INTERACTION BETWEEN PARAMETRIC AND FORCED OSCILLATIONS IN FUNDAMENTAL RESONANCE

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In nonlinear systems, the interaction between different oscillations is complicated and has attracted the attention of a lot of researches [1, 2]. Interesting results have been obtained, some aspects of this phenomenon can be found in a recent work [3].

The present paper is devoted to examine the interaction between parametric and forced oscillations in fundamental resonances. Some remark about the resolution of the equations determining the stationary oscillations will be given, some particularities of the resonance curve will be described.

§1. SYSTEM UNDER CONSIDERATION AND THE AVERAGING METHOD

Let us consider a quasi-linear oscillating system governed by the differential equation

\[ x + \omega^2 x = \varepsilon \left( -h \dot{x} - \gamma x^3 + \Delta x + 2p \cos 2\omega t + q \cos(\omega t + \phi) \right) \]  

where \( x \) - an oscillatory variable, \( \varepsilon > 0 \) - a small parameter, \( h \geq 0 \) - the damping viscous coefficient, \( \gamma \) - the cubic nonlinearity coefficient, \( 2p > 0, q > 0 \) and \( 2\omega, \omega \) - intensities and frequencies of the parametric and external excitations, respectively, \( \Delta = (\omega^2 - 1) \) - the detuning parameter \( (1 - \text{the natural frequency}) \), \( \phi (0 \leq \phi < 2\pi) \) - the dephase angle between two excitations.

Introducing slowly varying variables \( a, \theta \) (amplitude and dephase of the oscillatory regime).

\[ x = a \cos \varphi, \quad \dot{x} = -aw \sin \varphi, \quad \varphi = \omega t + \theta \]  

we establish the averaged equations:

\[ \dot{a} = -\frac{\varepsilon a}{2\omega} \left\{ hwa + pa \sin 2\theta + q \sin(\theta - \phi) \right\} \]  

\[ \dot{\theta} = -\frac{\varepsilon}{2a} \left\{ \left( \Delta - \frac{3\gamma a^2}{4} \right) a + pa \cos 2\theta + q \cos(\theta - \phi) \right\} \]  

Let \((a_0, \theta_0)\) be the amplitude and the dephase of the stationary oscillation. By vanishing the right hand sides of (1.3), we obtain two algebraic - trigonometrical equations for determining \((a_0, \theta_0)\):

\[ haw + pa \sin 2\theta + q \sin(\theta - \phi) = 0 \]  

\[ \left( \Delta - \frac{3\gamma a^2}{4} \right) a + pa \cos 2\theta + q \cos(\theta - \phi) = 0 \]  

or, in equivalent form:

\[ haw \sin \theta - \left[ \left( \frac{3\gamma a^2}{4} - \Delta \right) - p \right] a \cos \theta = -q \cos \phi \]  

\[ \left[ \frac{3\gamma a^2}{4} - \Delta \right] + p \] a \sin \theta + haw \cos \theta = q \sin \phi \]
As usual, first, (1.5) will be considered as two linear algebraic equations of two unknowns
\[ u = \sin \theta, \quad v = \cos \theta. \]
Then using trigonometrical formulae (for instance \( \sin^2 \theta + \cos^2 \theta = 1 \)) the amplitude - frequency relationship will be obtained.

Two cases must be distinguished:
1. The "ordinary" case where the determinant (of the algebraic linear equations (1.5a)) is different to zero
\[
D = \left| \begin{array}{cc}
\frac{\hbar \omega}{a} & -(\frac{3\gamma}{4}a^2 - \Delta) - p \end{array} \right| = a^2 D \neq 0
\]
or (since \( q \neq 0 \), the system considered does not admit the equilibrium regime \( \alpha = 0 \)):
\[
D = \left( \frac{3\gamma}{4}a^2 - \Delta \right)^2 + \hbar^2 \omega^2 - p^2 \neq 0
\]
2. The "critical" case where \( D = 0 \) or \( D = 0 \)
To illustrate this remark, we shall examine in detail the oscillating system without damping.

