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Optimization of variable-direction long-range trajectory for the unpowered flight vehicle

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Abstract. The study presents a new problem of trajectory for unpowered flight vehicles. The research objects are fixed-wing unpowered flight vehicles operating in the range of transonic and subsonic speeds. The purpose is to find the maximum range when unpowered flight vehicles have to perform a trajectory with changes in their direction. In this study, the flight direction angle at the initial point of the trajectory is a right angle to the target line and the angle at the final point is also a right angle but in an opposite direction. The research focuses on considering the trajectory optimization problem in the horizontal plane according to the criterion of maximum range. It is known that a maximum range of a hundred kilometers can be achieved by fixed-wing unpowered flight vehicles when dropped from an altitude of about ten kilometers at transonic speeds to the target directly. Based on a trajectory scheme and conditions of the trajectory in the vertical plane, this study has optimized the trajectory with direction changes in the horizontal plane. The result obtained is a maximum range of about eighty kilometers. Although this range may be ten kilometers shorter than that achieved in the previous study, it opens up new potential applications for fixed-wing unpowered flight vehicles such as their capability for multiple long-range trajectories.

Keywords: Unpowered flight vehicle, variable-direction trajectory, maximum range.

Classification numbers: 5.6.2, 5.10.2.

1. INTRODUCTION

Nowadays, the speeds of modern aircraft are around two Mach, but not far away, it is predicted that they can reach hypersonic speeds of around five Mach, which opens up many research directions on unpowered flight vehicles (UFVs) or unpowered gliding vehicles. Recently, regarding the subject of unpower glidings, Elandy *et al.* (2018) focused on modeling and simulating a gliding body in free fall [1]. Zheng *et al.* (2020) reported on solving the problem of tracking a non-cooperative gliding flight vehicle using the active switching multiple model method [2]. Al-Bakri *et al.* (2020) focused on solving the problem of guiding unpowered gliding vehicles during approach and landing [3]. El Tin *et al.* (2021) researched the technology of exploiting thermals in powered and unpowered flight autonomous gliders to enhance the vehicle's performance and flight capabilities [4]. Shen *et al.* (2022) investigated a penetration trajectory of a hypersonic gliding vehicle. The paper focuses on the scenario where a hypersonic gliding vehicle encounters two interceptors [5]. Ahmad *et al.* (2022) proposed the problem for a subsonic gliding vehicle by the non-uniform control vector parameterization method [6]. Mahmood *et al.* (2023) focused on solving the problem of range guidance for a subsonic unpowered gliding vehicle using integral action-based sliding mode control [7]. Zhou *et al.* (2023) proposed a control problem for a hypersonic vehicle in the gliding stage, using a guidance law to optimize the trajectory [8].

This study focuses on optimizing the maximum long-range trajectory of UFVs operating at transonic and subsonic speeds with the variable direction (Fig. 1). The purpose of the problem is to enable the carrying aircraft, which can drop the UFV, to follow a path perpendicular to the target line in order to avoid entering the enemy's protected zone. The problem is conducted under standard atmospheric conditions without considering wind effects.

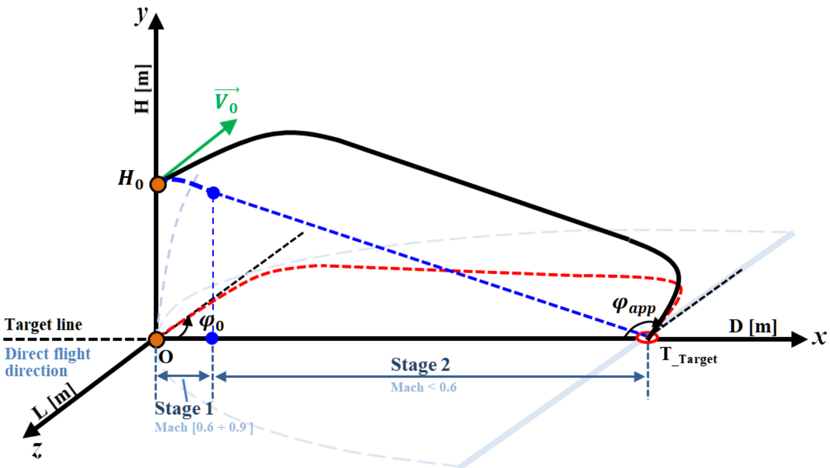


Figure 1. Variable-direction trajectory scheme of UFVs.

