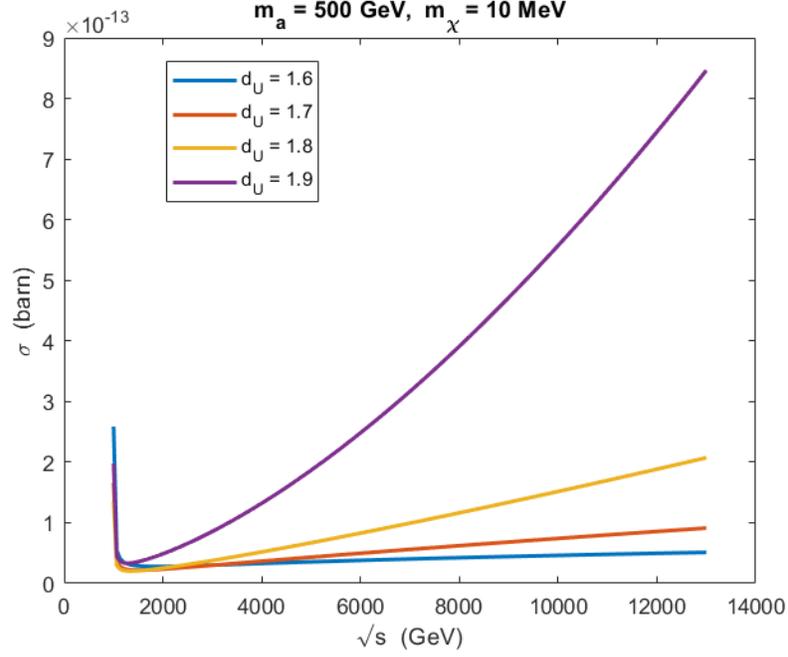
$$\sigma_\gamma = \frac{g_{a\gamma}^2}{9216\pi} \left[ s(\mu_\chi^2 - d_\chi^2) \left(1 + \frac{2m_\chi^2}{s}\right) + 12m_\chi^2 d_\chi^2 \right]. \quad (24)$$

### 3. Numerical results and discussion

Axion-like particles (ALPs) are hypothetical particles with a vast, largely unexplored mass range, from ultra-light (eV range) to heavy (GeV–TeV), constrained by experiments. Their mass dictates how they are searched for light ALPs via helioscopes, heavier ones via colliders, and astrophysical limits cover wide spans. Therefore, we choose a wide range of axion-like masses corresponding to the medium and heavy axion-like particles. Let us present the variation of the total cross-section  $\sigma$  as a function of  $\sqrt{s}$  with axion-like mass  $m_a = 300$  GeV (Fig. 2) and  $m_a = 500$  GeV (Fig. 3).



**Fig. 2.** (Color online) The total cross-section due to the unparticle contribution depending on the center of mass energy for  $m_a = 300$  GeV. Here, we assume  $m_\chi = 10$  MeV,  $\Lambda_U = 1$  TeV and  $d_U = 1.6, 1.7, 1.8, 1.9$ .



**Fig. 3.** (Color online) The total cross-section as a function of  $\sqrt{s}$  for  $m_a = 500$  GeV,  $m_\chi = 10$  MeV,  $\Lambda_U = 1$  TeV and  $d_U = 1.6, 1.7, 1.8, 1.9$ .

**Table 1.** The total cross-section with unparticle and photon effects, and their ratio  $\sigma/\sigma_\gamma$  at different energies for  $m_a = 300$  GeV.

$\sqrt{s}$ (GeV)	$\sigma$ (barn)	$\sigma_\gamma$ (barn)	$\sigma/\sigma_\gamma$
1000	$8.2304 \times 10^{-15}$	$1.2906 \times 10^{-43}$	$6.3774 \times 10^{28}$
2000	$9.1075 \times 10^{-15}$	$5.1622 \times 10^{-43}$	$1.7643 \times 10^{28}$
3000	$1.0428 \times 10^{-14}$	$1.1615 \times 10^{-42}$	$8.9785 \times 10^{27}$
4000	$1.1595 \times 10^{-14}$	$2.0649 \times 10^{-42}$	$5.6154 \times 10^{27}$
5000	$1.2625 \times 10^{-14}$	$3.2264 \times 10^{-42}$	$3.9132 \times 10^{27}$
6000	$1.3550 \times 10^{-14}$	$4.6672 \times 10^{-42}$	$2.9046 \times 10^{27}$
7000	$1.4393 \times 10^{-14}$	$6.3273 \times 10^{-42}$	$2.2762 \times 10^{27}$
8000	$1.5169 \times 10^{-14}$	$8.2596 \times 10^{-42}$	$1.8366 \times 10^{27}$
9000	$1.5892 \times 10^{-14}$	$1.0454 \times 10^{-41}$	$1.5202 \times 10^{27}$
10000	$1.6569 \times 10^{-14}$	$1.2492 \times 10^{-41}$	$1.3269 \times 10^{27}$
11000	$1.7207 \times 10^{-14}$	$1.5166 \times 10^{-41}$	$1.1019 \times 10^{27}$
12000	$1.7813 \times 10^{-14}$	$1.8548 \times 10^{-41}$	$9.5848 \times 10^{26}$
13000	$1.8389 \times 10^{-14}$	$2.1810 \times 10^{-41}$	$8.4311 \times 10^{26}$