§2. RESONANCE CURVE OF THE OSCILLATING SYSTEM WITHOUT DAMPING

For the system without damping, \( \hbar = 0 \), the equations (1.5) as the determinant (1.6) become more simple
\[
\left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) - p \right] a \cos \theta = q \cos \sigma
\]
\[
\left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) + p \right] a \sin \theta = q \sin \sigma
\]
\[
D = \left( \frac{3\gamma}{4}a^2 - \Delta \right)^2 - p^2 = \left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) - p \right] \left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) + p \right]
\]
It is noted that, in the plane \((\Delta, a^2)\), \( D = 0 \) is just the resonance curve of the pure - parametrically - excited system \( q = 0 \) which degenerates into two straight lines:
\[
D_1 : \frac{3\gamma}{4}a^2 = \Delta + p \quad \text{and} \quad D_2 : \frac{3\gamma}{4}a^2 = \Delta - p
\]
1. If \( D \neq 0 \) (the plane \((\Delta, a^2)\) after excluding \( D_1 \) and \( D_2 \)), we have:
\[
\cos \theta = \frac{q \cos \sigma}{\left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) - p \right] a} ; \quad \sin \theta = \frac{q \sin \sigma}{\left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) + p \right] a}
\]
and the amplitude - frequency relationship is of the form:
\[
W_1 = \frac{q^2 \cos^2 \sigma}{\left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) - p \right]^2 a^2} + \frac{q^2 \sin^2 \sigma}{\left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) + p \right]^2 a^2} - 1 = 0
\]
(2.5) give only some "parts" of the resonance curve, the "parts" lying out of the straight lines \( D_1 \) and \( D_2 \).
2. If \( D = 0 \) (either in \( D_1 \) or in \( D_2 \)) the two algebraic equation (2.1) can be resolved when \( \sigma = 0, \pi/2, \pi, 3\pi/2 \) Indeed
2a. In $D_1: \frac{3\gamma}{4}a^2 = \Delta + p$, from (2.1a) we deduce $\sigma = \pi/2$, $3\pi/2$ and $\nu_1 = \cos \theta$ arbitrary, then, from (2.1b):

$$u_1 = \sin \theta = \frac{\pm q}{2p}$$

The corresponding algebraic - trigonometrical equations (2.1) admit the solution:

$$\frac{3\gamma}{4}a^2 = \Delta + p, \quad a^2 \geq \frac{q^2}{4p^2}, \quad \sin \theta = \frac{\pm q}{2p}, \quad \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

(2.7)

2b. In $D_2: \frac{3\gamma}{4}a^2 = \Delta - p$, from (2.1b), we deduce $\sigma = 0$, $\pi$ and $u_2 = \sin \theta$ arbitrary, then, from (2.1a):

$$\nu_2 = \cos \theta = \frac{\pm q}{2p}$$

(2.8)

The corresponding algebraic - trigonometrical equations (2.1) in this subcase admit the solution:

$$\frac{3\gamma}{4}a^2 = \Delta - p, \quad a^2 \geq \frac{q^2}{4p^2}, \quad \cos \theta = \frac{\pm q}{2p}, \quad \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

(2.9)

Thus:

1. If $\sigma \neq 0, \pi/2, \pi, 3\pi/2$, the resonance curve - the entire resonance curve - is given by (2.5)
2. If $\sigma = 0, \pi$, the resonance curve consists of two branches: - the first branch is given by (2.5) - the second one is given by (2.9).
3. If $\sigma = \pi/2, 3\pi/2$, the resonance curve consists of two branches too: - the first also given by (2.5) and the second one by (2.7).

In figure 1, the heavy line represents the resonance curve corresponding to the values $\sigma = 0, \gamma = 0, 8, p = 0, 25, q = 0, 2$, given by (2.5) and (2.9).

For the same values $\gamma, p, q$, the resonance curve (2.5) corresponding to $\sigma = \pi/4$ is plotted in figure 2.