The main concern with UFVs is how to get them to fly as far as possible. According to the criterion of maximum range, a gliding flight of UFVs at subsonic speeds less than 0.6 Mach is known to achieve a maximum range when its flying state is maintained at the best aerodynamic quality, i.e. the ratio of lift to drag is maximum $(C_y/C_x)_{max}$. This issue has been mentioned since the 60s of the last century. In official international publications, it was also mentioned by Kelley *et al.* in 1981 [9] and more recently by Zhang *et al.* in 2015 [10].

In the studies of long-range UFVs by Hung *et al.* in 2019 and 2020 [11, 12], the flight state with the best aerodynamic quality at subsonic speeds is referred to as the K_{max} flight state and their studies show that the K_{max} segment may not be suitable when applied to transonic speeds because the aerodynamic properties are complexly changed. At transonic speeds, the coefficient of induced drag (A) and the zero-lift drag coefficient (C_{x0}) have a rapid change vs the Mach number. Therefore, optimizing the trajectory of UFVs is considered in two stages, including the first stage at transonic speeds and the K_{max} stage at subsonic speeds. The research focuses on optimizing the first stage according to the maximum range criterion. As a result, a new range of about ninety kilometers is created when a hypothetical UFV is throw-dropped to the target at transonic speeds and an altitude of around ten kilometers.

Continuing to consider the trajectory problem of UFVs at transonic and subsonic speeds, this paper addresses a new problem of variable-direction long-range trajectories of UFVs, in which the initial condition for the flight direction angle (φ) is set to a right angle ($|\varphi_0| = 90°$).

Assuming the initial condition is $\varphi_0 = 90°$, it is evident that UFVs could follow various trajectories to reach the target, and the final approach angle (φ_{app}) could have any value from 0^0 to -90°. However, only the desired maximum long-range of a variable-direction trajectory with a minimal number of direction changes may be considered, since the initial mechanical energy of UFVs is limited. The minimum number of changes required is two; the first one is to deviate from the target line, and the second one must be directed towards the target (Fig. 1). Therefore, φ_{app} must have the opposite sign to φ_0 to reach the target. If the initial angle is $\varphi_0 = 90°$, the approach angle condition should be $\varphi_{app} = -90°$. Because if the approach angle $|\varphi_{app}|$ is greater than 90°, it will inevitably result in a decrease in the range, which is undesirable. The less $|\varphi_{app}|$ than 90°, the less energy is lost in the final change of direction towards the target, and consequently the greater the long-range to the target. In this paper, the most severe case $\varphi_{app} = -90°$ is optimized from all possible long-range variable-direction trajectories.

Obviously, the direct flight path with a constant value of φ will reach the maximum range. Therefore, the variable-direction trajectory must take advantage of the straight flight stage for as long as possible. The trajectory must consist of three segments: The initial and final segments corresponding to two curves combined with a middle straight segment. The junctions between the segments must be continuous and smooth, without any breaks. The smoothness of the flight trajectory is derived from the limited possibility of any flight vehicle to change its flight direction.

2. OPTIMIZATION OF THE TRAJECTORY

Generally, the motion of UFVs with a variable-direction trajectory is a spatial motion, which means it is simultaneously described by a system of equations in both the vertical and horizontal planes [13]. In the present case, the long-range trajectories are achieved in the vertical plane with a small enough flight path angle [11, 12]. For simplification purposes, this study inherits the research results in the vertical plane [11, 12] and focuses on the problem of trajectory in the horizontal plane.

