A NUMERICAL SIMULATION OF MORPHOLOGICAL PROCESSES FOR A NAVIGATION CHANNEL

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SUMMARY. The finite difference method was applied to simulate the morphological processes for a navigation channel. Mathematical equations were based on the SUTRENCH and PROFILE models proposed by L. C. van Rijn. The difficulties of treatment for bed boundary conditions were overcome flexibly by using the interpolation functions with high accuracy. The results obtained showed that the simulation is very suitable and able to be applied to practical problems.

1. INTRODUCTION

The morphological process is a popular phenomenon happening in rivers, estuaries and coastal regions, especially in environments with a predominant suspended load. According to Ida Brooker Hedegaard et al. [2] that is the interaction between hydrodynamic conditions and bed level evolution. This problem has been interesting several scientists in the world. Up to now there are many mathematical models for this [7], but the model in which the turbulence of flow is taken into account, may be more suitable. Knowledge on this process is really necessary because several studies in practice are required, such as harbor project, in which the rate of sedimentation in the navigation channel may be one of the most important economic aspects, and can be the parameter that finally decides the optimal location of the harbour.

2. MATHEMATICAL MODEL

Using the assumptions of the quasi-steady flow, the longitudinal diffusion term being neglected in comparison with the vertical diffusion term and the one of unsteady concentration being relatively small in relation to the others, the system of equations describing sediment flow perpendicular to channel axis are as follows [5-6]

\[ u = A_1 u_h \ln \left( \frac{z}{z_0} \right) + u_h \left( 1 - A_1 h \ln \left( \frac{h}{z_0} \right) \right) \left( 2 \eta^2 - \eta^2 \right) \]  \hspace{1cm} (2.1)

\[ \frac{1}{b} \frac{\partial}{\partial z} \left( bu \right) + \frac{\partial w}{\partial z} = 0 \]  \hspace{1cm} (2.2)

\[ \frac{1}{b} \frac{\partial}{\partial z} \left( bu\xi \right) + \frac{\partial}{\partial z} \left( w - w_k \right) c - \frac{\partial}{\partial z} \left( \xi \frac{\partial c}{\partial z} \right) = 0 \]  \hspace{1cm} (2.3)

\[ \frac{h}{\partial t} \frac{\partial z}{\partial t} + \frac{1}{(1 - p)} \frac{\partial}{\partial \zeta} \left( z_h \right) = 0 \]  \hspace{1cm} (2.4)

in which \( u, w \) are the time-averaged fluid velocities in \( x, z \) direction, respectively, \( z_h \) the bed level with respect to horizontal datum, \( z \) height above bed (\( z = \bar{z} + z_h \)), \( u_h \) flow velocity at water surface (\( z = h \)), \( z_0 \) zero-velocity level (\( z_0 = 0.03k \)), \( k \), effective roughness height, \( h \) water depth, \( b \) the width of the flow, \( \eta = \left( \bar{z} - z_h \right)/\left( \bar{h} - z_h \right) \), \( A_1, \tau \) dimensionless variables to determine, \( c \) the
time-averaged sediment volume concentration, \( w \), the particle fall velocity, \( \varepsilon \), the sediment mixing coefficient, \( t \) the time, \( p \) the porosity of bed material, and \( s_s, s_b \), the cross section-integrated suspended load and bed load transports, respectively. The suspended load transport is defined as

\[
s_s = b \int_{z_0}^{z_0 + h} u c dz
\]

(2.5)

in which \( a \) is the reference level; and the bed load transport is determined by a stochastic function proposed by van Rijn [5-6]:

\[
s_b = 0.1b(\Delta g)^{0.6} \frac{d_{50}^{2}}{D_n^{0.5}} \frac{T_r^{2.1}}{C'_{cr}^{2}}
\]

(2.6)

with

\[
T_r^{2.1} = \frac{1}{(2\pi)^{0.5}} \left[ \left( \frac{\sigma'}{\sigma_{cr,1}} \right)^{2.1} I_3 + \left( \frac{\sigma'}{\sigma_{cr,2}} \right)^{2.1} I_4 \right],
\]

\[
I_3 = \int_0^\infty x^2 e^{-x^2} e^{-2x} dx, \quad I_4 = -\int_0^\infty x^2 e^{-2x} (x - p)^2/2 dx,
\]

\[
\Delta = \rho_s - \rho, \quad D_* = d_{50}(\Delta g/\nu^2)^{1/3}, \quad r = \tau / \sigma',
\]

\[
p = \tau / \sigma_{cr,2}, \quad \tau / \sigma_{cr,1} / \sigma', \quad \rho = \rho g(A/C')^2, \quad C' = 18 \log(4h/d_{50})
\]

\[
\tau_{cr,1} = \frac{\sin(\delta + \alpha)}{\sin \phi}, \quad \tau_{cr,2}^{n1} = \frac{\sin(\delta - \alpha)}{\sin \phi}, \quad \text{when } \frac{\partial \tau}{\partial z} \leq 0
\]

