DETERMINATION OF THE CONSTANT \( W_0 \) FOR LOCAL GEOID OF VIETNAM AND IT’S SYSTEMATIC DEVIATION FROM THE GLOBAL GEOID

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ABSTRACT: Constant \( W_0 \), defining the geoid, has important applications in the area of physical geodesy. With the development of artificial Earth satellite, constant \( W_0 \) for the global geoid approximating the oceans on Earth can be calculated from an expansion of spherical harmonics - Stokes constants determined by observation of perturbations in artificial satellite’s orbits. However, the Stokes constants are limited, therefore the geoid constant \( W_0 \) could not be calculated for local geoid (state geoid) from the mentioned expansion of spherical harmonics. In this paper, we present a method to determine the constant \( W_0 \) for local geoid of Vietnam, using generalized Bruns formula and Neyman boundary problem. The initial data used are Faye gravity anomalies surveyed on land and sea of Southern Vietnam. The constant \( W_0 \) is then used to calculate the systematic deviation of the local geoid of Vietnam from the global geoid EGM - 96.

Keywords: The geoid, Stokes constants, Bruns formula, Neyman boundary problem.

INTRODUCTION

The subject of the paper in the field of geodetic physics, related to gravity potential, and gravity anomalies are the original data to determine the geoid, which is the equatorial surface coinciding with the calm ocean surface, no wave, no wind, no tides, and no currents. The geoid shape is considered to be the shape of the Earth. Geoid is the standard surface for determining the standard elevation of territorial topography [1]. The elevation \( \zeta \) of the geoid surface was determined against the reference ellipsoid surface, it is referred to as the height anomaly. In this paper, we use the spheroid, that is approximative ellipsoid, which is normal potential \( U(\rho, \varphi) \) extracted from the serial of gravity potential \( W \) with spherical harmonics \( n \) and centrifugal potential [2]. The global geoid is approximately the ocean surface on Earth, determined by satellite method that does not approximate the sea surface of each country, including Vietnam. The traditional Stokes integral formula is used to determine the local geoid by using ground-based gravity anomalies. Since 1991, Lan P. H. has identified the local geoid for Viet Nam with accuracy of 1.5 - 2.0 m [3]. In 1998, Vo D. H. used the EGM-96 gravity model combination to build the geoid VN 2003, with details from 0.2 m to 0.5 m.

However, the Stokes formula considers the standard reference surface to calculate the geoid height as a sphere, not an ellipsoid, so the Stokes formula does not contain the constant \( U_0 \) of the reference ellipsoid and the constant \( W_0 \) of the local geoid [4]. To determine the systematic deviation (displacement) between the local geoid of Vietnam and the global geoid, it is necessary...
Determination of the constant $W_o$ for local...

To know the local geoid constant $W_o$ and the global geoid constant $W_o$. However, Pham Hoang Lan postulated that the local geoid constant $W_o$ cannot be determined [5]. This is a problem that this paper deals with.

To solve this problem, we used 3738 Faye gravity anomaly data in Southern Vietnam and sea of Southern Vietnam, at coordinates of $8.16^\circ\rightarrow 17^\circ$ latitude North, $104.5^\circ\rightarrow 112^\circ$ longitude East, to transform into ground-based potential anomalies $T$, by applying the Neyman boundary problem. In addition, we measure GPS at 20 specific locations along the coast of Vietnam to determine the standard geoid heights in Vietnam. Since then, we have determined the geoid constant $W_o$ for the local geoid of Vietnam by using the general Bruns formula.

Local geoid constant $W_o$ is important for determining the local geoid height $\zeta$ of Vietnam relative to any reference ellipsoid surface with the equation $U(\rho, \phi) = U_o$, and determining the systematic deviation of the local geoid of Vietnam from the global geoid EGM - 96 as we described in this paper.