For the trajectories of UFVs in the vertical plane (altitude channel): This study inherits the results of publications [11, 12], in which the trajectory is considered in two stages, including the first stage within transonic speeds (Mach > 0.6) and the second stage K_{max} (Mach ≤ 0.6).

For the trajectories of UFVs in the horizontal plane: The trajectory of the UFVs in the horizontal plane is the focus of this study. The solution for optimizing the projection trajectory in the horizontal plane is presented below.

To solve the optimization problem, one will try to describe the trajectory analytically, albeit only approximately.

2.1. Approximation of the variable-direction trajectory of UFVs

It is necessary to note that initial curve segment OB is performed at transonic speeds, resulting in significant drag and a rapid decrease in flight speed. Therefore, it can be assumed that the extension stage in the vertical plane [12] will be smaller than the first segment in the horizontal plane ($x_1 < x_2$), as shown in Fig. 2a. As a result, all straight segments of the optimal trajectory must be performed with a flight path angle of θ_{opt} at K_{max}.

As mentioned earlier, the trajectory comprises two curved segments at the initial (OB) and final (CT) stages based on the edge conditions, while the middle segment (BC) is the straight flight with a flight direction zero angle (Fig. 2a and Fig. 2b). The flight direction angle function $\varphi(x)$ is determined based on the conditions at points O, B, C, and T, where $\varphi_0 = 90°$, $\varphi_B = 0°$, $\varphi_C = 0°$, and $\varphi_{app} = -90°$ (Fig. 2c).

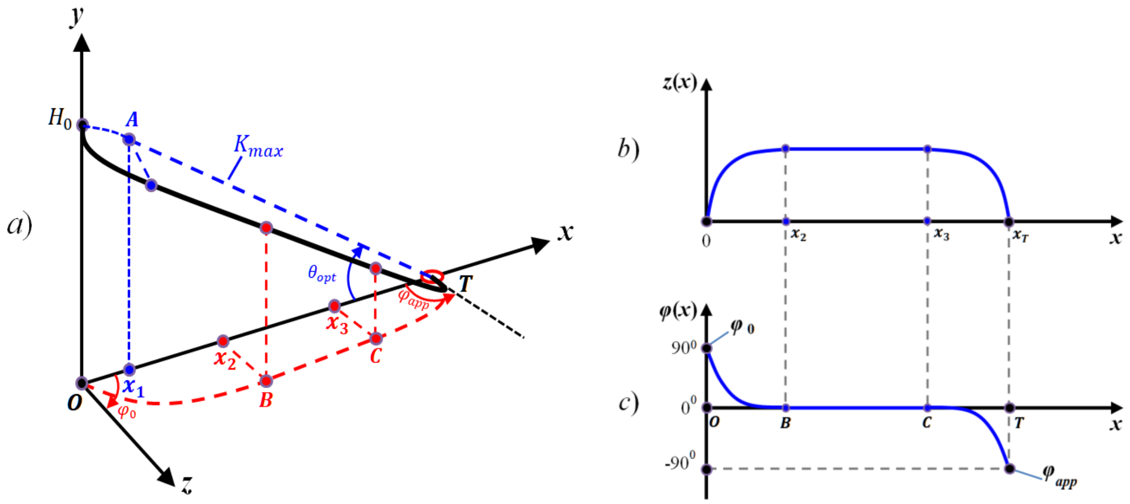


Figure 2. Approximation of variable-direction trajectory of UFVs.

a) The trajectories in horizontal and vertical planes; b) The trajectory in the horizontal plane;

c) The flight direction angle φ(x) vs x-coordinates.

The functions z(x) and φ(x) must be continuous and smooth. So one may approximate as higher-order polynomials with unknown coefficients which depend on known edge conditions.

