NEW DISSIMILARITY MEASURES ON PICTURE FUZZY SETS AND APPLICATIONS

LE THI NHUNG*, NGUYEN VAN DINH, NGOC MINH CHAU, NGUYEN XUAN THAO

Faculty of Information Technology, Vietnam National University of Agriculture

Abstract. The dissimilarity measures between fuzzy sets/intuitionistic fuzzy sets/picture fuzzy sets are studied and applied in various matters. In this paper, we propose some new dissimilarity measures on picture fuzzy sets. These new dissimilarity measures overcome the restrictions of all existing dissimilarity measures on picture fuzzy sets. After that, we apply these new measures to the pattern recognition problems. Finally, we introduce a multi-criteria decision making (MCDM) method that uses the new dissimilarity measures and apply them in the supplier selection problems.

Keywords. Picture fuzzy set; Dissimilarity measure; MCDM.

1. INTRODUCTION

The ranking of subjects is very important in the decision-making process. The ranking can be based on measures such as the similarity measures, the distance measures or dissimilarity measures. In practical problems, fuzzy set and intuitionistic fuzzy set have been widely used [3, 9, 12, 18, 19, 21, 22]. The dissimilarity measures between them were also studied and applied in various matters [10, 14, 16, 17, 20, 23].

In 2014, Picture fuzzy set was introduced by Cuong [4]. It has three memberships: a degree of positive membership, a degree of negative membership, and a degree of neutral membership. Picture fuzzy set is a generality of fuzzy set [42] and intuitionistic fuzzy set [1]. Today, picture fuzzy set has been studied and applied widely in many fields [2, 6, 8, 11, 24, 25, 26, 37], especially in clustering problems [13, 15, 27, 28, 29, 32, 33, 31, 36]. Hoa et al. [13] used picture fuzzy sets to apply for Geographic Data Clustering. Thao and Dinh approximated the picture fuzzy set on the crisp approximation spaces to give results as rough picture fuzzy sets and picture fuzzy topologies [30]. Dinh et al. investigated the picture fuzzy set database [35]. Cuong and Hai [5] studied some fuzzy logic operators for picture fuzzy sets. The cross-entropy and similarity measures on picture fuzzy sets were studied by Wei and applied in MCDM [38, 41, 39, 40]. As opposed to the similarity measures, the dissimilarity measures on picture fuzzy sets were first introduced by Dinh et al. in 2017 [7, 34]. But these

*This paper is selected from the reports presented at the 11th National Conference on Fundamental and Applied Information Technology Research (FAIR’11), Thang Long University, 09 - 10/08/2018.

© 2018 Vietnam Academy of Science & Technology
dissimilarity measures have certain restrictions (detail in Example 1, Section 3). To continue
with the idea of the dissimilarity measures on picture fuzzy sets in practical applications, we
propose some new dissimilarity measures to overcome the mentioned restrictions and apply
them in practical problems (detail in Example 1 and Example 2, Section 3). In the similarity
measure, if the value of the similarity measure between two objects is greater, the two objects
are more likely to be identical. On the contrary, in the dissimilarity measure, if the value of
the dissimilarity measure between two objects is smaller, the two objects are considered to
be the same.

In this paper, we introduce some new dissimilarity measures on picture fuzzy sets. The
paper is organized as follows: the concept of picture fuzzy set is recalled in Section 2.
The dissimilarity measures on PFS-sets are defined in Section 3. After that, we introduce
an application of the dissimilarity measures between PFS-sets for the pattern recognition in
Section 4. We also propose a multi-criteria decision making using new dissimilarity measures
and apply this MCDM to select the supplier in Section 5.

2. BASIC NOTIONS

**Definition 1.** (see [4]) Picture fuzzy set on a universe \( U \) is an object of the form \( A = \{(u, \mu_A(u), \eta_A(u), \gamma_A(u)) | u \in U \} \), where \( \mu_A \) is a membership function, \( \eta_A \) is neutral mem-
bership function, \( \gamma_A \) is non-membership function of \( A \) and \( 0 \leq \mu_A(u) + \eta_A(u) + \gamma_A(u) \leq 1 \)
for all \( u \in U \).

Further, we denote by \( PFS(U) \) the collection of picture fuzzy sets on \( U \) with \( U = \{(u, 1, 0, 0) | u \in U \} \) and \( \emptyset = \{(u, 0, 0, 1) | u \in U \} \) for all \( u \in U \).