\[
\tau_{cr,1} = \frac{\sin(\delta - \alpha)}{\sin \phi}, \quad \tau_{cr,2} = \frac{\sin(\delta + \alpha)}{\sin \phi}, \quad \text{when } \frac{\partial \tau}{\partial z} > 0
\]

\[
\tau_{cr}^{n1} = 1.5 \tau_{cr}
\]

where \( D_* \) is the dimensionless diameter of particle, \( d_{50} \) and \( d_{90} \) the particle diameters of bed material, \( C' \) the Chezy coefficient related to grains, \( \Delta \) the relative density, \( \nu \) the kinematic viscosity coefficient, \( \alpha' \) the standard deviation of bed-shear stress (after van Rijn \( \alpha' = 0.41 \tilde{\tau} \) with \( \tilde{\tau}_{b,0} \) is effective bed-shear stress at \( z = 0 \), \( \tau \) the time-averaged bed-shear stress, \( \tau_{cr,1} \) and \( \tau_{cr,2} \) the instantaneous critical bed-shear stress in and against flow direction, respectively, \( \tau_{cr}^{n1} \) the mean critical bed shear stress at a horizontal bed according to Shields, \( \delta \) the angle of internal friction, \( \alpha \) the bed slope angle, and \( u \) is the depth-averaged fluid velocity in \( x \) direction.

The formula (2.1) was obtained by L.C. van Rijn on the base of Coles' result [1965] for determination of the velocity profile in nonuniform flow. As it depends on three unknown variables \( A_1, r, u_b \), so three more equations are needed to close the system of equations. They are

\[
Q = b \int_{z_0}^h u d\bar{z}
\]

(2.7)

\[
\frac{-1 + \ln(h/z_0)}{\ln(0.5h/z_0)} = \frac{3r + 1}{(2r^2 + 3r + 1)(2(0.5)^r - (0.5)^{2r})} \cong 0.16r^2 - 0.29r + 1.02
\]

(2.8)

\[
\frac{d\tau_h}{dz} = \alpha_1 \frac{\tau_{kr}}{h} - \alpha_2 \frac{\tau_h}{h} - \alpha_3 \frac{u_h}{b}
\]

(2.9)
with

\[ u_{bc} = \frac{g^{0.5} \ln(h/z_0)}{\kappa C} \lambda \bar{u} = \frac{Q}{kh} \quad C = 18 \log(12h/k) \]
\[ \alpha_1 = 0.28 + 0.11 \tanh(6(dh/dx) - 0.15) \]
\[ \alpha_2 = 0.235 + 0.065 \tanh(17(dh/dx - 0.035)) \]
\[ \alpha_3 = 0.1 \tanh(10dh/dx) \]

in which eq. (2.7) is the continuity equation, eq. (2.8) parameter equation established by using the equilibrium mid-depth velocity of logarithmic distribution in eq. (2.1), eq. (2.9) the water surface velocity equation, \( u_{bc} \) the surface velocity for equilibrium flow, \( \bar{u} \) the cross-section-averaged flow velocity, \( C \) the Chezy coefficient, \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) the empirical coefficients, \( g \) the acceleration of gravity, and \( \kappa \) the constant of Von Karman.

In the case of consideration the sediment mixing coefficient, \( \varepsilon_s \), is approximately equal to the fluid mixing coefficient, \( \varepsilon_f \), that is used usually from parabolic-constant distribution

\[ \varepsilon_f = \begin{cases} 
\varepsilon_{f,\text{max}} - \varepsilon_{f,\text{max}} (1 - 2z/h)^2, & \quad \text{for } \frac{z}{h} < 0.5 \\
\varepsilon_{f,\text{max}}, & \quad \text{for } \frac{z}{h} \geq 0.5
\end{cases} \]  

\[ \frac{d\varepsilon_{f,\text{max}}}{dz} = \frac{\alpha_4}{h} (\varepsilon_{f,\text{max},e} - \varepsilon_{f,\text{max},s}) - \alpha_5 \frac{d(u_h - \bar{u})}{dz} \exp(-15 \frac{dh}{dx}) \]

where \( \varepsilon_{f,\text{max},e} \) is the maximum fluid mixing coefficient for equilibrium conditions, \( u_{*e} \) the equilibrium bed-shear velocity, and \( \alpha_4 \) and \( \alpha_5 \) the empirical coefficients