As shown in Fig. 2c, φ = 0 is represented on segment BC which has two edges at x_2 and x_3. And the function φ(x) is approximated by a 2nd-order polynomial, as shown below.

$$\varphi(x) = \begin{cases} a_0 + a_1 x + a_2 x^2 \\ 0 \\ b_0 + b_1 x + b_2 x^2 \\ \varphi_{app} \end{cases} \quad \text{when} \quad \begin{matrix} x < x_2 \\ x_2 \le x \le x_3 \\ x_3 < x < x_T \\ x = x_T \end{matrix} \tag{1}$$

where a_0, a_1, a_2, b_0, b_1, and b_2 are unknown coefficients of polynomials.

We have:

$$\frac{dz}{dx} = \tan(\varphi) \tag{2}$$

Therefore, the function z(x) is written:

$$z(x) = \int \tan(\varphi(x))dx + z_0 \quad (3)$$

Calculating the trajectory parameters:

With φ_0 and φ_{app} are known, we have trajectory conditions: $\varphi(x_B) = 0^0$ at the x_2 coordinate, $\varphi(x_C) = 0^0$ at the x_3 coordinate, as shown in Fig. 2. As a result, the trajectory parameters can be determined as:

$$\begin{cases} a_0 = \varphi_0; \quad a_1 = -2\varphi_0 / x_2; \quad a_2 = \varphi_0 / x_2{}^2; \\ b_0 = b_2 x_3^2; \quad b_1 = -2b_2 x_3; \quad b_2 = \varphi_{app} / \left(x_T - x_3\right)^2. \end{cases} \quad (4)$$

2.2. The optimization problem

In [4], the trajectory in the vertical plane was approximated by a high-order polynomial. Similarly, in this study, trajectories in the horizontal plane were also approximated using polynomials. Therefore, when applying the classical equations that describe the motion of the center of mass of flight vehicles [13] to the problem of variable-direction trajectories, they can be reduced to combine with approximation polynomials in both horizontal and vertical planes, as shown below:

$$\begin{cases} \dot{V} = -\left(X + G\sin\theta\right)/m \\ n_y = \left(\dfrac{V\dot{\theta}}{g} + \cos\theta\right)/\cos\gamma \\ \sin\gamma = -\dfrac{mV\cos\theta}{Y}\dot{\varphi} \\ \dot{x} = V\cos\theta\cos\varphi \end{cases} \quad (5)$$

where X, Y, m, g, n_y , x are drag, lift, mass, gravitational acceleration, the normal load factor, and coordinate x, respectively.

Formulation of the optimization problem:

According to (4), the parameters a_0, a_1, a_2, b_0, b_1, and b_2 of the function z(x) depend on parameters x_2 and x_3. Therefore, the problem of finding the optimal trajectory based on the criterion of maximum range (x_{T_max} or D_{max}) can be solved by finding the two optimal variables $(x_2, x_3)_{opt}$. From the conditions: $\varphi_0 = 90^0$ và $\varphi_{app} = -90^0$, find the variable parameters $(x_2, x_3)_{opt}$ that provide the maximum range:

$$D = x_T = f(x_2, x_3) \to \max \quad (6)$$

with the constraints:

$$\begin{cases} C_y \le C_{y\max} \\ V \ge V_{\min} \end{cases} \quad (7)$$

The constraints (7) must be enforced on the whole trajectory [11].

***Method of solving the optimization problem*:**

It can be recognized that the range D is the value of the coordinate x at zero altitude (D = $x_{|y=0}$), and the function D depends on the time functions V(t), θ(t), φ(t). Obviously, solving the system of equations (5) and (6) by analytical methods is not feasible, and the optimization problem by analytical methods is also impossible. They are only feasible with numerical methods. In this study, the numerical method of optimization is used through the Particle Swarm Optimization algorithm (PSO) [14]. According to this algorithm, a software in Matlab/Simulink has been built.

3. RESULTS AND DISCUSSION

3.1. Input data used in calculation

The optimization is performed on a hypothetical UFV model with the characteristics and initial conditions of trajectory shown in Table 1. The coefficient of induced drag (A) and the zero-lift drag coefficient (C_{x0}) of the UFV are used according to [11]. The data are very close to reality for both the carrying aircraft and the UFV.