As we can observe from Figs. 2 and Fig. 3, the cross-section  $\sigma$  strongly increases with increasing  $\sqrt{s}$  for  $d_U = 1.9$ . In the cases of  $d_U = 1.6, 1.7, 1.8$ , the cross-sections much more weakly increase with  $\sqrt{s}$ .

**Table 2.** The total cross-section with unparticle and photon effects, and their ratio  $\sigma/\sigma_\gamma$  at different energies for  $m_a = 500$  GeV.

$\sqrt{s}$ (GeV)	$\sigma$ (barn)	$\sigma_\gamma$ (barn)	$\sigma/\sigma_\gamma$
1000	–	$1.2906 \times 10^{-43}$	–
2000	$2.7867 \times 10^{-14}$	$5.1622 \times 10^{-43}$	$5.3982 \times 10^{28}$
3000	$3.0104 \times 10^{-14}$	$1.1615 \times 10^{-42}$	$2.5919 \times 10^{28}$
4000	$3.2889 \times 10^{-14}$	$2.0649 \times 10^{-42}$	$1.5927 \times 10^{28}$
5000	$3.5535 \times 10^{-14}$	$3.2264 \times 10^{-42}$	$1.1014 \times 10^{28}$
6000	$3.7978 \times 10^{-14}$	$4.6460 \times 10^{-42}$	$8.1753 \times 10^{27}$
7000	$4.0351 \times 10^{-14}$	$6.3237 \times 10^{-42}$	$6.3643 \times 10^{27}$
8000	$4.2351 \times 10^{-14}$	$8.2596 \times 10^{-42}$	$5.1275 \times 10^{27}$
9000	$4.4320 \times 10^{-14}$	$1.0454 \times 10^{-41}$	$4.2397 \times 10^{27}$
10000	$4.6173 \times 10^{-14}$	$1.2942 \times 10^{-41}$	$3.5778 \times 10^{27}$
11000	$4.7925 \times 10^{-14}$	$1.5166 \times 10^{-41}$	$3.0690 \times 10^{27}$
12000	$4.9590 \times 10^{-14}$	$1.8588 \times 10^{-41}$	$2.6684 \times 10^{27}$
13000	$5.1177 \times 10^{-14}$	$2.1810 \times 10^{-41}$	$2.3464 \times 10^{27}$

Next, we give the numerical values of the total cross-section with unparticle and photon effects, and the ratio  $\sigma/\sigma_\gamma$  at different energies in the Tables 1, 2 for  $m_a = 300$  GeV,  $m_a = 500$  GeV, respectively. The input parameters are:  $\Lambda_U = 1$  TeV,  $g_{a\gamma} = 10^{-10}$  GeV $^{-1}$ ,  $\mu_\chi = \frac{1}{8.4 \times 10^6}$  TeV $^{-1}$ ,  $d_\chi = \frac{1}{8.7 \times 10^6}$  TeV $^{-1}$ ,  $m_\chi = 10$  MeV,  $d_U = 1.6$ .

From numerical results, we have found that the cross sections are about 10-50 fb. Therefore, the effects of the unparticle on the cross-sections for the creation of heavy dark matter fermions from photon–axion-like can be very strong. If the measurement is carried out at  $\sqrt{s} = 1$  TeV – 13 TeV then the  $\sigma$  for the process  $a\gamma \rightarrow \chi\bar{\chi}$  should be detectable. Notably, if unparticle physics is disregarded, the cross-section of this process is very small. The above results show that the cross-section  $\sigma$  is larger than  $\sigma_\gamma$  by 26–28 orders of magnitudes. Therefore, the unparticle effect gives us hope that we can find dark matter from photon and axion-like.

#### 4. Conclusion

Within the framework of the Standard Model, our process involving photon exchange, has a cross section of approximately  $10^{-42}$ , which is very small. Hence, this process is undetectable. However, the situation is completely different in the same process, but through unparticle exchange. In summary, this paper shows that the interaction vertex of unparticle to photon–axion-like can give rise to observable signatures of DM fermions.

Along with recent papers on the creation of DM fermions in collisions [8, 9], our work may have important implications for dark matter searches at future colliders. Our work can be extended for other processes, for example,  $e^+e^- \rightarrow \nu\bar{\nu}$  via the unparticle.

#### Conflict of interest

The authors have no conflict of interests to declare.

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