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ON MODELLING AND SIMULATION OF A MANIPULATOR UNDER CONSIDERATION OF A JAMMED JOINT

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Abstract. In this paper, the modelling of a jamming process of a joint during the operation of a manipulator is presented. Based on the kinematic property of a jamming process, a motion law of the jammed joint is chosen. By introducing a matrix coressponding to jammed joint, the equation of motion of the system is restructured without re-deriving. Some numerical simulations are carried out to illustrate the proposed algorithm.

1. INTRODUCTION

During the operation of a manipulator or of a mechanical system failures can be occurred. One of these failures is jamming of a joint. The joint can be locked due to a jammed brake, the stuck of gear transmittion,. . . As consequence, there will be no relative motion between the two links that are connected through the jammed joint, i.e., the corresponding links and the joint will be reduced to a single link. The length of the resultant single link can be calculated from the lengths of two original links, and the relative position of these links after jamming occurs. A jammed joint will result in the reduction of a DOF of the manipulators. The issue of fault detection and isolation for serial and parallel manipulators have been investigated recently [2-6, 10,14,15]. However, the modelling of a jamming process for simulation has not been discussed yet. The investigation on kinematics and dynamics of a manipulator in case a joint getting stuck can be carried out by examining of each case of the jammed joint. The approach method requires time and calculating burden.

In this paper, the dynamic problem of a manipulator in case of a joint getting stuck is considered. If the jamming time and the jammed joint are known, the equation of motion of a manipulator can be determined by introduction of the so-called structure matrix. This paper is organized as follows: in section 2 the dynamics model in case of a jammed joint is described. Section 3 shows some results of numerical simulations.

2. MODELLING OF A JAMMING PROCESS

The dynamic equations of a robot manipulator can be derived by using Lagrange's theory [1, 7- 9, 11-13] as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u}, \tag{1}$$

where $\mathbf{q} \in \mathrm{R}^n$ is the joint position vector, $\mathbf{M}(\mathbf{q}) \in \mathrm{R}^{n\times n}$ is the symmetric positive definite inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathrm{R}^{n\times n}$ is the Coriolis and centripetal matrix, $\mathbf{g}(\mathbf{q}) \in \mathrm{R}^n$ is the gravitational torque vector, $\mathbf{D}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ is damping moments, and $\mathbf{u} \in \mathrm{R}^n$ is the applied torque vector.

The jamming process can occur suddenly and it is difficult to describe. In this paper, it is assumed that k^{th} joint starts jamming at time $t = t_1$, after a time interval $\Delta t = t_2 - t_1$ this joint is completely jammed. During this time the relative velocity decreases to zero $\dot{q}_k \to 0$ and the joint variable is blocked at $q_k \to q_k(t_2)$. Mathematically, the motion of the jammed joint can be described as follows:

If $t_1 \le t < t_2$: $\ddot{q}_k(t) = -\dot{q}_k(t_1)/\Delta t, \dot{q}_k(t) = \dot{q}_k + \ddot{q}_k(t - t_1)$.

If $t_2 \le t$: $\ddot{q}_k(t) = 0, \dot{q}_k(t) = 0$.

Here, in the jamming process the joint acceleration is assumed to be constant. In order to simulate, the operation of the manipulator can be devided into three phases:

- Normal phase (without jam), $0 < t < t_1$

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{q})[\mathbf{u} - \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{d}], \tag{2}$$

with $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{d}$.

- Jamming phase (joint k get stuck), $t_1 \le t < t_2$

$$\begin{aligned} &\mathbf{M}_a = \mathbf{E}_k^T \mathbf{M} \mathbf{E}_k, \quad \mathbf{h}_a = \mathbf{E}_k^T \mathbf{h} + \mathbf{E}_k^T \mathbf{M} \mathbf{e}_k \ddot{q}_k, \quad \mathbf{u}_a = \mathbf{E}_k^T \mathbf{u}, \\ &\ddot{\mathbf{q}}_a = \mathbf{M}_a^{-1}(\mathbf{q})[\mathbf{u}_a - \mathbf{h}_a(\mathbf{q}, \dot{\mathbf{q}})], \ddot{\mathbf{q}} = \mathbf{E}_k \ddot{\mathbf{q}}_a + \mathbf{e}_k \ddot{q}_k. \end{aligned} \tag{3}$$