Table 1. Characteristics of the hypothetical UFVand initial conditions of trajectory.

m	S_{ref}	C_{ymax}	H_0	V_0	θ_0	θ_{opt}	x_0	z_0	φ_0	φ_{app}
kg	m^2	-	m	m/s	deg	deg	m	m	deg	deg
250	0.5	1.5	12,000	270	0	-8.5	0	0	+90	-90

3.2. Software Verification

The software was verified qualitatively and quantitatively. Qualitative verification was implemented by increasing or decreasing some input parameters while the output parameters must be changed accordingly. Quantitative verification was also implemented by comparison with the theoretical solution in a special case (“benchmark”).

The program consists of three interconnected computational components: Solving dynamical equations, optimizing the trajectory, and approximating the trajectory. The trajectory approximation is done analytically and is easy to be verified. The dynamical equation system has been validated by comparing it with some published solutions in certain special cases [13]. The numerical optimization algorithm, PSO, has been validated and verified [14].

The combination of solving dynamical equations and trajectory optimization has been verified as follows: The theoretical solution for the variable-direction trajectory optimization problem exhibits a rectangular shape consisting of three segments. The flight direction angle (φ) changes from 90^0 to 0^0 in the initial segment and from 0^0 to -90^0 in the final segment while the mid-segment represents a straight flight segment. To maximize the range for the variable-direction trajectory, the straight segment must be as long as possible.

However, the theoretical trajectory requires two breaks of the trajectory (the flight direction angle φ must be changed immediately from 90^0 to 0^0 and from 0^0 to -90^0), and at the time, dφ/dt (the flight vehicle maneuverability) must be of an infinite value. It can be observed that dφ/dt approaches positive or negative infinity as C_y approaches infinity. Consequently, the flight speed

(V) approaches zero (Fig. 3) when the drag is extremely high. Therefore, we can verify the combination of the numerical methods and computational program by increasing the value of C_{ymax} within the approach of the trajectories to the theoretical trajectory (the "benchmark") as shown in Fig. 4.

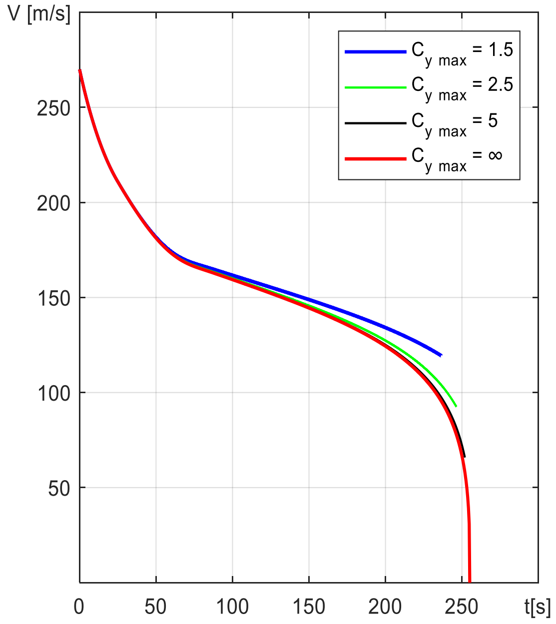


Figure 3. Changes in the flight speed.

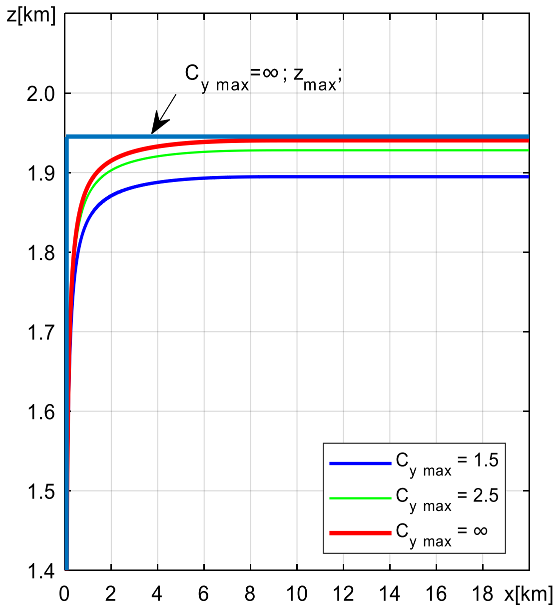


Figure 4. Approach of the variable-direction trajectories to the theoretical trajectory ("benchmark").