Remark - (2.5) is often replaced by:

$$W = q^2 \left\{ \left[ \frac{3\gamma}{4}a^2 - \Delta \right] + p \right\}^2 \cos^2 \sigma + \left\{ \left[ \frac{3\gamma}{4}a^2 - \Delta \right] - p \right\}^2 \sin^2 \sigma \right\} - a^2 \left\{ \left[ \frac{3\gamma}{4}a^2 - \Delta \right]^2 - p^2 \right\}^2 = 0$$

(2.10)
It is necessary to note that (2.5) and (2.10) are equivalent only if \( D \neq 0 \). In the critical case where \( D = 0 \), the relationship (2.10) gives us also "parts" (2.7), (2.9) but not the inequality \( a^2 \geq q^2/4p^2 \).

§3. RESONANCE CURVE OF THE SYSTEM WITH DAMPING

For the system with damping, \( h > 0 \).

1. If \( D \neq 0 \), from (1.5), we deduce:

\[
\sin \theta = -\frac{q}{aD} \left\{ h \omega \cos \sigma - \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) - p \right] \sin \sigma \right\}
\]

\[
\cos \theta = \frac{q}{aD} \left\{ h \omega \sin \sigma + \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) + p \right] \cos \sigma \right\}
\]

and the amplitude - frequency relationship is of the form:

\[
W_1 = \frac{q^2}{a^2D^2} \left\{ \left( h \omega \cos \sigma - \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) - p \right] \sin \sigma \right)^2 + \left( h \omega \sin \sigma + \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) + p \right] \cos \sigma \right)^2 \right\} - 1 = 0
\]

or

\[
W_1 = \frac{q^2}{a^2D^2} \left\{ \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) + p \cos 2\sigma \right]^2 + \left[ h \omega + p \sin 2\sigma \right]^2 \right\} - 1 = 0
\]

As it has been in §2, under condition \( D = 0 \), (3.2) can be replaced by

\[
W = q^2 \left\{ \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) + p \cos 2\sigma \right]^2 + \left[ h \omega + p \sin 2\sigma \right]^2 \right\} - a^2D^2 = 0
\]

2. If

\[
D = \left( \frac{3\gamma}{4} a^2 - \Delta \right) + h^2 \omega^2 - p^2 = 0
\]

the two linear algebraic equations (1.5) is in "critical" situation respectively by \([D]\) and \([\tilde{D}]\), we denote the coefficient matrix and the extended one:

\[
[D] = \begin{vmatrix}
     h \omega & -\left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) - p \right] a \\
     \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) + p \right] a & h \omega
\end{vmatrix}
\]

\[
[\tilde{D}] = \begin{vmatrix}
     h \omega & -\left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) - p \right] a & -q \cos \sigma \\
     \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) + p \right] a & h \omega & q \sin \sigma
\end{vmatrix}
\]

Since \( h > 0, \omega = 1 \), if \( D = 0 \) we have:

\[
\text{rang} \ [D] = 1
\]

Hence, the algebraic equations (1.5) can be resolved only if:

\[
\text{rang} \ [\tilde{D}] = 1
\]

This requirement leads to two equations:

\[
\left| \begin{array}{cc}
     h \omega a & -q \cos \sigma \\
     \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) + p \right] a & q \sin \sigma
\end{array} \right| = a(q \left( h \omega \sin \sigma + \left[ \left( \frac{3\gamma}{4} a^2 - \Delta \right) + p \right] \cos \sigma \right) = 0
\]
\[
\left| -\left[ \frac{3\gamma}{4}a^2 - \Delta \right] + p \right| = aq \left\{ h\omega \cos\sigma - \left[ \left( \frac{3\gamma}{4}a^2 - \Delta \right) - p \right] \sin\sigma \right\} = 0
\]  
(3.9b)

or, in equivalent form:

\[
\begin{align*}
&h\omega + p \sin 2\sigma = 0 \quad (3.10a) \\
&\left( \frac{3\gamma}{4}a^2 - \Delta \right) + p \cos 2\sigma = 0 \quad (3.10b)
\end{align*}
\]

It is easy to prove that the equations (3.10) admit an unique solution \((\Delta_*, a_*)\) satisfying (3.4). It means that, in the curve \(D = 0\) (the resonance curve of the pure parametrically excited system), there always exists a point \(C_*(\Delta_*, a_*^2)\) at which the algebraic equations (1.5) can be resolved and can be reduced - for instance - to (1.5a):

\[
h\omega a_* u - \left( \frac{3\gamma}{4}a_*^2 - \Delta_* \right) - p \alpha = -q \cos \sigma
\]

