THE FORMS OF THE SHELL WITH ZERO BENDING STRESSES SUBJECTED TO HYDROSTATIC PRESSURE AND OTHER LOADS

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1. Introduction

The shells without bending stress usually are used in practice. The problems for determination of the forms of these shells when external loads are given, have been investigated in many works [1, 2].

In reality, these problems are inverse mathematical problems and for some simple cases of loading, analytical and numerical solutions are derived in [3, 4].

In this paper we consider the shell subjected to hydrostatic pressure and axially symmetrical load, which is parallel to axis of revolution. The differential equations for determination of meridian forms of the shell are obtained. Analytical and numerical solution of these equations are presented. The forms of the shell are shown by program written by FTN77 on PC.

These problems are solved according to geometrical linear theory of the shell.

2. Equations of linear membrane theory for shells of revolution under axially-symmetrical loading

The equation of equilibrium for a shell of revolution in an axisymmetric membrane state of stress are [5]:

\[ \frac{dT_s}{ds} + (T_\rho - T_s) \frac{\sin \theta}{r} = -X, \]

\[ \frac{T_s}{R_1} + \frac{T_\rho}{R_2} = Z, \]

where \( T_s, T_\rho \) - stress resultants; \( X, Z \) - the surface load components; \( R_1, R_2 \) - radius of curvatures of middle surface; \( r \) - radius of the hoop circle; \( \theta \) - angle of the meridional and axis z.

If \( \eta = \sec \theta \) then solution of (2.1) will be written [1]:

\[ T_s = \frac{1}{r} \left[ \int_{r_0}^{r} r \left( Z + \frac{X}{\sqrt{\eta^2 - 1}} \right) dr + C \right] \eta, \]

\[ T_\rho = r \eta Z + \left[ \int_{r_0}^{r} r \left( Z + \frac{X}{\sqrt{\eta^2 - 1}} \right) dr + C \right] \frac{d\eta}{dr}. \]

Note that \( C \) may be found from loading in boundary of the shell. If \( Q \) is axially symmetrical load, which is parallel to axis of revolution at \( r = r_0 \) then

\[ C = T_s(r_0) \times r_0 \times \cos \eta_0 = -Q \times r_0. \]
From the condition of zero bending stresses, the curvature change is equal to zero, we have

\[ x_\pi = -\frac{d}{ds} \left( \frac{du}{ds} - \frac{u}{R_1} \right) = 0, \]
\[ \chi_\varphi = \sin \theta \left( \frac{du}{ds} - \frac{u}{R_k} \right) = 0, \]

hence

\[ \frac{dw}{ds} - \frac{u}{R_1} = 0. \]

Substituting this relation into the compatibility deformation equation [5]

\[ r \frac{d\phi}{ds} - (\varepsilon_\phi - \varepsilon_x) \sin \theta - \left( \frac{dw}{ds} - \frac{u}{R_1} \right) \cos \theta = 0, \]

we obtain the condition of zero bending stresses in the form

\[ r \frac{d\phi}{ds} - (\varepsilon_\phi - \varepsilon_x) \frac{dr}{ds} = 0 \Rightarrow \frac{d(r \tau_\phi)}{dr} = \varepsilon_x. \]  

The deformation components can be expressed by stress resultants

\[ \varepsilon_x = \frac{1}{Eh} (T_x - \nu T_\phi), \]
\[ \varepsilon_\phi = \frac{1}{Eh} (T_\phi - \nu T_x), \]  

where \( E \) - the Young's modulus, \( \nu \) - the poisson's ratio and \( h \) - the thickness of the shell.

### 3. Integro-differential equation for determination of shell forms

Substituting (2.4) and (2.2) into (2.3) we obtain integro-differential equation for determination of shell forms:

\[
r^2 \left[ \int_{r_0}^{r} \left( Z + \frac{X}{\sqrt{\eta^2 - 1}} \right) dr + C \right] \frac{d^2 \eta}{dr^2} + \]
\[
r \left[ \int_{r_0}^{r} \left( Z + \frac{X}{\sqrt{\eta^2 - 1}} \right) dr + C \right] \times \left( 1 - \frac{rh'}{h} \right) + r \left( rZ + \frac{\int_{r_0}^{r} \left( Z + \frac{X}{\sqrt{\eta^2 - 1}} \right) dr + C }{dr} \right) \frac{d \eta}{dr} + \]
\[
r^2 Z \left( 2 - \frac{rh'}{h} + \nu \right) + \int_{r_0}^{r} \left( r \left( Z + \frac{X}{\sqrt{\eta^2 - 1}} \right) dr + C \right) \left( \frac{rh'}{h} \nu - 1 \right) - \]
\[
r \left[ \int_{r_0}^{r} \left( Z + \frac{X}{\sqrt{\eta^2 - 1}} \right) dr + C \right] \frac{dZ}{dr} + r^2 \frac{dZ}{dr} \eta = 0. \]  