- Joint k is blocked, $t_2 < t$

$$\begin{aligned} &\ddot{q}_k(t) = 0, \quad \dot{q}_k(t) = 0, \mathbf{M}_a = \mathbf{E}_k^T \mathbf{M} \mathbf{E}_k, \quad \mathbf{h}_a = \mathbf{E}_k^T \mathbf{h}, \quad \mathbf{u}_a = \mathbf{E}_k^T \mathbf{u}, \\ &\ddot{\mathbf{q}}_a = \mathbf{M}_a^{-1}(\mathbf{q})[\mathbf{u}_a - \mathbf{h}_a(\mathbf{q}, \dot{\mathbf{q}})], \quad \ddot{\mathbf{q}} = \mathbf{E}_k \ddot{\mathbf{q}}_a + \mathbf{e}_k \ddot{q}_k, \end{aligned} \tag{4}$$

where $\mathbf{E}_k \in \mathrm{R}^{n\times(n-1)}$ is the so-called structrure matrix obtained by deleting k^{th} colume of the identical matrix of zise $n \times n$, and $\mathbf{e}_k$ is a $n \times 1$-vector with $e_k(k) = 1$, $e_k(i) = 0$, $(i \neq k)$.

Noting that when the joint k gets stuck the degree of freedom of the manipulator is reduced by one. Hence, the inertia matrix $\mathbf{M}_a$, vectors $\mathbf{h}_a$, and $\mathbf{u}_a$ have a zise of $(n - 1)$, e.g. $\mathbf{M}_a \in \mathrm{R}^{(n-1)\times(n-1)}$, $\mathbf{h}_a \in \mathrm{R}^{(n-1)\times 1}$, $\mathbf{u}_a \in \mathrm{R}^{(n-1)\times 1}$. Moreover, the matrix $\mathbf{M}_a$ is a symmetric positive definite one, because this is the inertia matrix of a mechanical system obtained by rigidening one joint of the original system. The value of $k \in (1, .., n)$, jamming time moment $t_1 > 0$, and jamming duration $\Delta t = t_2 - t_1 > 0$ can be given by a random number.

3. NUMERICAL SIMULATION

In this section, some simulations in universal software Matlab is implemented to illustrate the proposed algorithm. In this simulation, a 3-DOF planar manipulator moving in the vertical plane is considered. The i^{th} link has a length of L_i, a mass of m_i, and the inertia moment respect to C_i is J_{Ci} (C_i is a center of mass). The distance from C_i to the joint connected to $(i\text{-}1)^{th}$ link is a_i. The torque of motor fixed on link (i-1) acts on link i

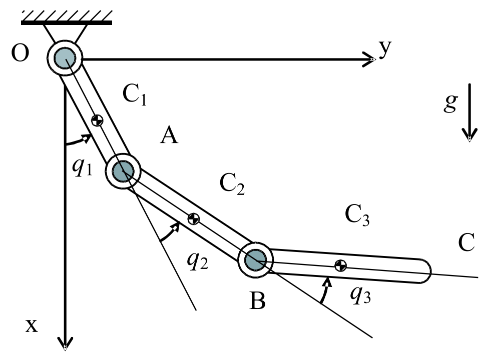


Fig. 1. A 3-DOF planar manipulator

is u_i (i= 1, ..., 3). The model of the manipulator and its parameters are shown in Fig. 1 and Table 1.