**THE GENERAL BRUNS FORMULA, NEYMAN BOUNDARY PROBLEM, GEOID CONSTANT AND THE SYSTEMATIC DEVIATION BETWEEN TWO GEOIDS**

**The general Bruns formula**

The general Bruns formula has the form [6]:

$$\zeta = \frac{T}{\gamma} + \frac{U_o - W_o}{\gamma}$$

With: $\zeta$ - the geoid height relative to the reference ellipsoid has an equation $U(\rho, \phi) = U_o$; $T$ - the disturbed potential is potential anomaly of satellite gravity method, random variation, depending on latitude and longitude:

$$T(\rho, \phi, \lambda) = W(\rho, \phi, \lambda) - U(\rho, \phi)$$

$\gamma$ - normal gravity values change slowly in latitude $\phi$.

Formula (1) is the general Bruns formula, where $T/\gamma$ is the fast variable component, set:

$$\zeta_o = \frac{U_o - W_o}{\gamma}$$

$\zeta_o$ - the component changes slowly with normal gravity (latitude $\phi$).

This is the deviation of approximately optimal spheroid surface, which is determined by equation $U(\rho, \phi) = W_o$ ($U_o = W_o$, also known as the common spheroid), with reference ellipsoid surface $U_o$.

**Neyman boundary problem**

The Neyman boundary problem [7]: There is derivative $V_z$ of the gravitational potential $V$ for $z$-dimension ($V_z$ - gravitational force), distributed on the plane of observation Oxy. We need to find the potential $V$ in out space that satisfies the equation Laplace and the boundary conditions, mentioned above, and is regular in infinity.

Applying the Poisson formula (in the Oxyz coordinate system, with the upward axis Oz) for the derivative $V_z$, that is identical to the gravity anomaly $\Delta g$:

$$V_z(x, y, z) = \frac{z}{2\pi} \int_0^\infty \frac{V_z(\xi, \eta, \zeta) d\xi d\eta}{\left[(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2\right]^{3/2}}$$

To multiply the two sides with $-dz$, and integrate by $z$, $z \rightarrow \infty$, $\zeta = 0$ (on the plane of observation Oxy):

$$-\int_{z}^{\infty} \frac{\partial V(x, y, z)}{\partial z} dz = -\frac{1}{2\pi} \int_0^\infty V_z(\xi, \eta, 0) \left[\frac{dz}{\left[(x-\xi)^2 + (y-\eta)^2 + z^2\right]^{3/2}}\right] d\xi d\eta$$

139
\[
\Leftrightarrow -V(x, y, \infty) + V(x, y, z) = \frac{1}{2\pi} \int_{\mathcal{S}} \frac{V_z(\xi, \eta, 0) d\xi d\eta}{\left[(x - \xi)^2 + (y - \eta)^2 + z^2 \right]^{1/2}}
\]

(6)

Since \(V(x, y, \infty) = 0\), regular in infinity, we have the Neyman boundary problem, with \(z = 0\):

\[
V(x, y, 0) = \frac{1}{2\pi} \int_{\mathcal{S}} \frac{V(z(\xi, \eta, 0)) d\xi d\eta}{\left[(x - \xi)^2 + (y - \eta)^2 \right]^{1/2}}
\]

(7a)

Applying (7a) with \(V = T\), the disturbed potential (potential anomaly) and \(V_z\) is gravity anomaly \(V_g\). We have the formula to calculate disturbed potential \(T\) from gravity anomalies \(V_g\):

\[
T(x, y, 0) = \frac{1}{2\pi} \int_{\mathcal{S}} \frac{\Delta g(\xi, \eta, 0) d\xi d\eta}{\left[(x - \xi)^2 + (y - \eta)^2 \right]^{1/2}}
\]

(7b)

**Geoid constant \(W_o\)**

When the spheroid satisfies the equation \(U(\rho, \phi) = W_o\) (the geoid constant \(W_o\) instead of \(U_o\)), we obtain the equation of the approximately optimal spheroid of geoid [8].

