Prediction of maximum earthquake magnitude for northern Vietnam region based on the gev distribution

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ABSTRACT

The present work is a continuation and improvement of the application of the generalized extreme value distribution to study the seismicity of the Southeast Asia. We have applied the generalized extreme value distribution (GEV) method to estimate maximum magnitude value (M_max) for the earthquake catalog of Northern Vietnam. Using this method, we obtain the distribution of maximum earthquake magnitude values. This distribution can be characterized by its quantile Q_q(τ) at any desirable statistical level q. The quantile Q_q(τ) provides a much more stable and robust characteristic than the traditional absolute maximum magnitude M_max (M_max can be obtained as the limit of Q_q(τ) as q → 1, τ → ∞). The parameters have been obtained: ζ = -0.178 ± 0.08; σ = 0.23 ± 0.08; μ = 4.39 ± 0.16; M_max = 6.8 with the probability of 98% for period 2014 - 2064.

Keywords: Maximum magnitude (M_max), generalized extreme value distribution (GEV), earthquake prediction, seismic hazard.

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1. Introduction

The Northern Vietnam region is the most active tectonic and high potential risk area of Vietnam. The parameter M_max represents the maximum of possible earthquake magnitude in the study region. This parameter plays a very important role in seismic hazard assessment and mitigation of the seismic risk. Giving a reliable estimate of M_max, it is comparatively easy to take adequate decisions on the construction standards of buildings or on the insurance policy (Pisarenko et al., 2014b). Therefore, the maximum magnitude earthquake prediction is not only the task with the scientific sense but also an imperative task for the seismic practice of Vietnam.

There are many methods to assess maximum earthquake magnitude including the geological extrapolation (Phan et al., 2012, 2013), calculation of M_max base on size of earthquake source zone (Nguyen N.T et al., 2005; Bui et al., 2013), probabilistic methods... (Gumbel, 1958; Nguyen H.P, 1991, Nguyen N.T et al., 2005, Nguyen H.P et al.,
One of the probabilistic methods is based on the generalized extreme value distribution (GEV). This method is introduced by Pisarenko et al. for the Harvard catalog (Pisarenko et al., 2007, 2008), the catalogs of Japan (Pisarenko et al., 2010) and Vietnam (Pisarenko et al., 2012). We used this method to assess $M_{\text{max}}$ for Southeast Asia and obtained $235,8$ for period 2013 - 2063 with probability 98% (Vu et al., 2014).

In this work we continue to use this method to assess $M_{\text{max}}$ for the Northern Vietnam and obtained $8,6$ for period 2014 - 2064 with probability 98%.

2. Methodology and used data

2.1. Used data

The study area is limited by the coordinates $\phi = 17^\circ \div 24^\circ$ N; $\lambda = 102^\circ \div 110^\circ$ E (Figure 1).

We collect data from various sources: the Department of the seismological survey, the Earthquake Information and Tsunami Warning Centre, the previously published earthquake catalog on the territory of Vietnam and the data from International Seismological Center - ISC. In the data from ISC, an earthquake can have 4 types of magnitude: Local magnitude ($M_L$), body - wave magnitude ($m_b$), surface - wave magnitude ($M_s$), moment magnitude ($M_w$). However, as $M_L$ is the most common magnitude used in Vietnam, the $M_L$ values were chosen for the entire catalog. It is possible to convert $m_b, M_s, M_w$ values to $M_L$.

The collected data have 1376 earthquakes with magnitudes $M = 1.7$ - $7.5$. After separation of foreshocks and aftershocks from this earthquake catalog, we get independent earthquake catalog including 1196 independent earthquakes with magnitude $1.7 \leq M \leq 7.5$ for Northern Vietnam and surrounding regions.

The data in this catalog are continuous on time since 1972, so we chose the period from 1972 to 2014 for estimation of $M_{\text{max}}$. There are 349 earthquakes with $M \geq 4.1$ in the period.

2.2. Prediction method

The distribution function generalized extreme value is defined as follows (Pisarenko et al., 2007, 2008, 2010):

$$GEV(x | \sigma, \mu, \zeta) = \begin{cases} \exp\left(-\left(1 + \left(\frac{\zeta}{\sigma}\right)(x - \mu)\right) - \frac{1}{\zeta} \right) & \text{if } 0 < \sigma > 0; \; \zeta > 0; \; x > \mu - \sigma/\zeta; \; \zeta \neq 0 \\ \exp\left(-\exp\left[-\frac{x - \mu}{\sigma}\right]\right) & \text{if } \zeta = 0 \end{cases}$$

Where $x$ is variable representing the magnitude earthquake value, $\sigma$ is the scale parameter, $\mu$ is the location parameter, $\zeta$ is the form parameter.