Let $q^T = [q_1,\ \ q_2,\ \ q_3]$ be the generalized coordinates of the manipulator, q_1 is the angle of link 1, bar OA; q_2 is the relative angle of link 2 respective to link 1; and q_3 is the relative angle of link 3 respective to link 2. The kinetic and potential energy of the system are obtained as follows:

$$\begin{aligned} T = \frac{1}{2}[&J_{C_1} + m_1a_1^2 + J_{C_2} + m_2a_2^2 + m_2l_1^2 + 2m_2l_1a_2\cos q_2 + 2m_2l_2a_3\cos q_3 \\ &+ J_{C_3} + m_3a_3^2 + m_3l_1^2 + m_3l_2^2 + 2m_3l_1a_3\cos(q_2+q_3) + 2m_3l_1l_2\cos q_2]\dot{q}_1^2 \\ &+ [J_{C_2} + m_2a_2^2 + m_2l_1a_2\cos q_2 + J_{C_3} + m_3a_3^2 + m_3l_2^2 \\ &+ m_3l_1a_3\cos(q_2+q_3) + m_3l_1l_2\cos q_2 + 2m_3l_2a_3\cos q_3]\dot{q}_1\dot{q}_2 \\ &+ [J_{C_3} + m_3a_3^2 + m_3l_2a_3\cos q_3 + m_3l_1a_3\cos(q_2+q_3)]\dot{q}_1\dot{q}_3 \\ &\frac{1}{2}(J_{C_2} + m_2a_2^2 + J_{C_3} + m_3a_3^2 + m_3l_2^2 + 2m_3l_2a_3\cos q_3)\dot{q}_2^2 \\ &+ \left(J_{C_3} + m_3a_3^2 + m_3l_2a_3\cos q_3\right)\dot{q}_2\dot{q}_3 + \frac{1}{2}(J_{C_3} + m_3a_3^2)\dot{q}_3^2 \end{aligned} \tag{5}$$

$$\begin{aligned} \Pi = -\sum_{i=1}^{3} m_i g x_{Ci} = -\{&m_1ga_1\cos q_1 + m_2g[l_1\cos q_1 + a_2\cos(q_1+q_2)]+ \\ &+m_3g[l_1\cos q_1 + l_2\cos(q_1+q_2) + a_3\cos(q_1+q_2+q_3)]\}; \end{aligned} \tag{6}$$

The applied torques of motors acting on links and damping at each joint are given by the principle of virtual work:

$$Q_1 = -b_1\dot{q}_1 + u_1, Q_2 = -b_2\dot{q}_2 + u_2, Q_3 = -b_3\dot{q}_3 + u_3 \tag{7}$$

Applying the Lagrangian equation of the second kind one obtains the equation of motion in matrix form as (1). The differential equation of motion (1) will be implemented in Matlab simulink for simulation as in Fig. 2.

In order to check the simulation results, the case of the free motion of the manipulator is investigated. It is clearly that there are no torques applying on the manipulator,

Table 1. Some parameters of the manipulator

Link	1	2	3
m [kg]	9.00	7.00	5.00
J_C [kgm^2]	1.50	1.00	0.60
L [m]	0.80	0.70	0.60
a [m]	0.40	0.35	0.30

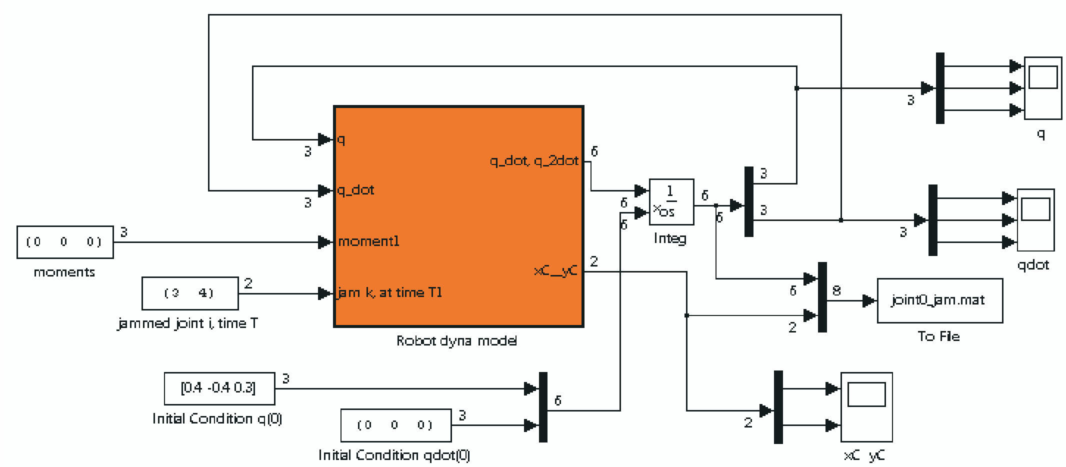


Fig. 2. The simulink diagram

so the manipulator will move to stable equilibrium $\mathbf{q}_e = [0\ 0\ 0]^T$ from any arbitrary initial position due to gravitational forces and damping at each joint. In simulations the coordinates of the center of mass is also given to check the reliability of the presented approach.

