NUMERICAL AND EXPERIMENTAL MODELING OF INTERACTION BETWEEN A TURBULENT JET FLOW AND AN INLET

H. D. LIEN*, I. S. ANTONOV**

* Agricultural University - Hanoi, Faculty of Mechanization & Electrification
** Technical University of Sofia, Bulgaria, Hydroaerodynamics Department

ABSTRACT. In ventilation devices to get rid of harmful substances out of working places, we use sucking devices. The local sources of pollution are evacuated by them. A basic element when creating the model of sucking device is: the source of harmful substances is discussed as a rising convective flow, which is ejected out of sucking spectrum, created by a sucking apparatus. In the present work, the flow is a whole one with variable quantity of motion and kinetic energy along it’s length. The change in those two parameters is caused by and is in dependent function of the inlet spectrum. There has been discussed a two-component flow of air and gas in ventilation devices. A two-velocity scheme of flow is used to realise the numerical method. An integral method of investigation is used, based on the conditions of conservation of mass contents, quantity of motion and kinetic energy. It’s been accepted that quantity of motion and energy change in function of inlet action. A comparison of numerical results and natural experiment are made for two conditions: full suck and not full suck. Conclusion is that the present model is precise and can be unset for engineering calculations.

Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_c$</td>
<td>capacity in initial section</td>
</tr>
<tr>
<td>$Q_i$</td>
<td>capacity in inlet</td>
</tr>
<tr>
<td>$L$</td>
<td>distance between outgoing section of jet and inlet</td>
</tr>
<tr>
<td>$r_0$</td>
<td>initial radius of jet</td>
</tr>
<tr>
<td>$r_i$</td>
<td>initial radius of inlet</td>
</tr>
<tr>
<td>$u_g$</td>
<td>velocity of air (carrier phase)</td>
</tr>
<tr>
<td>$u_{po}$</td>
<td>initial velocity of admixture</td>
</tr>
<tr>
<td>$u_{go}$</td>
<td>initial velocity of air</td>
</tr>
<tr>
<td>$R_u$</td>
<td>dynamic boundary layer</td>
</tr>
<tr>
<td>$R_p$</td>
<td>diffusion boundary layer</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>density of admixture</td>
</tr>
<tr>
<td>$u_p$</td>
<td>velocity of admixture (smoke)</td>
</tr>
<tr>
<td>$u_{gn}$</td>
<td>maximum velocity of air</td>
</tr>
<tr>
<td>$u_{pm}$</td>
<td>maximum velocity of admixture</td>
</tr>
<tr>
<td>$Re_p$</td>
<td>Reynolds’s number</td>
</tr>
<tr>
<td>$\chi$</td>
<td>concentration of admixture</td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>initial concentration of admixture</td>
</tr>
<tr>
<td>$F_x$</td>
<td>inter-phase forces</td>
</tr>
<tr>
<td>$\nu_{tp}$</td>
<td>turbulent viscosity of admixture</td>
</tr>
<tr>
<td>$\nu_{tg}$</td>
<td>turbulent viscosity of air</td>
</tr>
<tr>
<td>$S\bar{c}_t$</td>
<td>schmidt’s turbulent number</td>
</tr>
<tr>
<td>$S_{ij}$</td>
<td>complexes of constants</td>
</tr>
<tr>
<td>$\rho_g$</td>
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<td>$\rho_{go}$</td>
<td>initial density of air</td>
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</table>
1. Introduction

The applications of some methods of calculating such devices are given in [1] and others. Some well-known works about this problem [1, 2, 3] etc., when developing a numerical model of the flow, discuss it often by method summing up the flows (superposition). Allthrough the last given satisfying results, by a theoretical point of view it’s not very precise. It has been presumed summing up a real turbulent jet flow to a potential one, created by an inlet (Sucking) Spectrum. in order to avoid this moment, in the present work, the flow is a whole one with variable quantity of motion and kinetic energy along its length. The change in those two parameters is caused by and is in dependent function of the inlet spectrum. This shown in experimental studies [4, 5, 8]. In the present model, the unreliable summing up of the flows is avoided and has given a solution of complex interaction of jet and inlet spectrum, using the usual methods in the dynamics of real fluids.

2. Basic of the numerical model

There has been discussed a two-component flow of air-smoke gases. To realise the numerical model a two-velocity scheme of the flow is used and it has been accepted that velocities of two components do not coincide [6, 7].