3. INITIAL AND BOUNDARY CONDITIONS

In general the following conditions are required

\[ z_i(x,t)|_{t=0} = f_i(x), \quad Q = f_2(t), \]
\[ u(x,z,t)|_{z=0} = f_3(x,t), \quad c(x,z,t)|_{z=0} = f_4(x,t), \]
\[ \frac{\partial c}{\partial z} = 0 \quad \text{at } z = z_0 + h \]
\[ -\varepsilon_s \frac{\partial c}{\partial z} = E_{a,c} \quad \text{at } z = z_0 + h \]

in which \( f_i \) (\( i = 1, 2, 3, 4 \)) are known functions, \( E_{a,c} \) is the equilibrium upwards diffusive sediment flux established by van Rijn as follows [6]:

\[ E_{a,c} = 0.03u_* \frac{d\mu}{a} \frac{T_{1.5}^{1.5}}{D_{1.3}^{1.3}} \]

where

\[ T_{1.5}^{1.5} = \frac{1}{(2\pi)^{0.5}} \left[ \left( \frac{\sigma'}{\sigma_{cr,1}} \right)^{1.5} I_1 + \left( \frac{\sigma'}{\sigma_{cr,2}} \right)^{1.5} I_2 \right] \]

\[ I_1 = \int_0^\infty x^{1.5} e^{-|x-n|^{3/2}dz}, \quad I_2 = -\int_0^\infty x^{1.5} e^{-|x-n|^{3/2}dz} \]
4. NUMERICAL PROCEDURE AND RESULTS

The mathematical model was applied to simulate numerically the morphological processes for an experiment flume. The numerical solution was obtained by using Runge Kutta Method for equations determining $u_h$, $c_{f,max}$, then difference method with Crank Nicolson and Spline's interpolation for diffusion equation [1], and Lax scheme for bed deformation.

At the inflow boundary the equilibrium sediment concentration profiles and the logarithmic distribution velocity were used as the boundary conditions [1, 4, 6]:

$$c_e = \begin{cases} \left( \frac{a}{h - \frac{\varepsilon}{Z_0}} \right)^Z & \text{for } \frac{\varepsilon}{Z_0} < 0.5 \\ \left( \frac{a}{h - \frac{\varepsilon}{Z_0}} \right)^Z \exp(-4Z(\frac{\varepsilon}{Z_0} - 0.5)) & \text{for } \frac{\varepsilon}{Z_0} \geq 0.5 \end{cases}$$

in which $Z = \frac{u_c}{\beta k u_c}$ = suspension parameter, the equilibrium concentration at the reference level, $c_{ac}$, is determined according to experiment formula proposed by Van Rijn

$$c_{ac} = 0.015 \frac{d_{so}}{a} T^{1.5} T = \frac{(u'_c)^2 - (u_{acr})^2}{(u_{acr})^2}, \quad u'_c = \frac{g^{0.5}}{C'} \tilde{u}$$

where $u_{ac}$ is the critical bed shear velocity according to Shields, and corresponding velocity profile

$$u_c = \frac{g^{0.5}}{C'} \ln\left(\frac{z}{z_0}\right)\tilde{u}$$

Input data

- water depth at inlet $z = 0$: $h_0 = 0.39m$
- mean current velocity at inlet $z = 0$: $\tilde{u}_0 = 0.51m/s$
- ratio of sediment and fluid mixing coefficient: $\beta = 1$
- particle diameters of bed material: $d_{mn} = 160\mu m$
- $d_{pp} = 200\mu m$
- particle fall velocity of suspended sediment: $u_s = 0.013m/s$
- effective bed roughness: $k_s = 0.025m$
- sediment density: $\rho_s = 2650kg/m^3$
- fluid density: $\rho = 1000kg/m^3$
- angle of repose of bed material: $\phi = 35^\circ$
- porosity of bed material: $\rho = 0.4$
- constant of Von Karmann: $\kappa = 0.4$
- reference level: $a = 0.0125m$

Some results of computation were illustrated in the figures 1 - 7. Fig.1 and Fig.2 present the field of velocity after 5 and 10 hours, Fig.3 - Fig.6 present the concentration distributions at $z = 6m$ corresponding to different time. It is seen that suspended sediment concentration is approximately equal to zero upwards the surface. Finally, the prediction of bed deformation due to deposition in turn after 5, 9 and 15 hours in comparision with the initial form was presented in Fig.7.

4. CONCLUSION

The computational results showed that the mathematical model is posible to apply to the practice. However, because of the stability and the accuracy of solution, the time and space steps chosen is quite small. Therefore the computing process is rather slow. It takes 2.5 minutes per a computational step for AT286, 1 minute for AT386 DX and 15 seconds for AT486 DX to predict the deformation of bed after 3 minutes.
Fig. 1. Velocity field after 5 hours

Fig. 2. Velocity field after 10 hours

Fig. 3. Suspended concentration distribution at \( z = 6 \text{m} \) after 1 hour

Fig. 4. Suspended concentration distribution at \( z = 6 \text{m} \) after 5 hour

Fig. 5. Suspended concentration distribution at \( z = 6 \text{m} \) after 10 hour

Fig. 6. Suspended concentration distribution at \( z = 6 \text{m} \) after 15 hour
Fig. 7. Bed levels after 0, 5, 9 and 15 hours

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MÔ PHÒNG SÓ QUẢ TRÌNH BIẾN ĐÀNG
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