Three numerical simulations are carried out: In the first case, there are no fault (no jammed joint), the second case joint 1 gets stuck at time $t_1 = 2$ s. And in the last simulation, joint 2 gets stuck at time $t_1 = 0.5$ s. The simulation results are shown in Fig. 3 to Fig. 6.

Fig. 3 present the simulation results of the first case: the manipulator moves to the equilibrium $\mathbf{q}_e = [0\ 0\ 0]^T$ after about 15s. In this case, the center of mass is located at

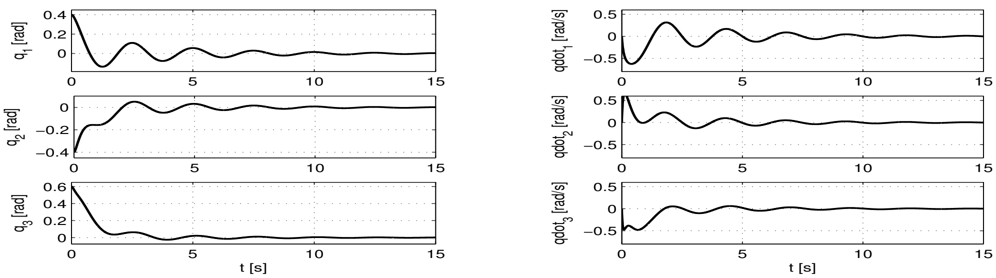


Fig. 3. Time history of generalized coordinates (1. case, no jammed joint)

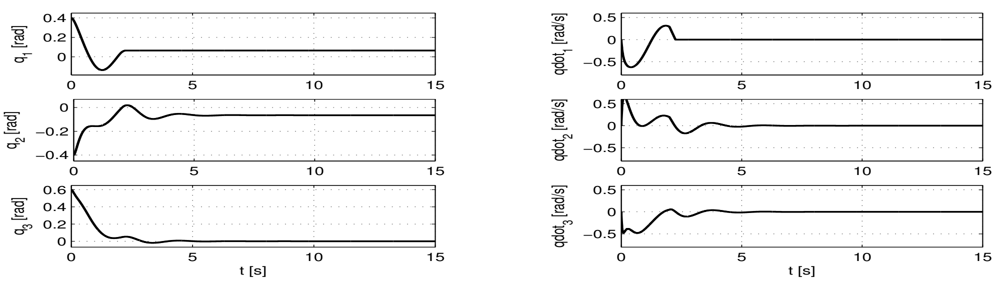


Fig. 4. Time history of generalized coordinates (2. case, joint 1 gets stuck)

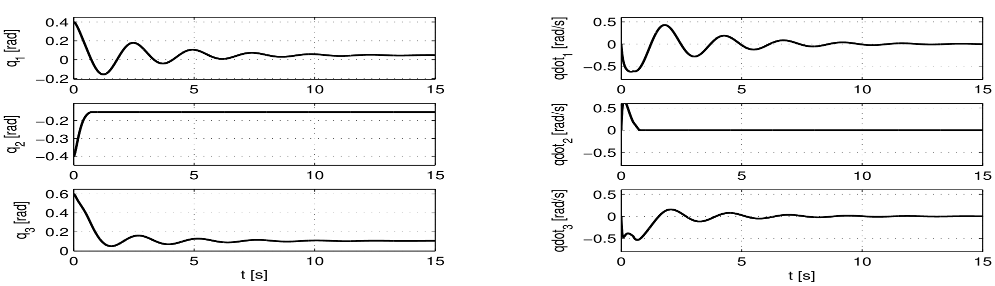
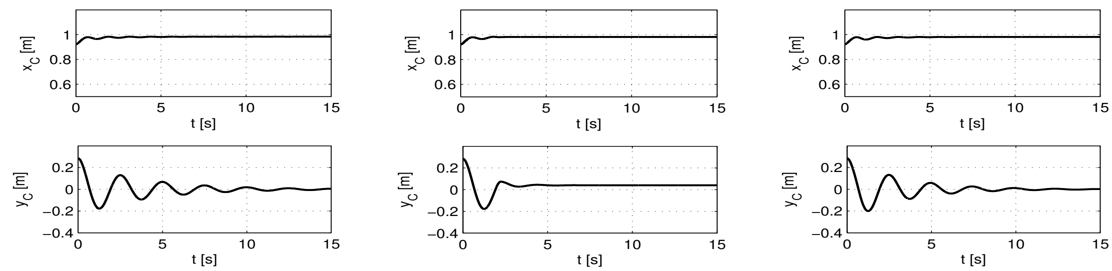