Then, reference ellipsoid will duplicate with approximately optimal spheroid of geoid and geoid will fluctuate around approximately optimal spheroid of geoid, geoid heights obtain negative values and positive values, according to traditional Bruns formula:

\[
\zeta = \frac{T}{\gamma}
\]

(8)

After transforming the observed gravity anomaly to the potential anomaly, \(T\) combines with geoid height \(h\), measured by GPS in the coast of Vietnam as a boundary condition. We determine the local geoid constant \(W_o\) in formula (1). At the coast, the standard height \(H = 0\), so \(\zeta = h - \text{GPS receiver.}\)

Applying (1) to local geoid by re-symbolizing: \(W_o = W'_o\), \(T = T'\), \(\zeta = \zeta'\) so that:

\[
\zeta' = \frac{T'}{\gamma} + \frac{U_o - W'_o}{\gamma}
\]

(9)

From (9): \(W'_o = \frac{T' - \gamma \zeta'}{\gamma} + U_o\)

(10)

The local geoid constant \(W'_o\) is calculated by the values \(T', \gamma, U_o, \zeta'; U_o = 62636851.71 - \text{ellipsoid constant of normal gravity WGS - 84}; \gamma - \text{normal gravity formula of normal gravity WGS - 84.}\)

\[
\gamma = \frac{9.7803267714(1 + 0.001931851386\sin^2 \phi)}{\sqrt{1 - 0.0066943799013\sin^2 \phi}}
\]

(11)

\(\zeta'\): obtained from GPS observation to measure geodetic height in the coastal area of Southern Vietnam, we have: \(\zeta' = h\); \(T'\): calculated from gravity anomalies by integral method (7b) (solution of Neyman boundary problem).

**The systematic deviation between two geoids**

Apply the formula (1) to the global geoid and local geoid: \(\zeta = \frac{T}{\gamma} + \frac{U_o - W'_o}{\gamma}\)

(12)

\[
\Delta \zeta = \left(\frac{T}{\gamma} + \frac{U_o - W'_o}{\gamma}\right) - \left(\frac{T}{\gamma} + \frac{U_o - W_o}{\gamma}\right) = \frac{\Delta T}{\gamma} + \frac{W_o - W'_o}{\gamma}
\]

(14)
Symbol:
\[
\Delta \zeta_\omega = \frac{W_o - W'_o}{\gamma} \tag{15}
\]

\(\Delta \zeta_\omega\): the systematic deviation between two geoids, systematically varies with \(\gamma\).

\(\Delta \zeta_\omega = \frac{W_o - W'_o}{\gamma}\) is the systematic deviation between two spheroids, that are approximately optimal spheroids of geoids (dotted line) as fig. 1.

Fig. 1. The systematic deviation between two geoids is the systematic deviation between two spheroids that are approximately optimal spheroids of geoids

**CALCULATION RESULTS**

**Faye gravity anomaly map**

The data used to process in this paper is the Faye gravity anomaly data in Southern Vietnam and sea of Southern Viet Nam, at coordinates of 8.16°→17° latitude North, 104.5°→112° longitude East, with 3738 points. These include ground-based gravity data and satellite sea-based gravity, provided by Southern Vietnam Geological Mapping Division.

Use the Surfer to interpolate data and Matlab to calculate data.

Data are interpolated by Surfer with size-grid 0.9’ × 0.9’, i.e. 1.6 km × 1.6 km. The size-grid is (0.9’ × 0.9’) to retain the real data at the sea in the interpolation data (fig. 2).

Determining the local disturbed potential from gravity anomaly \(\Delta g\) according to the Neyman problem

Applying formula (7b) to calculate the local disturbed potential \(T’\) from the gravity anomaly \(\Delta g\) at 9409 points distributed on the grid in the study area. We establish the map of the contour lines of the local disturbed potential \(T’\) (fig. 3).

Fig. 2. Contour lines of gravity anomaly, interpolated with size-grid 0.9’ × 0.9’ (contour lines are separated with 4 mGal)

Fig. 3. Contour lines of local disturbed potential \(T’\) with 9409 data (contour lines are separated with 2 m\(^2\)s\(^{-2}\))
The coastal area measured approximately flat. Here local geoid height is determined by the Garmin Montana 650 GPS meter - on September 25, 2015 - (at the coast, we have elevation terrain H = 0 so the geodetic height is measured by GPS: h = ζ - local geoid height). Measurement is operated on 2000 m long straight, linear northsouth, line along the coastline in relatively flat terrain, interval between points is 100 m. Measurement is conducted at medium tide (water level between the highest and lowest tide from the coast) (table 1).