To determine the GEV function we need to identify 3 parameters $\zeta, \sigma, \mu$ in formula (1). These parameters $\zeta, \sigma, \mu$ are determined in each period $T$, by solving the set of three equations below:

$$\begin{align*} \frac{1}{n} \sum_{k=1}^{n} x_k &= \mu - \frac{\sigma}{\zeta} + \frac{\sigma}{\zeta} \Gamma(1 - \zeta) = M1 \\ \frac{1}{n} \sum_{k=1}^{n} (x_k - M1)^2 &= (\frac{\sigma}{\zeta})^2 \left[\Gamma(1 - 2\zeta) - (\frac{\zeta}{\zeta})(\frac{\zeta}{\zeta})\right] = M2 \\ \frac{1}{n} \sum_{k=1}^{n} (x_k - M1)^3 &= (\frac{\sigma}{\zeta})^3 \left[\Gamma(1 - 3\zeta) - \frac{\zeta}{\zeta} \Gamma(-3\zeta)\right] = M3 \end{align*}$$

where $\Gamma(x)$ is the Gamma function: $\Gamma(t) = \int_{0}^{\infty} x^{t-1} e^{-x} dx$ , $n$ is the number of earthquakes in each $T$-intervals, $x_k$ is magnitude of $k^{th}$ earthquake.

It is important to determine $T$-intervals to suit each catalog because $T$-intervals have the influence on the values of the three parameters $\zeta, \sigma, \mu$ of the GEV function. To
find T-intervals, we need to determine the density Poisson distribution (\(\lambda\)) of the magnitude earthquake values:

\[ \lambda = \frac{N}{t}, \text{ where } N \text{ is the number of independent earthquakes, } t \text{ is the time between the first event and the last event.} \]

The chosen T-values (days) must satisfy three conditions:

- All T-intervals are non-empty.
- Value 1 / \(\lambda T\) \(\rightarrow\) 0 (with \(\lambda\) is the frequency earthquakes with magnitude \(M \geq m\)).
- Value of parameter \(\xi\) is stable enough to determine the GEV function.

The following steps should be taken:

- Choose an interval of values \((T_L; T_H)\) for time interval durations \(T\), for which the catalog still contains a sufficient number of T-intervals (with \(T_L\) is the lowest time; \(T_H\) is the highest time);
- Choose in this interval \((T_L; T_H)\) a finite set of \(u\) time-interval durations \(T\) \((T_L \leq T_1 < T_2 < \ldots < T_u \leq T_H)\);
- The GEV parameters are estimated by the method of moments (Pisarenko et al., 2007, 2008, 2010) for each of the \(u\) time - interval durations \(T\), which yields the following set of parameters:
  \[ \zeta(T_1), \zeta(T_2), \ldots, \zeta(T_u), \sigma(T_1), \sigma(T_2), \ldots, \]
  \[ \sigma(T_h), \mu(T_1), \mu(T_2), \ldots, \mu(T_u); \]
- To estimate the average values \(\overline{\zeta}, \overline{\sigma}, \overline{\mu}\) of the GEV parameters \(\zeta, \sigma, \mu\);
- The \(\tau\) is the predicted period (from the time of the earthquake event was chosen as supporting event). The parameters \(\zeta, \sigma, \mu\) are represented as the functions of \(\tau\) by the formulas (5-7) below:

\[ \zeta(\tau) = \zeta(T); \]
\[ \sigma(\tau) = \sigma(T); (\tau/T)^{\zeta}; \]
\[ \mu(\tau) = \mu(T) + (\sigma(T)/\xi).((\tau/T)^{\zeta} - 1); \]

- The quantile in this period is:

\[ Q_q(\tau) = h + (s/\xi) - (\sigma/(\lambda T)^{\zeta} - 1); \]

where:

\[ a = (\log(1/q))^{\zeta}, \]
\[ h = \mu + (\sigma/\xi) - (\lambda T)^{\zeta} - 1; \]
\[ s = \sigma. (\lambda T)^{\zeta}. \]

When \(\tau \rightarrow \infty\), then \(Q_q(\tau) \approx M_{\max}(\tau) \rightarrow M_{\max}:\)

\[ M_{\max} \text{ predict } = \lim_{t \rightarrow \infty} Q_q(\tau) \]

Thus, after finding the appropriate T-intervals, three parameters \(\zeta, \sigma, \mu\) can be found in each time period. The obtained results can be used to determine the content of GEV, decile point value of \(Q_q(\tau)\), and to assess the \(M_{\max}\) value.

3. Calculation results

In this section, we present the calculation results for the given data set.

**Step 1: Calculate the density Poisson distribution (\(\lambda\))**

The period from 23/1/1972 \((t_1)\) to 20/8/2014 \((t_u)\) used with the daily unit. The total time are 15518.71 days. The number of T-intervals is \(n\): \(n = \text{integer} \frac{t_u - t_1}{T}\)

Density \(\lambda\) Poisson distribution is calculated as follows:

\[ \lambda = \frac{N}{t} = \frac{349/15518.71}{15518.71} = 0.002249 \]

**Step 2: Select the jump (\(T\))**

According to the data in the catalog, to satisfy the condition (a) above, the smallest value of T-intervals is 250 days. The T-intervals in the corresponding product \(\lambda T\) are the following:

<table>
<thead>
<tr>
<th>(T)</th>
<th>255</th>
<th>265</th>
<th>275</th>
<th>285</th>
<th>295</th>
<th>305</th>
<th>315</th>
<th>325</th>
<th>335</th>
<th>345</th>
<th>355</th>
<th>365</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1/\lambda T)</td>
<td>0.174</td>
<td>0.168</td>
<td>0.162</td>
<td>0.156</td>
<td>0.151</td>
<td>0.146</td>
<td>0.141</td>
<td>0.137</td>
<td>0.133</td>
<td>0.129</td>
<td>0.125</td>
<td>0.122</td>
</tr>
</tbody>
</table>

From this table, the greater T-intervals are, the smaller value of the ratios \((1/\lambda T)\) are. In principle, the closer values \((1/\lambda T)\) are to the value "0", the better T-intervals are. However, to satisfy the condition (c), Figure 2 shows an approximate "stabilization" of the \(\zeta\) estimates...
in the range 300 and 350 days. Therefore, to satisfy the above conditions, the value of $T$-interval is 350 days. With $T = 350$ days, then $n = \text{integer} \left( \frac{T_{m-1}}{T} \right) = 44$.

**Step 3: Determine the parameters $\zeta, \sigma, \mu$**

In each $u$ time-interval durations $T$ ($T_L \leq T_1 < T_2 < \ldots < T_u \leq T_H$), the parameters $\zeta, \sigma, \mu$ are determined in each period $T$, by solving the set of three equations (2.4).

$\zeta(T_1), \zeta(T_2), \ldots, \zeta(T_u), \sigma(T_1), \sigma(T_2), \ldots, \sigma(T_u), \mu(T_1), \mu(T_2), \ldots, \mu(T_u)$;

To estimate the average these values:

$\overline{\zeta} = -0.178; \overline{\sigma} = 0.23; \overline{\mu} = 4.39$.

In order to estimate the Mean Square Error (MSE) of these estimates, we use formulas (Pisarenko et al., 2008):

$$MSE_\zeta = \left( \frac{1}{n} \sum_{j=1}^{n} (\zeta_j - \overline{\zeta})^2 \right)^{1/2}$$

$$MSE_\sigma = \left( \frac{1}{n} \sum_{j=1}^{n} (\sigma_j - \overline{\sigma})^2 \right)^{1/2}$$

$$MSE_\mu = \left( \frac{1}{n} \sum_{j=1}^{n} (\mu_j - \overline{\mu})^2 \right)^{1/2}$$

Therefore, the parameters are:

$\zeta = -0.178 \pm 0.08; \sigma = 0.23 \pm 0.08; \mu = 4.39 \pm 0.16$.

**Step 4: Determine predicted $M_{\text{max}}$**

In the earthquake catalog used, the last strongest earthquake, which occurred 29.06.2014 with magnitude $M = 4.4$, has satisfied above specified conditions. So we have chosen this event as supporting event.

$M_{\text{max}}^{\text{predict}} = \lim_{T \to \infty} Q_T(\zeta(T))$

With predicted probability 98%, we get the graph of the function $Q_T(\zeta)$ in Figure 3.

From figure 3, we have:

$M_{\text{max}}^{\text{predict}} = \lim_{T \to 10} Q_T(\zeta(T)) = 6.67$;

$M_{\text{max}}^{\text{predict}} = \lim_{T \to 20} Q_T(\zeta(T)) = 6.72$;

$M_{\text{max}}^{\text{predict}} = \lim_{T \to 30} Q_T(\zeta(T)) = 6.75$;

$M_{\text{max}}^{\text{predict}} = \lim_{T \to 40} Q_T(\zeta(T)) = 6.78$;

$M_{\text{max}}^{\text{predict}} = \lim_{T \to 50} Q_T(\zeta(T)) = 6.8$.
4. Discussions

Largest earthquake is predicted to occur in the Northern Vietnam by GEV method is $M_{max}^{Predict} = 6.8$ in the next 50 years. This result is quite consistent with the results obtained in the work (Nguyen Ngoc Thuy, 2005), but there are differences compared to the results in the works (Cao Dinh Trong, 2013) ($M_{max} = 6.7$), (Ngo Thi Lu, 2012) ($M_{max} = 7.0$); (Nguyen Hong Phuong 1991) ($M_{max} = 7.0$); Phan Trong Trinh et al., 2012) ($M_{max} = 7.0$); Pham Van Thuc and Kijko ($M_{max} = 7.2$); (Nguyen Hong Phuong, 1997) ($M_{max} = 7.3$). Such differences may be due to the different studied zones, the methods used and the limitations of the length of data period considered (only in 42 years (1972-2014)).

5. Conclusions

On the basis of the catalog of independent earthquakes in period 1972-2014, the maximum earthquake magnitude value was assessed for the Northern Vietnam using GEV method.

We obtained the following sample estimates for this catalog with $T = 350$ days: $\zeta = -0.178 \pm 0.08$; $\sigma = 0.23 \pm 0.08$; $\mu = 4.39 \pm 0.16$;

This distribution can be characterized by its quantile $Q_q(\tau)$ at any desirable statistical level $q$. With predicted probability 98%, we obtained $M_{max}^{Predict} = \lim_{\tau \to \infty} Q_q(\tau) = 6.8$ for period 2014 - 2064.

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