Fig. 5. Time history of generalized coordinates (3. case, joint 2 gets stuck)

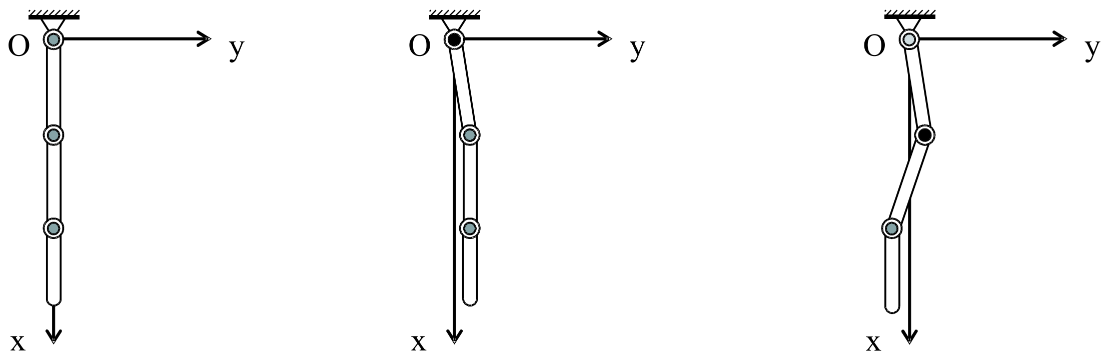
the lowest position, and $y_C = 0$ (Fig. 6). The results of the second case are shown in Fig. 4. The joints 1 gets jamming at the time $t_1 = 2$s, after that q_1 becomes a constant, and the system moves to the equilibrium $\mathbf{q}_e = [0.0645,\ -0.0645,\ 0.0]^T$ after about 10 s. The results show that in the equilibrium $q_1 = -q_2$, and $q_3 = 0.0$.

The results of the last case are shown in Fig. 5. Joints 2 gets jamming at time $t_1 = 0.5$ s, after that q_2 becomes a constant, and the system moves to the equilibrium $\mathbf{q}_e = [0.0473\ -0.1525\ 0.1051]^T$ after about 15 s. The center of mass in the equilibrium lies on the x axis, $y_C = 0$.



Case 1 Case 2 Case 3

Fig. 6. Coordinates of the center of mass



Case 1: No fault $\mathbf{q}_e = [0, 0, 0]^T$

Case 2: Jam at joint 1 $\mathbf{q}_e = [0.0645, -0.0645, 0.0]^T$

Case 3: Jam at joint 2 $\mathbf{q}_e = [0.0473, -0.1525, 0.1051]^T$

Fig. 7. Some manipulator configurations in equilibrium

Fig. 7 shows the configurations of the manipulator in equilibrium in case of no-fault, jamming at joint 1, and jamming at joint 2. Visually, these configurations are close to reality, because the center of mass is always located at the lowest position due to gravitation.

4. CONSLUSION

In reality, when a joint is completely jammed, the manipulator structure and the number of degrees of freedom will be changed. This leads to change the forward kinematics. Thus we have to solve the problem for each case of jam of joints. This work requires time and calculating burden. The presented algorithm in this study has overcome these difficulties. By choosing a motion law of the jammed joint and introducing the so-called restructured matrix, the equation of motion of the system is restructured without rederiving. The effectiveness of the proposed method is demonstrated by means of numerical experiments.