For \( A, B \in PFS(U) \) and for all \( u \in U \) consider some algebraic operators for picture fuzzy
sets as follows:

- Union of \( A \) and \( B \): \( A \cup B = \{(u, \mu_{A \cup B}(u), \eta_{A \cup B}(u), \gamma_{A \cup B}(u)) | u \in U \} \), where

\[
\mu_{A \cup B}(u) = \max\{\mu_A(u), \mu_B(u)\},
\eta_{A \cup B}(u) = \min\{\eta_A(u), \eta_B(u)\} \quad \text{and}
\gamma_{A \cup B}(u) = \min\{\gamma_A(u), \gamma_B(u)\}.
\]

- Intersection of \( A \) and \( B \): \( A \cap B = \{(u, \mu_{A \cap B}(u), \eta_{A \cap B}(u), \gamma_{A \cap B}(u)) | u \in U \} \), where

\[
\mu_{A \cap B}(u) = \min\{\mu_A(u), \mu_B(u)\},
\eta_{A \cap B}(u) = \min\{\eta_A(u), \eta_B(u)\}, \quad \text{and}
\gamma_{A \cap B}(u) = \max\{\gamma_A(u), \gamma_B(u)\}.
\]

- Subset: \( A \subset B \) iff \( \mu_A(u) \leq \mu_B(u), \eta_A(u) \leq \eta_B(u) \) and \( \gamma_A(u) \geq \gamma_B(u) \).

3. NEW DISSIMILARITY MEASURES ON PICTURE FUZZY SETS

In this section, we introduce concept of dissimilarity measure on picture fuzzy sets.

**Definition 2.** A function \( DM : PFS(U) \times PFS(U) \to R \) is a dissimilarity measure on
PFS-sets if it satisfies the following properties:
Assume that there are two patterns denoted by picture fuzzy sets on $U$. We define some dissimilarity measures on picture fuzzy sets as follows:

$$DM_C(A, B) = \frac{1}{3n} \sum_{i=1}^{n} \left[ |S_A(u_i) - S_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| \right]$$

where $S_A(u_i) = |\mu_A(u_i) - \gamma_A(u_i)|$ and $S_B(u_i) = |\mu_B(u_i) - \gamma_B(u_i)|$.

$$DM_H(A, B) = \frac{1}{3n} \sum_{i=1}^{n} \left[ |\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)| \right].$$

$$DM_L(A, B) = \frac{1}{5n} \sum_{i=1}^{n} \left[ |S_A(u_i) - S_B(u_i)| + |\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)| \right].$$

$$DM_O(A, B) = \frac{1}{\sqrt{5n}} \sum_{i=1}^{n} \left[ |\mu_A(u_i) - \mu_B(u_i)|^2 + |\eta_A(u_i) - \eta_B(u_i)|^2 + |\gamma_A(u_i) - \gamma_B(u_i)|^2 \right]^{\frac{1}{2}}.$$
This drawback suggests us to improve the dissimilarity measure on picture fuzzy sets. Suppose \( U = \{u_1, u_2, \ldots, u_n\} \) is an universe set. For any \( A, B \in PFS(U) \), we denote
\[
R_A(u_j) = \mu_A(u_j) - \gamma_A(u_j), R_B(u_j) = \mu_B(u_j) - \gamma_B(u_j),
\]
\[
S_A(u_j) = \eta_A(u_j) - \gamma_A(u_j), S_B(u_j) = \eta_B(u_j) - \gamma_B(u_j),
\]
and
\[
D_j(A, B) = \frac{|R_A(u_j) - R_B(u_j)| + |S_A(u_j) - S_B(u_j)|}{4}
\]
for all \( j = 1, 2, \ldots, n \).

**Definition 4.** Let \( U = \{u_1, u_2, \ldots, u_n\} \) be an universal set. For any \( A, B \in PFS(U) \) the dissimilarity measure \( DM_N : PFS(U) \times PFS(U) \to [0, 1] \) is defined by
\[
DM_N(A, B) = \frac{1}{n} \sum_{j=1}^{n} D_j(A, B).
\]

**Theorem 1.** Let \( U = \{u_1, u_2, \ldots, u_n\} \) be a universal set. For any \( A, B \in PFS(U) \), a function \( DM_N : PFS(U) \times PFS(U) \to \mathbb{R} \) defined by \( DM_N(A, B) = \frac{1}{n} \sum_{j=1}^{n} D_j(A, B) \) satisfies

(i) \( 0 \leq DM_N(A, B) \leq 1 \);
(ii) \( DM_N(A, B) = DM_N(B, A) \);
(iii) \( DM_N(A, A) = 0 \);
(iv) If \( A \subset B \subset C \) then \( DM_N(A, C) \geq \max\{DM_N(A, B), DM_N(B, C)\} \) for all \( A, B, C \in PFS(U) \).