Table 1. Data of geoid height ζ' và disturbed potential T' at 20 points

<table>
<thead>
<tr>
<th>Longitude (°)</th>
<th>Latitude (°)</th>
<th>ζ' (m)</th>
<th>T' (m²s⁻²)</th>
<th>Longitude (°)</th>
<th>Latitude (°)</th>
<th>ζ' (m)</th>
<th>T' (m²s⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>106.6902</td>
<td>10.02133</td>
<td>3.5</td>
<td>-2.7443</td>
<td>106.6869</td>
<td>10.01225</td>
<td>2.0</td>
<td>-2.7411</td>
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<td>-2.7415</td>
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<td>1.0</td>
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<td>106.6849</td>
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<td>1.0</td>
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</tr>
<tr>
<td>106.689</td>
<td>10.01685</td>
<td>3.0</td>
<td>-2.7389</td>
<td>106.6843</td>
<td>10.00795</td>
<td>1.0</td>
<td>-2.7473</td>
</tr>
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<td>106.6887</td>
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<td>-2.7393</td>
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<td>106.6822</td>
<td>10.00495</td>
<td>2.5</td>
<td>-2.7535</td>
</tr>
</tbody>
</table>

Applying (10) with disturbed potential T', ellipsoid constant U_0, normal gravity γ and geoid height ζ' at 20 points, we have 20 values of local geoid constants W'_o (table 2).

Table 2. Values of local geoid constants W'_o

<table>
<thead>
<tr>
<th>Longitude (°)</th>
<th>Latitude (°)</th>
<th>W'_o (m²s⁻²)</th>
<th>Longitude (°)</th>
<th>Latitude (°)</th>
<th>W'_o (m²s⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>106.6902</td>
<td>10.02133</td>
<td>62636815</td>
<td>106.6869</td>
<td>10.01225</td>
<td>62636829</td>
</tr>
<tr>
<td>106.69</td>
<td>10.0204</td>
<td>62636829</td>
<td>106.6864</td>
<td>10.01133</td>
<td>62636839</td>
</tr>
<tr>
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<td>62636839</td>
</tr>
<tr>
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<td>10.00958</td>
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</tr>
<tr>
<td>106.6893</td>
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<td>62636825</td>
<td>106.6849</td>
<td>10.0088</td>
<td>62636839</td>
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<tr>
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<td>106.6887</td>
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<td>106.6874</td>
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<td>62636825</td>
<td>106.6822</td>
<td>10.00495</td>
<td>62636825</td>
</tr>
</tbody>
</table>

To average W'_o we have result:

W'_o ± ΔW'_o = 62636830 ± 7.8 \( m²s⁻² \)

With ΔW'_o is the accuracy of W'_o.

Determining the systematic deviation Δζ between two geoids

Both global geoid and local geoid are randomly variable, very complex in terms of latitude and longitude. If we want to investigate the systematic deviation between the two geoid surfaces, we must express two approximately optimal spheroids of geoids with the reference ellipsoid on one diagram,
Determination of the constant \( W_o \) for local geoid surfaces.

Choosing ellipsoid WGS-84 that has an ellipsoid constant \( U_o \) as a reference face for calculating the systematic deviation of two approximately optimal spheroids of two geoids (global geoid and local geoid). From (12) and (13) we have two formulas for the two systematic deviations between approximately optimal spheroids of geoids with the reference ellipsoid surface:

\[
\zeta_o = \frac{U_o - W_o}{\gamma} \quad \text{và} \quad \zeta'_o = \frac{U_o - W'_o}{\gamma}
\]