3.3. The results of optimization

The optimization results show the maximum range D_{max} = 81 [km] by two optimal variables x_{2_opt} = 8,298 [m]; x_{3_opt} = 72,919 [m] and the variable-direction trajectory is shown in Fig. 5:

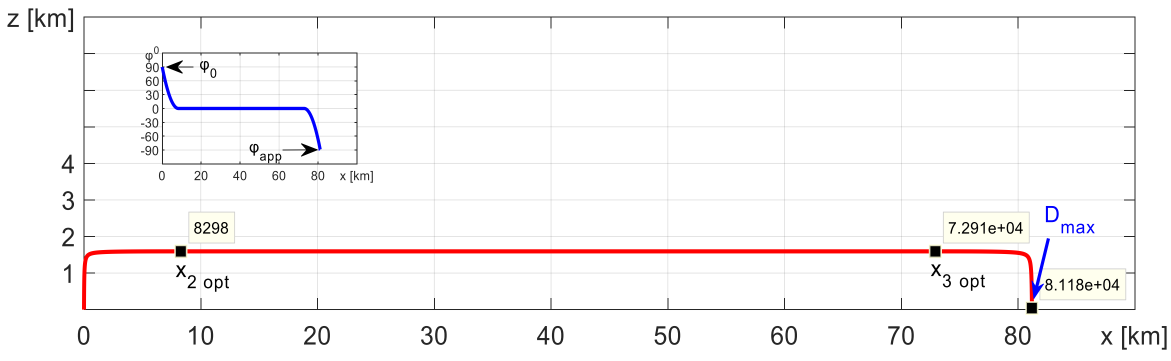


Figure 5. The variable-direction trajectory of the hypothetical UFV (its horizontal projection).

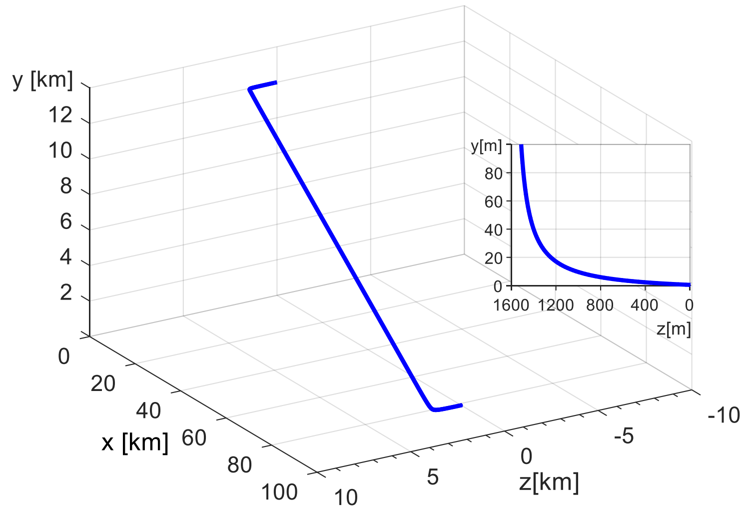
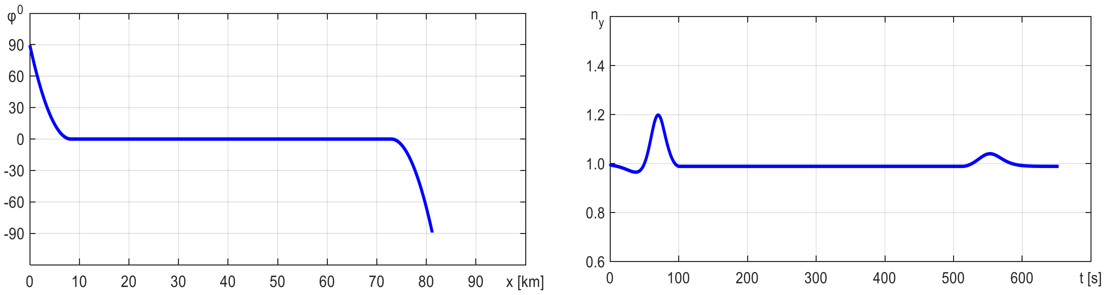