(3.11)

However, \(C_*\) is acceptable only if \(a_*^2 > 0\) and if \(\Delta_*\) is in the neighbourhood of zero (in the resonance region), if not, \(C_*\) must be rejected. Moreover, even in the case where \(C_*\) is acceptable, it may be that the trigonometrical equation corresponding to (3.11) i.e. the following one

\[
h\omega a_* \sin\theta - \left( \frac{3\gamma}{4}a_*^2 - \Delta_* \right) - p \alpha \cos\theta = -q \cos \sigma
\]

(3.12)

does not admit any solution. Trigonometrical solutions exist only if:

\[
a_*^2 \left\{ h^2\omega^2 + \left[ \left( \frac{3\gamma}{4}a_*^2 - \Delta_* \right) - p \right] \alpha^2 \right\} \geq q^2 \cos^2 \sigma
\]

(3.13)

or, by using (3.13):

\[
a_*^2 \geq \frac{q^2}{4p^2}
\]

(3.14)

Thus, if (3.14) is satisfied, the point \(C_*\) corresponds to determined stationary oscillations so that \(C_*\) is the second "branch" of the resonance curve.

The point \(C_*\) plays a special role:

\[
W \bigg|_{C_*} = 0, \quad \frac{\partial W}{\partial \Delta} \bigg|_{C_*} = 0, \quad \frac{\partial W}{\partial a_*} \bigg|_{C_*} = 0 \quad \text{i.e. } C_* \text{ is a critical point}
\]

(3.15)

of the curve \(C\) determined by the relationship \(W = 0\)

\[
\begin{align*}
\frac{\partial^2 W}{\partial \Delta^2} \bigg|_{C_*} &= 2 \left\{ q^2 + \frac{h^2}{4\omega_*^2} - h^4 a_*^2 - 4h^2 p a_*^2 \cos 2\sigma - 4p^2 a_*^2 \cos^2 2\sigma \right\} \\
\frac{\partial^2 W}{\partial \Delta \partial a_*} \bigg|_{C_*} &= 2 \left( \frac{3\gamma}{4} \right) \left\{ -q^2 + 2h^2 p a_*^2 \cos 2\sigma + 4p^2 a_*^2 \cos^2 2\sigma \right\} \\
\frac{\partial^2 W}{\partial (a_*^2)^2} \bigg|_{C_*} &= 2 \left( \frac{3\gamma}{4} \right)^2 \left\{ q^2 - 4p^2 a_*^2 \cos^2 2\sigma \right\}
\end{align*}
\]

(3.16)

so that, in the neighbourhood of \(C_*\), by neglecting the terms of powers greater than 2 relative to \(x = \Delta - \Delta_*\), \(y = a^2 - a_*\), the curve \(C\) is given by:

\[
W = \left( \frac{3\gamma}{4} \right)^2 \{ q^2 - 4p^2 a_*^2 \cos^2 2\sigma \} y^2 + 2 \left( \frac{3\gamma}{4} \right) \{ 4p^2 a_*^2 \cos 2\sigma + 2h^2 p a_*^2 \cos 2\sigma - q^2 \} yx + \\
+ \left\{ q^2 + \frac{h^2 q^2}{4\omega_*^2} - h^4 a_*^2 - 4h^2 p a_*^2 \cos 2\sigma - 4p^2 a_*^2 \cos^2 2\sigma \right\}^2 = 0
\]

(3.17)
The quadratic form of the left hand side of (3.17) has as discriminant:

\[
D = \left(\frac{3\gamma}{4}\right)^2 \left\{4pa^2 \cos^2 2\sigma + 2ph^2 a^2 \cos 2\sigma - q^2 \right\}^2 - \\
- \left(\frac{3\gamma}{4}\right)^2 \left\{q^2 - 4p^2 a^2 \cos^2 2\sigma \right\} \left\{q^2 + \frac{h^2 q^2}{4\omega^2} - h^2 a^2 - 4ph^2 a^2 \cos 2\sigma - 4p^2 a^2 \cos^2 2\sigma \right\} = \\
= \left(\frac{3\gamma}{4}\right)^2 \frac{h^2 q^2}{4\omega^2} (4p^2 a^2 - q^2) \tag{3.18}
\]