(3.1)

If the thickness of the shell changes on rule

\[ \frac{h}{h_0} = \left( \frac{r}{r_0} \right)^n, \]
where \( r_0 \) and \( h_0 \) are respectively the values of \( r \) and \( h \) at \( r = r_0 \), and \( n \) is real. The loading consists of hydrostatic pressure and loads parallel to axis \( z \) at the boundary of the shells [5]:

\[
X = 0, \quad Z = q \left( 1 - \frac{s}{L} \right) = q \left( \frac{r - r_1}{r_0 - r_1} \right), \quad C = -Q r_0,
\]

where \( q \) - pressure at the bottom of the shell, \( s \) - coordinate of the any point of the meridian, \( L \) - the length of the shell meridian. If \( p \) is axial load at the boundary \( r = r_1 \) then from equilibrium condition we have relation

\[
2\pi r_0 Q = 2\pi r_1 p = -\pi r_0^2 q \Rightarrow C = -r_0 Q = -pr_1 = qr_0^2/2.
\]

The basic equation (3.1) then reduces to:

\[
r^2 f(r) \frac{d^2 \eta}{dr^2} + r g(r) \frac{d\eta}{dr} + k(r) \eta = 0,
\]

where

\[
\begin{align*}
f(r) &= \frac{q}{r_0 - r_1} \left( \frac{r^3}{3} - r_1^3 \left( \frac{r^2}{2} + \frac{r_1^2}{2} - \frac{r_0^2}{3} \right) \right) + C, \\
g(r) &= \left[ \frac{q}{r_0 - r_1} \left( \frac{r^3}{3} - r_1^3 \left( \frac{r^2}{2} + \frac{r_1^2}{2} - \frac{r_0^2}{3} \right) \right) + C \right] (1 - n) + 2q r^2 \left( \frac{r - r_1}{r_0 - r_1} \right), \\
k(r) &= q r^2 \left( \frac{r - r_1}{r_0 - r_1} \right) (2 - n) + (n \nu - 1) \left[ \frac{q}{r_0 - r_1} \left( \frac{r^3}{3} - r_1^3 \left( \frac{r^2}{2} + \frac{r_1^2}{2} - \frac{r_0^2}{3} \right) \right) + C \right] + r^2 q \left( \frac{r - r_1}{r_0 - r_1} \right) .
\end{align*}
\]

The equation (3.2) is differential equation of Fock type with regularity point \( r = 0 \).

If \( \rho = \rho_1 - \rho_2 \) is not natural, solution is found in the form:

\[
\eta_1 = r^{\rho_1} \sum_{i=0}^{\infty} C_i r^i, \quad \eta_2 = r^{\rho_2} \sum_{i=0}^{\infty} C_i r^i, \tag{3.3}
\]

\( \rho_1 \) and \( \rho_2 \) are solutions of the characteristic equation:

\[
\rho (\rho - 1) f(0) + \rho g(0) + k(0) = 0,
\]

where

\[
\begin{align*}
f(0) &= \frac{q}{r_0 - r_1} \left( \frac{r_1^2}{2} - \frac{r_0^2}{3} \right) + C, \\
g(0) &= (1 - n) f(0), \\
k(0) &= (n \nu - 1) f(0), \\
\rho_{1,2} &= \frac{1}{2} \left[ n \pm \sqrt{n^2 - 4(n \nu - 1)} \right].
\end{align*}
\]

The coefficients \( C_i \) in (3.3) are determined by substituting (3.3) into (3.2) and giving zero coefficient with the same degree \( r^k \), we have

\[
\begin{align*}
C_0 &= 1, \quad C_1 = 0, \\
C_2 &= \frac{r_1 q}{2 f(0) (r_0 - r_1)}, \\
C_m &= \frac{1}{f(0)} \left[ C_{m-2} \frac{r_1 q}{2 (r_0 - r_1)} - C_{m-3} \frac{q}{3 (r_0 - r_1)} \right] \text{ with } m \geq 3.
\end{align*}
\]
Coefficients in the series (3.3) then can be reduced to function:

\[
\eta_1 = \frac{\rho^1}{1 - \frac{q}{f(0)(r_0 - r_1)} \left( \frac{r_1^2}{2} - \frac{r^2}{3} \right)}, \quad \eta_2 = \frac{\rho^2}{1 - \frac{q}{f(0)(r_0 - r_1)} \left( \frac{r_1^2}{2} - \frac{r^2}{3} \right)}.
\]  

(3.4)

General solution is:

\[
\eta = A\eta_1 + B\eta_2,
\]  

(3.5)

\(A, B\) - constants, which are given from boundary conditions of \(\eta\) in the \(r = r_1\) and \(r = r_0\).