The system equation of motion for axis-symmetrical two-phase turbulent jets can be received by development of theory of turbulent jet of Abramovich and in cartesian-coordinate has a form:

$$\frac{\partial u_g}{\partial x} + \frac{1}{y} \frac{\partial (u_g y)}{\partial y} = 0,$$

$$\frac{\partial u_p}{\partial x} + \frac{1}{y} \frac{\partial (u_p y)}{\partial y} = 0,$$

$$u_g \frac{\partial u_g}{\partial x} + u_g \frac{\partial u_g}{\partial y} = -\frac{1}{\rho_g} \frac{\partial}{\partial y} (\rho_g u_g' w_g') - \frac{u_g'^2}{y} \frac{F_x}{\rho_g},$$

$$u_p \frac{\partial u_p}{\partial x} + u_p \frac{\partial u_p}{\partial y} = -\frac{1}{\rho_p} \frac{\partial}{\partial y} (\rho_p u_p' w_p') - \frac{u_p'^2}{y} \frac{F_x}{\rho_p} + F_x,$$

$$u_p \frac{\partial \chi}{\partial x} + u_p \frac{\partial \chi}{\partial y} = -\frac{1}{\rho_p} \frac{\partial}{\partial y} (\rho_p u_p' \chi') - \frac{u_p' \chi'}{y},$$
where $\chi = \frac{\rho_p}{\rho_g}$, $F_x$ - inter-phase forces [9]:

$$F_x = k_x \rho_p (u_g - u_p)^2.$$  

The coefficient $k_x$ is contrary to [10], that’s function of Reynold’s number, formed by experimental formula as follows:

$$k_x = B(1 + b_1 Re_p^{0.5} + b_2 Re_p)$$

where $B = 0.01 \div 0.03$; $b_1 = 0.179$; $b_2 = 0.013$; $Re_p = \frac{(u_g - u_p)D_p}{\nu_p}$.  

It’s necessary to give the boundary conditions ($y = 0; y = R_u$), when solving over system of equations.

In the equations of movement, the double correlation of velocity, concentration $u'\nu'_g$, $u'_p\nu'_p$, $\nu'_p\chi'$, we can define these correlations, using turbulent viscosity and the field of mean parameters:

$$\frac{u'_g\nu'_g}{\nu_g} = -\nu_{tg} \frac{\partial u_g}{\partial y}; \quad \frac{u'_p\nu'_p}{\nu_p} = -\nu_{tp} \frac{\partial u_p}{\partial y}; \quad \frac{\nu'_p\chi'}{Sc_t} = -\frac{\nu_{tp}}{Sc_t} \frac{\partial \chi}{\partial y}.$$  

An integral method of investigation is used, based on the conditions of conservation of mass contents, for total quantity of motion, for kinetic energy of two-phases, conditions from higher rank of concentration and additional relations between parameters. It has been accepted that quantity of motion and energy change in function of inlet action [8].

The numerical model is developed on the basis of following integral conditions [5]:

$$\int_0^\infty \rho_g u_g y dy = G_1, \quad (2.6)$$

$$\int_0^\infty \rho_g u'_g y dy + \int_0^\infty \rho_p u'_p y dy = I(x), \quad (2.7)$$

$$\frac{\partial}{\partial x} \int_0^\infty \rho_g u'_g y dy = -2 \int_0^\infty \rho_{tg} \left( \frac{\partial u_g}{\partial y} \right)^2 y dy - 2 \int_0^\infty u_g F_x y dy + E(x), \quad (2.8)$$

$$\frac{\partial}{\partial x} \int_0^\infty \rho_p u'_p y dy = -2 \int_0^\infty \rho_{tp} \left( \frac{\partial u_p}{\partial y} \right)^2 y dy + 2 \int_0^\infty u_p F_x y dy + E(x), \quad (2.9)$$

31
In our equations above a model of turbulence analogies to Schetz’s model is suggested [7] as follow:

\[ \nu_{t0} = k_x R_u u_{gm}, \]

\[ \nu_{tp} = k_x R_u u_{pm}. \]

On the right side of equations (2.7), (2.8) and (2.9) standing the quantity of motion and flow energy are variables along the stream. According to experimental studies [4], [5], [8]. They can be presented in the following:

\[ I(x) = I_1 (1 + k_1 x^n), \]

\[ E(x) = E_1 (1 + k_2 x^n). \]

The equations (2.12), (2.13) are numerically investigated when inputting suitable for the solution values of \( k_1, k_2, n \) and \( m \) for the corresponding regime [4], [5].