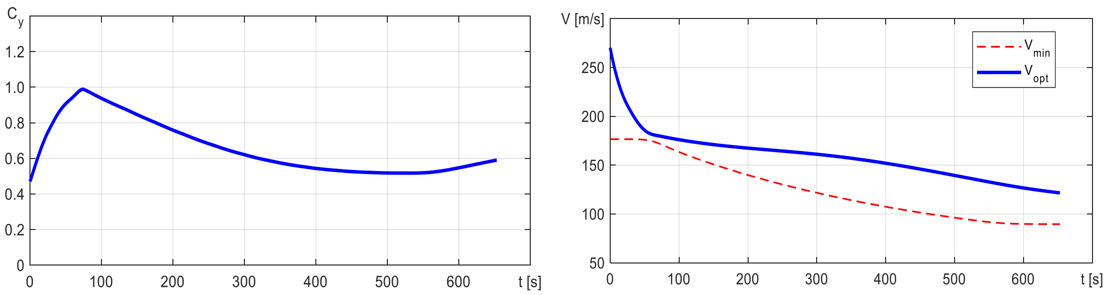
Figure 6. The trajectory in spatial reference system.



a) The flight direction angle vs x-coordinate

b) The normal load factor

Figure 7. Correlation between input parameter of flight direction angle and required normal load factor.



a) The required lift coefficient

b)The velocity

Figure 8. The flight parameters of the UFV.

Discussion: The optimal trajectory results were obtained using the optimal variables $(x_2, x_3)_{opt}$, which created a trajectory with the maximum range of D_{max} = 81 kilometers, as shown by the curve in Fig. 5 and Fig. 6. The results of the functions z(x) and φ(x), as shown in Fig. 5 and

Fig. 7a, respectively, are consistent with two initial and final curve segments represented by continuous and smooth curves. Regarding the flight dynamics characteristics, the trajectories are feasible when $C_y < C_{ymax}$ and $V > V_{min}$, as shown in Fig. 8. The results for the normal load factor (n_y) are also within a realistic scope, as shown in Fig. 7b. We attempted to approximate the variable-direction long-range trajectory of UFVs using a higher-order polynomial, but the results were similar and not better.

4. CONCLUSIONS

The variable-direction trajectory of UFVs and the law of the flight direction angle are built to reach the maximum range. The study used polynomial approximation for the initial and final curve segments and a straight middle segment to form a smooth trajectory that leads to a two-parameter optimization problem which is solved numerically using the PSO optimization algorithm, in which the computational program has verified the theoretical trajectory on the basis of C_{ymax} infinity. As a result, the maximum range of the variable-direction long-range trajectory of UFVs was found to be about eighty kilometers. The results were obtained under realistic conditions. It can be used as a reference trajectory intended for long-range UFVs. Although the results obtained from this work have a range smaller than those obtained from the case of direct flight to the target [12], it opens up new applications for UFVs such as multi-trajectory solutions for gliding bodies and gliding vehicles.

CRediT authorship contribution statement. Pham Tuan Hung: Problem solving method, formal analysis, codes programming, writing and editing original draft. Nguyen Duc Cuong: General orientation, review and editing.

Declaration of competing interest. The authors declare no conflict of interest.

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