- If \(a^2 < q^2/4p^2\), then \(D < 0\), therefore \(C_*\) is an isolated point of the curve \(C\) determined by \(W = 0\) but does not correspond to any stationary oscillation. In this case, the resonance curve is obtained from \(C\) after excluding \(C_*\).

- If \(a^2 = q^2/4p^2\), then \(D = 0\), therefore in the neighbourhood of \(C_*\), there exists two branches of \(C\), connecting themselves at \(C_*\), having at \(C_*\) the common tangent and form so a sharp cap (this remark can be deduced by retaining the terms of powers 3 in the expression of \(W\) in the neighbourhood of \(C_*\)).

- If \(a^2 > q^2/4p^2\), then \(D > 0\), therefore in the neighbourhood of \(C_*\), there exists two branches of \(C\), intersecting themselves at \(C_*\). In two last cases, \(C_*\) corresponds to determined stationary oscillations and the resonance curve is given by \(C (W = 0)\) including \(C_*\).

Let us fix \(\gamma = 0.8, p = 0.25, h = 0.22\). For \(\sigma = 0\), from (3.10) we deduce \(\Delta_* = -1, a^2 = -3/3\gamma (1 + p)\) so that \(C_*\) is not acceptable. The resonance curves corresponding to \(q = 0.007\) and \(q = 0.05\) are presented in the figure 3 by the curves \(a, b\) respectively. If \(q\) is small enough, the resonance curve \((a)\) consists of two branches, separated by the curve \(D = 0\) (the resonance curve of the pure - parametrically excited system). Increasing \(q\), the branch lying under \(D = 0\) is restricted then disappears and the resonance curve consists of unique branch lying upon \(D = 0\).

For \(\sigma = 3\pi/4\), from (3.11) we deduce \(\frac{3\gamma}{4} a^2 = \Delta_* = \frac{p^2 - h^2}{h^2} \approx 0.29\) and the critical point \(C_*\) is acceptable. In figure 4 the curves \(a, b\) represent the resonance curves corresponding to the values \(q = 0.1, q = 0.2\), respectively: the resonance curve consists of two branches, connecting at \(C_*\), lying respectively under and upon the curve \(D = 0\); increasing \(q\), the branched lying under \(D = 0\) is restricted then disappears.
§4. STABILITY OF STATIONARY OSCILLATIONS

The stability study of the stationary oscillations \((a_0, \theta_0)\) will be based on the following variational equations:

\[
\delta \dot{a} = -\frac{\epsilon}{2\omega} \left( h\omega + p \sin 2\theta_0 \right) \delta a - \frac{\epsilon}{2\omega} \left( 2pa_0 \cos 2\theta_0 + q \cos(\theta_0 - \sigma) \right) \delta \theta
\]
\[
\delta \dot{\theta} = -\frac{\epsilon}{2\omega a_0} \left( p \cos 2\theta_0 + \left( \Delta - \frac{9\gamma}{4} a_0^2 \right) \right) \delta a + \frac{\epsilon}{2\omega a_0} \left( 2pa_0 \sin 2\theta_0 + q \sin(\theta_0 - \sigma) \right) \delta \theta
\]  

(4.1)

where \(\delta a = a - a_0, \delta \theta = \theta - \theta_0\) are the variations of the amplitude and the dephase, respectively.