Using equation (2.11) we get the connection between diffusion boundary layer \( R_p \), dynamic boundary layer \( R_u \) and Schmidt’s turbulent number \( S\sigma_t \)

\[ S\sigma_t = S\sigma_0 (1 + \sqrt{1 + \xi_0}) \]

\( S\sigma_0 = 0.75 \) (in the investigation of Abramovich), \( \xi_0 \) is an adjusted initial particle concentration which is expressed by the following ratio:

\[ \xi_0 = \frac{x_0}{1 + x_0}. \]

In the system of equation (2.6) ÷ (2.10) the marked integrals are done using the similarity of cross velocity and concentration distribution of the kind:

\[ \frac{u_g}{u_{gm}} = \frac{u_p}{u_{pm}} = \exp(-K_u \eta^2), \]

\[ \frac{\chi}{\chi_m} = \exp(-K_x \eta^2), \]

where \( \eta = \frac{y}{x} \), \( K_u = 92 [6] \), \( K_x = S\sigma_t K_u \).

Having done the integrals after some revision and normalization, we obtain the following system of algebraical-differential equations:
where

\[ \bar{x} = \frac{x}{y}, \quad \bar{u}_{pm} = \frac{u_{pm}}{u_g}, \quad \bar{u}_{gm} = \frac{u_{gm}}{u_g}, \quad \bar{R}_u = \frac{R_u}{y}, \quad \bar{R}_p = \frac{R_p}{y}. \]

In which the values of \( A_{ij} \) integrals given in Table 1. Normalisation is done with the initial parameters of the flow. The system of equations (2.14) – (2.18) is solved numerically using a suitable algorithm. The joint solution of (2.14) – (2.18) comes to an equation regarding \( u_{gm} \) of the kind:

\[ S_38\bar{u}_{gm}^9 + S_37\bar{u}_{gm}^8 + S_36\bar{u}_{gm}^7 + S_35\bar{u}_{gm}^6 + S_34\bar{u}_{gm}^5 + S_33\bar{u}_{gm}^4 
+ S_32\bar{u}_{gm}^3 + S_31\bar{u}_{gm}^2 + S_30\bar{u}_{gm} + S_{29} = 0, \quad (2.19) \]

where \( S_{ij} \) is complex of constants, which given in Table 2.

**Table 1**

<table>
<thead>
<tr>
<th></th>
<th>( A_{11} )</th>
<th>( A_{21} )</th>
<th>( A_{22} )</th>
<th>( A_{31} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \int )</td>
<td>( \frac{1}{2(K_x + K_u)} )</td>
<td>( \frac{1}{4K_u} )</td>
<td>( \frac{1}{2(K_x + 2K_u)} )</td>
<td>( \frac{1}{6K_u} )</td>
</tr>
</tbody>
</table>
\[
A_{32} = 2K_z \int_0^\infty \left[ \frac{\partial}{\partial \eta} \left( \frac{u_g}{u_{gm}} \right) \right]^2 \eta d\eta \quad K_x
\]

\[
A_{33} = 2K_z \int_0^\infty \left( \frac{u_g}{u_{gm}} \right)^2 \eta d\eta \quad \frac{K_x}{2K_u}
\]

\[
A_{41} = \int_0^\infty \left( \frac{\chi}{\chi_m} \right) \left( \frac{u_p}{u_{pm}} \right)^2 \eta d\eta \quad \frac{1}{2(K_x + 3K_u)}
\]

\[
A_{42} = 2K_z \int_0^\infty \left( \frac{\chi}{\chi_m} \right) \left[ \frac{\partial}{\partial \eta} \left( \frac{u_g}{u_{gm}} \right) \right]^2 \eta d\eta \quad \frac{4K_x K^2_u}{(K_x + 2K_u)^2}
\]

\[
A_{43} = 2K_z \int_0^\infty \left( \frac{u_p}{u_{pm}} \right)^2 \eta d\eta \quad \frac{K_x}{3K_u}
\]

\[
A_{51} = \int_0^\infty \left( \frac{\chi}{\chi_m} \right)^2 \left( \frac{u_p}{u_{pm}} \right) \eta d\eta \quad \frac{1}{2(2K_x + K_u)}
\]

\[
A_{52} = \frac{2K_x}{Sc_t} \int_0^\infty \left[ \frac{\partial}{\partial \eta} \left( \frac{\chi}{\chi_m} \right) \right]^2 \eta d\eta \quad \frac{K_x}{Sc_t}
\]