If \(d = \rho_1 - \rho_2\) is natural, solution has the form:

\[
\eta_1 = \frac{\rho^1}{1 - \frac{q}{f(0)(r_0 - r_1)} \left( \frac{r_1^2}{2} - \frac{r^2}{3} \right)}, \quad \eta_2 = \eta_1 \ln r + ar^\rho_1 - d.
\]  

(3.6)

If \(z\) denotes the distance along the axis of revolution, then a relationship between \(r\) and \(z\) may be obtained by integrating the equation with \(r_0 > r\):

\[
\frac{dr}{dz} = -\sqrt{1 - \eta^2}.
\]  

(3.7)

The general shapes of shell obtained from equation (3.4) - (3.7) are illustrated in numerical results Tab.1 and Fig.1. The type of curve depends on the \(n, A, B, q, r_1, r_0\). These numerical results are given by program written by FTN77 on PC for calculating \(\eta\) and integrating (3.7).

Tab. 1

<table>
<thead>
<tr>
<th>(R_i)</th>
<th>(Z_1)</th>
<th>(Z_2)</th>
<th>(Z_3)</th>
<th>(Z_4)</th>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>8.484</td>
</tr>
</tbody>
</table>

Example. The shells with various boundary values of \(\eta(r_0)\) and \(\eta(r_1)\)

Let us consider the shells with \(r_0 = 14m, r_1 = 6m, q = -100 T/m^2, n = 2, \nu = 0.33, \gamma = -r_0^2 q\) is the axial load at boundary \(r_1\) and given from equilibrium condition of the shell. The boundary values of \(\eta(r_0)\) and \(\eta(r_1)\) are following:

1. The meridian form \(Z_1\) according to \(\eta(r_0) = 1.1; \eta(r_1) = 1.3\)
2. The meridian form \(Z_2\) according to \(\eta(r_0) = 1.1; \eta(r_1) = 1.5\)
3. The meridian form \(Z_3\) according to \(\eta(r_0) = 1.3; \eta(r_1) = 1.5\)
4. The meridian form \(Z_4\) according to \(\eta(r_0) = 1.3; \eta(r_1) = 1.7\)

The radius of the cross circles is given in column \(R\) of the Table 1. The distances \(z\) (from shell bottom to circle \(R_i\)) are presented in each columns \(Z_1\), \(Z_2\), \(Z_3\), \(Z_4\). The meridian forms of these shells are shown on figure 1.
Conclusion

The equation for determining meridian form of the shell is given. The solution of equation is obtained for the shells subjected to hydrostatic pressure and axially symmetrical load, which is parallel to axis of revolution. Many examples are considered to illustrate the method of determination of the shell forms. Using this method, the other meridian curves may be obtained by executing given program with other values of \( n, A, B, q, r_1, r_0 \).

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References


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ĐÀNG VÔ KHÔNG CHỦ ỨNG SUẤT UỒN DUỘI TÁC DỤNG CỦA ÁP LỤC THỦY TÍNH VÀ CÁC ĐẲNG TẠI KHÁC

Trong bài báo đã đưa ra phương trình vi phân xác định dạng xung sinh của vò tròn xoay không chịu Ứng suất dưới tác dụng của áp lực thủy tĩnh và tài đối xứng tác dụng song song với trực quay trên biên vò. Giải phương trình đã được tìm ra dưới dạng hàm tỉ lệ. Kết quả số cụ thể của các điểm trên xung sinh cho các trường hợp gốc của xung sinh tại biên vò khác nhau và hình dạng xung sinh vò đã được biểu thị trên đồ thị nhô corroborate trên thực nghiệm thực nghiệm theo FTN77. Các dạng khác của vò có thể được tìm bằng phương pháp và chương trình này với các điều kiện khác với áp lực và đầu kiến hình học trên biên vò. Kết quả nghiên cứu có thể áp dụng trong việc thiết kế các vò tròn xoay không chịu Ứng suất.