APPENDIX

- Kinetic energy of the maniupulator: $T = \frac{1}{2}J_{O_1}\omega_1^2 + \frac{1}{2}\sum_{i=2}^{3}(m_i v_{Ci}^2 + J_{Ci}\omega_i^2)$

Link 1: $J_{O_1} = J_{C1} + m_1 a_1^2, \quad \omega_1 = \dot{q}_1.$

Link 2: $x_{C2} = l_1\cos q_1 + a_2\cos(q_1+q_2), \quad \dot{x}_{C2} = -l_1\dot{q}_1\sin q_1 - a_2(\dot{q}_1+\dot{q}_2$

$y_{C2} = l_1\sin q_1 + a_2\sin(q_1+q_2), \quad \dot{y}_{C2} = l_1\dot{q}_1\cos q_1 + a_2(\dot{q}_1+\dot{q}_2)\cos(q_1+q_2),$

$\omega_2 = (\dot{q}_1+\dot{q}_2), \quad v_{C2}^2 = \dot{x}_{C2}^2 + \dot{y}_{C2}^2 = l_1^2\dot{q}_1^2 + a_2^2(\dot{q}_1+\dot{q}_2)^2 + 2l_1a_2\dot{q}_1(\dot{q}_1+\dot{q}_2)\cos q_2.$

Link 3: $x_{C3} = l_1\cos q_1 + l_2\cos(q_1+q_2) + a_3\cos(q_1+q_2+q_3),$

$y_{C3} = l_1\sin q_1 + l_2\sin(q_1+q_2) + a_3\sin(q_1+q_2+q_3),$

$\dot{x}_{C3} = -l_1\dot{q}_1\sin q_1 - l_2(\dot{q}_1+\dot{q}_2)\sin(q_1+q_2) - a_3(\dot{q}_1+\dot{q}_2+\dot{q}_3)\sin(q_1+q_2+q_3),$

$\dot{y}_{C3} = l_1\dot{q}_1\cos q_1 + l_2(\dot{q}_1+\dot{q}_2)\cos(q_1+q_2) + a_3(\dot{q}_1+\dot{q}_2+\dot{q}_3)\cos(q_1+q_2+q_3),$

$\omega_2 = (\dot{q}_1+\dot{q}_2+\dot{q}_3), \quad v_{C3}^2 = \dot{x}_{C3}^2 + \dot{y}_{C3}^2.$

- The inertia matrix $\mathbf{M}(\mathbf{q}) \in \mathrm{R}^{n\times n}$:

$M_{11} = 2m_3l_1l_2\cos q_2 + 2m_3l_1a_3\cos(q_2+q_3) + 2m_2l_1a_2\cos q_2 + J_{C2} + m_2a_2^2 +$
$\quad + m_1a_1^2 + 2m_3l_2a_3\cos q_3 + J_{C1} + m_3l_2^2 + J_{C3} + m_3a_3^2 + m_3l_1^2 + m_2l_1^2,$

$M_{12} = J_{C2} + m_3l_1a_3\cos(q_2+q_3) + m_3a_3^2 + J_{C3} + m_2l_1a_2\cos q_2 +$
$\quad + 2m_3l_2a_3\cos q_3 + m_2a_2^2 + m_3L_2^2 + m_3l_1l_2\cos q_2,$

$M_{13} = m_3l_2a_3\cos q_3 + J_{C3} + m_3l_1a_3\cos(q_2+q_3) + m_3a_3^2,$

$M_{21} = M_{12} \quad M_{23} = J_{C3} + m_3l_2a_3\cos q_3 + m_3a_3^2,$

$M_{22} = J_{C3} + 2m_3l_2a_3\cos q_3 + J_{C2} + m_2a_2^2 + m_3a_3^2 + m_3l_2^2,$

$M_{31} = M_{13}, \quad M_{32} = M_{23}, \quad M_{33} = m_3a_3^2 + J_{C3}$

- The Coriolis and centripetal matrix $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) \in \mathrm{R}^{n\times n}$:

$C_{11} = -m_3l_1a_3\dot{q}_2\sin(q_2+q_3) - m_3l_1l_2\dot{q}_2\sin q_2 - m_2l_1a_2\dot{q}_2\sin q_2$
$\quad - m_3l_1a_3\dot{q}_3\sin(q_2+q_3) - m_3l_2a_3\dot{q}_3\sin q_3,$

$C_{12} = -m_2l_1a_2\dot{q}_2\sin q_2 - m_3l_1l_2\dot{q}_2\sin q_2 - m_3l_1a_3\dot{q}_2\sin(q_2+q_3) - m_3l_1a_3\dot{q}_3\sin(q_2+q_3)$
$\quad - m_3L_2a_3\dot{q}_3\sin q_3 - m_3l_1a_3\dot{q}_1\sin(q_2+q_3) - m_3l_1l_2\dot{q}_1\sin q_2 - m_2l_1a_2\dot{q}_1\sin q_2,$

$C_{13} = -m_3a_3[l_1\dot{q}_2\sin(q_2+q_3) + l_2\dot{q}_3\sin q_3 + l_1\dot{q}_3\sin(q_2+q_3)$
$\quad + l_1\dot{q}_1\sin(q_2+q_3) + l_2\dot{q}_1\sin q_3 + l_2\dot{q}_2\sin q_3],$

$C_{21} = -m_3L_2a_3\sin(q_3)\dot{q}_3 + m_3L_1a_3\sin(q_2+q_3)\dot{q}_1 + \dot{q}_1m_3L_1L_2\sin(q_2) + \dot{q}_1m_2L_1a_2\sin(q_2),$

$C_{22} = -m_3L_2a_3\sin(q_3)\dot{q}_3, \quad C_{23} = -m_3L_2a_3\sin(q_3)(\dot{q}_3+\dot{q}_1+\dot{q}_2),$

$C_{31} = m_3a_3(\dot{q}_1L_1\sin(q_2+q_3) + \dot{q}_1L_2\sin(q_3) + L_2\sin(q_3)\dot{q}_2),$

$C_{32} = m_3L_2a_3\sin(q_3)(\dot{q}_1+\dot{q}_2), \quad C_{33} = 0.$

- The gravitational torque vector $\mathbf{g}(\mathbf{q}) \in \mathrm{R}^n$:

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} m_1g(a_1\sin q_1) + m_2g[l_1\sin q_1 + a_2\sin(q_1+q_2)] + \ldots \\ m_3g[l_1\sin q_1 + l_2\sin(q_1+q_2) + a_3\sin(q_1+q_2+q_3) \\ m_2ga_2\sin(q_1+q_2) + m_3g[l_2\sin(q_1+q_2) + a_3\sin(q_1+q_2+q_3)] \\ m_3ga_3\sin(q_1+q_2+q_3) \end{bmatrix}$$

- The applied torque vector $\mathbf{u} \in \mathrm{R}^n$: $\mathbf{u} = [u_1,\ \ u_2,\ \ u_3]^T.$

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VỀ VIỆC MÔ HÌNH HÓA VÀ MÔ PHỎNG CHUYỂN ĐỘNG CỦA TAY MÁY CÓ CHÚ Ý ĐẾN SỰ CỐ KẸT KHỚP

Trong bài báo này, việc mô hình hóa quá trình kẹt của khớp trong khi tay máy vận hành được trình bày. Trên cơ sở phân tích tính chất động học của quá trình kẹt, một luật chuyển động của khớp kẹt được chọn để mô tả động học quá trình kẹt. Bằng cách đưa vào ma trận cấu trúc tương ứng với khớp kẹt, phương trình vi phân chuyển động của hệ được cấu trúc lại mà không cần phải thiết lập lại. Từ đó giảm được thời gian phân tích động học và động lực học của hệ khi cấu trúc thay đổi do khớp bị kẹt. Các mô phỏng số được thực hiện bằng phần mềm đa năng Matlab để minh họa cho thuật toán. Kết quả mô phỏng đó khẳng định tính đúng đắn và phù hợp của phương pháp tiếp cận trong bài báo.