Using (4.1), we write (4.1) in the form:

\[
\delta \dot{a} = -\frac{\epsilon}{2\omega} \left( h\omega + p \sin 2\theta_0 \right) \delta a - \frac{\epsilon}{2\omega} \left( \left( \frac{9\gamma}{4} a_0^2 - \Delta \right) a_0 \right) \delta \theta
\]
\[
\delta \dot{\theta} = -\frac{\epsilon}{2\omega a_0} \left( p \cos 2\theta_0 - \left( \frac{9\gamma}{4} a_0^2 - \Delta \right) \right) \delta a - \frac{\epsilon}{2\omega} \left( h\omega - p \sin 2\theta_0 \right) \delta \theta
\]

(4.2)

and the characteristic equation can be established:

\[
p^2 + \epsilon h\omega + \frac{\epsilon^2}{4\omega^2} \left( D + 2 \left( \frac{3\gamma}{4} a_0^2 \right) a_0^2 \left[ \left( \frac{3\gamma}{4} a_0^2 - \Delta \right) + p \cos 2\theta_0 \right] \right) = 0
\]

(4.3)

The first stability condition \(h > 0\) is satisfied for the system with damping, the second one is given by the inequality:

\[
D + 2 \left( \frac{3\gamma}{4} a_0^2 \right) a_0^2 \left[ \left( \frac{3\gamma}{4} a_0^2 - \Delta \right) + p \cos 2\theta_0 \right] > 0
\]

(4.4)

Using again (1.4) and (3.3) we find

\[
p \cos 2\theta_0 = \left( \frac{3\gamma}{4} a_0^2 - \Delta \right) - \frac{q^2}{a_0^2 D} \left[ \left( \frac{3\gamma}{4} a_0^2 - \Delta \right) + p \cos 2\theta_0 \right]
\]

(4.5)

So, (4.4) can be written as:

\[
D + 2 \left( \frac{3\gamma}{4} a_0^2 \right) \left( 2 \left( \frac{3\gamma}{4} a_0^2 - \Delta \right) - \frac{q^2}{a_0^2 D} \left[ \left( \frac{3\gamma}{4} a_0^2 - \Delta \right) + p \cos 2\theta_0 \right] \right) =
\]
\[
= D + 4 \left( \frac{3\gamma}{4} a_0^2 \right) \left( \frac{3\gamma}{4} a_0^2 - \Delta \right) - 2 \left( \frac{3\gamma}{4} a_0^2 \right) \frac{q^2}{D} \left[ \left( \frac{3\gamma}{4} a_0^2 - \Delta \right) + p \cos 2\theta_0 \right] = -\frac{1}{D} \frac{\partial W}{\partial a_0^2} > 0
\]

(4.6)

Analyzing the signs of \(D\) and \(\partial W/\partial a_0^2\), from (4.6) we can easily determine in the resonance curve, the "parts" corresponding to stable stationary oscillations and those corresponding to unstable stationary oscillations.

Since \(\frac{\partial W}{\partial a_0^2} \bigg|_{a_0} = 0\), the stability of the stationary oscillations corresponding to the critical point \(C_\ast\) cannot be deduced from the variational equations (4.1).

CONCLUSION

Using the asymptotic method, we have examined the interaction between parametric and forced oscillations in fundamental resonance. We have concentrated our attention on the critical situation and the critical point in the resonance curve has been analyzed in detail. Depending on this critical point, diverse shapes of the resonance curve have been obtained.
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REFERENCE


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TUONG TÁC GIỮA DAO ĐỘNG THÔNG SỐ VÀ CUỐNG BỨC Ở CÔNG HƯỚNG CO BÀN

Bài báo xét tương tác giữa dao động thông số và cuồng bức ở công hưởng cơ bản (trong ứng tần số kích động lớn cạnh đời và bằng tận số riêng). Dữ phân biết trường hợp thường và trường hợp tối hạn khi giải phương trình đại số - luồng giấc đề xác định biên độ và pha của dao động đúng. Dữ phân tích điểm lạ trên đường công hưởng và thấy mối liên quan giữa tính chất điểm lạ với các dạng đường công hưởng.

CONVECTION IN BINARY MIXTURE ...
(tếp trang 4)

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CHUYÊN ĐỘI LƯU TRONG HỒNG HỘP HAI THÀNH PHẦN CÓ MẬT THƯỢNG

Trong bài báo nghiên cứu mới đã trình bày và duy nhất nghiêm suy rộng của bài toán về chuyển động đối lưu nhiệt trong hỏng hộp hai thành phần có mật thô lượng