In which: \( \gamma(\phi) \) is selected as \( \gamma = 9.7827 \text{ ms}^{-2} \) at latitude \( \phi = 12.5^\circ \) (latitude \( \phi \) varies between 8.16\(^\circ\)→16\(^\circ\) latitude North, corresponding to the latitude of the South pole and Central Vietnam). We have \( W'_o = 62636830 \text{ m}^2\text{s}^{-2} \) (local geoid constant); \( W_o = 62636856.88 \text{ m}^2\text{s}^{-2} \) (global geoid constant - EGM96); \( U_o = 62636851.71 \text{ m}^2\text{s}^{-2} \) (ellipsoid WGS-84), instead of the above formulas, we have:

\[
\zeta_o = -0.5 \text{ m} \quad \text{và} \quad \zeta'_o = 2.2 \text{ m}
\]

Thus, the approximately optimal spheroid of local geoid is shifted upward relative to the ellipsoid WGS-84 about 2.2 m. Also, the approximately optimal spheroid of global geoid is shifted downward relative to the ellipsoid WGS-84 about 0.5 m. So, it is synonymous with the displacement of the two corresponding geoids, because the geoid bonds to the approximately optimal spheroid. The systematic deviation varies slowly in terms of \( \gamma \) (latitude \( \phi \)). We find that \( \zeta_o \) and \( \zeta'_o \) change very slowly in the study area.

Using the local value \( W'_o \) and the global value \( W_o \) (EGM-96) to (15), giving the systematic deviation between the two geoid surfaces.

\[
\Delta \zeta_o = \zeta'_o - \zeta_o = \frac{W_o - W'_o}{\gamma}
\]

The latitude \( \phi \) in the formula \( \gamma \) (15) receives 8.16\(^\circ\)→16\(^\circ\) latitude North (corresponding to the latitude of the South pole and Central Vietnam), with step \( \Delta \phi = 0.5^\circ \) we find that the systematic deviation \( \Delta \zeta_o \) varies slowly in terms of latitude \( \phi \) (table 3).

### Table 3. Values of \( \Delta \zeta_o \) varies slowly in terms of latitude \( \phi \)

<table>
<thead>
<tr>
<th>Latitude (°)</th>
<th>8.6</th>
<th>9</th>
<th>9.5</th>
<th>10</th>
<th>10.5</th>
<th>11</th>
<th>11.5</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \zeta_o ) (m)</td>
<td>2.748058</td>
<td>2.748019</td>
<td>2.747979</td>
<td>2.747937</td>
<td>2.747893</td>
<td>2.747846</td>
<td>2.747798</td>
<td>2.747747</td>
</tr>
<tr>
<td>Latitude (°)</td>
<td>12.5</td>
<td>13</td>
<td>13.5</td>
<td>14</td>
<td>14.5</td>
<td>15</td>
<td>15.5</td>
<td>16</td>
</tr>
<tr>
<td>( \Delta \zeta_o ) (m)</td>
<td>2.747695</td>
<td>2.74764</td>
<td>2.747584</td>
<td>2.747525</td>
<td>2.747465</td>
<td>2.747403</td>
<td>2.747338</td>
<td>2.747272</td>
</tr>
</tbody>
</table>

Because \( \Delta \zeta_o \) varies slowly in terms of latitude \( \phi \), we can select \( \Delta \zeta_o = 2.74 \text{ m} \) as specific value of study area (fig. 4).

### CONCLUSION

The local geoid constant \( W'_o \) for Vietnam is first determined by applying the Bruns formula and Neyman boundary problem for the local area with GPS measurement at the Vietnamese coast.

Calculating the constant \( W'_o \) for the local geoid of the Vietnamese state is important for geodetic physics such as:
Determining the systematic deviation between the local geoid surface of Vietnam and the global geoid surface. This quantity varies very slowly, gradually increasing to the equator, valued at over 2.74 m in the study area.

The relative position of two approximately optimal spheroids of global geoid and local geoid is compared to the reference ellipsoid WGS-84.

Open up the possibility to investigate systematic deviation between the local geoid in Vietnam and the global geoid nationally, from Hon Dau to Ca Mau.

Open up the possibility to establish exactly local geoid of Vietnam to interrelate any reference ellipsoid, which has real geoid waves.

REFERENCES