**Table 2**

\[
S_1 = I_1(1 + k_1 x^n) \quad S_2 = E_1(1 + k_2 x^n)
\]

\[
S_3 = \frac{S_1 A_{11}}{G_1 A_{22}} \quad S_4 = \frac{A_{21} A_{11} x^2}{G_1 A_{22}}
\]

\[
S_5 = \frac{4A_{41} A_{51} x}{2A_{41} A_{52} + A_{42} A_{51}} \quad S_6 = \frac{A_{11} A_{43} A_{51} x^2}{G_1(2A_{41} A_{52} + A_{42} A_{51})}
\]

\[
S_7 = \frac{S_2 A_{11} A_{51} x^2}{G_1(2A_{41} A_{52} + A_{42} A_{51})} \quad S_8 = \frac{n I_1 k_1 x^{n-3}}{2A_{21}}
\]

\[
S_9 = \frac{G_1 A_{22} A_{52}}{2A_{11} A_{21} A_{51} x^4} \quad S_{10} = \frac{2A_{22} G_1}{A_{11} A_{21} x^3}
\]

34
Equation (2.19) is solved by the method of Newton. The determined \( u_{ij} \) is replaced consecutively in the rest equations and demanded quantities are given. With the initial conditions of flow: The initial concentration and velocities components, specific weight, quantity of motion and flow energy are used as input data:

\[
X = 0, \quad u_g = u_{g0}, \quad u_p = u_{p0}, \quad \chi = \chi_0, \quad G = G_1, \quad I = I_1, \quad E = E_1
\]

\[
X = L, \quad I = I_1(1 + k_1x^n), \quad E = E_1(1 + k_2x^m).
\]

The distance between outgoing section of jet and inlet is \( \bar{L} = \frac{L}{r_0} = 20 \) and the
relation of capacities in initial section and in the inlet is:

$$Q_c = \frac{Q_c}{Q_i}$$

where

$$Q_c = 2\pi \int_0^{r_0} u_p r dr, \quad Q_i = 2\pi \int_0^{r_i} u_p r dr$$

with two cases: A case with full such ($\bar{Q}_c = 3.8$) and a case with not full such ($\bar{Q}_c = 1.6$), where $I_1, E_1, k_1, n, k_2$ and $m$ are followed [4], [8].

<table>
<thead>
<tr>
<th>$\bar{L}$</th>
<th>$\bar{Q}_c$</th>
<th>$I_1$</th>
<th>$E_1$</th>
<th>$k_1$</th>
<th>$n$</th>
<th>$k_2$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.6</td>
<td>0.4447</td>
<td>0.4019</td>
<td>6.10^{-9}</td>
<td>7.3300</td>
<td>1.09\cdot10^{-13}</td>
<td>11.4319</td>
</tr>
<tr>
<td>20</td>
<td>3.8</td>
<td>0.3113</td>
<td>0.2010</td>
<td>9.11\cdot10^{-13}</td>
<td>10.7720</td>
<td>8.275\cdot10^{-19}</td>
<td>16.0380</td>
</tr>
</tbody>
</table>

The following integral parameters of jet are results of solution: the change of maximum velocities components ($u_{pm}, u_{gm}$), concentration ($\chi$) and borderlines of diffusion $R_p$ and dynamic $R_u$ jet boundary layers. Results of calculation about two conditions-full suck and not full suck given on Fig. 1 and Fig. 2, where there is comparison with experimental data [4]. In the experimental the second component as an admixture is a smoke gas. To determine the diffusion border of flow. We can make a comparison of numerical results and experimental results $R_p$.

Fig. 1. A case with full suck

36
Some conclusions are drawn by checking with the experimental data:

- the above model is more reliable and can be used for engineering calculations;
- the considerable contraction of diffusion boundary layer speaks about a great security in realising such devices. Being enveloped by a zone filled with air of environment, does no allow any harms to come out into the working places. This, of course, is possible when the sucking installation works in a condition of full suck or not full suck.

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MÔ HÌNH SỐ VÀ THỰC NH combos ĐINH GÌA ĐONG PHUN RÔI VÀ MIĘNG HÚT


Tóm lại mô hình thực tại là đảm bảo độ chính xác và có thể được ứng dụng để tính toán trong kỹ thuật.