**Proof.**

(i) We have \( 0 \leq R_A(u_j), R_B(u_j), S_A(u_j), S_B(u_j) \leq 1 \). Hence, \( 0 \leq D_j(A, B) \leq 1 \). Therefore, from eq.(6) we have \( 0 \leq DM_N(A, B) \leq 1 \).

(ii) It is obvious.

(iii) It is obvious.

(iv) If \( A \subset B \subset C \) then \( \mu_A(u_j) \leq \mu_B(u_j) \leq \mu_C(u_j) \), \( \eta_A(u_j) \leq \eta_B(u_j) \leq \eta_C(u_j) \) and \( \gamma_A(u_j) \geq \gamma_B(u_j) \geq \gamma_C(u_j) \) for all \( u_j \in U \).

So that, \( R_A(u_j) \leq R_B(u_j) \leq R_C(u_j) \) and \( S_A(u_j) \leq S_B(u_j) \leq S_C(u_j) \).

Hence, \[ |R_C(u_j) - R_A(u_j)| \geq \max\{|R_C(u_j) - R_B(u_j)|, |R_B(u_j) - R_A(u_j)|\} \text{ and } |S_C(u_j) - S_A(u_j)| \geq \max\{|S_C(u_j) - S_B(u_j)|, |S_B(u_j) - S_A(u_j)|\}. \]

Hence, \( DM_N(A, C) \geq \max\{DM_N(A, B), DM_N(B, C)\} \). It means PF-Diss 4 is satisfied.

Now, we assign to \( u_j \) a weight \( \omega_j \in [0, 1] \) such that \( \sum_{j=1}^{n} \omega_j = 1 \). We can define a new dissimilarity measure between two picture fuzzy sets as follows.

**Definition 5.** Let \( U = \{u_1, u_2, \ldots, u_n\} \) be a universal set. For any \( A, B \in PFS(U) \), a dissimilarity measure \( DM_N^\omega : PFS(U) \times PFS(U) \to [0, 1] \) is defined by
\[
DM_N^\omega(A, B) = \sum_{j=1}^{n} \omega_j D_j(A, B).
\]
Definition 6. Let \( U = \{u_1, u_2, ..., u_n\} \) be a universal set. For any \( A, B \in PFS(U) \), a dissimilarity measure \( DM_\omega^\omega : PFS(U) \times PFS(U) \rightarrow [0, 1] \) is defined by
\[
DM_\omega^\omega(A, B) = \sum_{j=1}^{n} \omega_j D_j^\omega(A, B)
\]
(8)
where
\[
D_j^\omega(A, B) = \frac{\left| |R_A(u_j) - R_B(u_j)|^p + |S_A(u_j) - S_B(u_j)|^p \right|}{4}
\]
(9)
for all \( j = 1, 2, ..., n; \ p \in N^* \).

Theorem 2. Let \( U = \{u_1, u_2, ..., u_n\} \) be a universe set. Then for any \( A, B \in PFS(U) \)
\[
DM_\omega^N(A, B) = \sum_{j=1}^{n} \omega_j D_j^N(A, B)
\]
and
\[
DM_\omega^P(A, B) = \sum_{j=1}^{n} \omega_j D_j^P(A, B)
\]
are the dissimilarity measures on picture fuzzy sets.

Proof. It is easy.

Example 2. We consider the problem in Example 1. In that example, we cannot determine whether sample \( B \) belongs to the class of pattern \( A_1 \) or \( A_2 \) if we use the dissimilarity measures in expressions eq.(1), eq.(2), eq.(3) and eq.(4). Now, we consider this problem with the new dissimilarity measures in eq.(6) and eq.(8) with \( \omega_1 = \omega_2 = 0.5 \) and \( p = 2 \).

+ Using the dissimilarity measure in eq.(6), we have
\[
DM_N(A_1, B) = 0.05 \quad \text{and} \quad DM_N(A_2, B) = 0.0375.
\]
+ Using the dissimilarity measure in eq.(8), we have
\[
DM_\omega^P(A_1, B) = 0.04045 \quad \text{and} \quad DM_\omega^P(A_2, B) = 0.03018.
\]
We can easily see that using two new measures we can conclude that the sample \( B \) belongs to the class of pattern \( A_2 \).

4. APPLYING THE PROPOSED DISSIMILARITY MEASURE IN PATTERN RECOGNITION

In this section, we will give some examples using dissimilarity measures in the pattern recognition. Given for \( m \) patterns \( A_1, A_2, ..., A_m \) are picture fuzzy sets in the universal set \( U = \{u_1, u_2, ..., u_n\} \). If we have a sample \( B \) is also a picture fuzzy set on \( U \).

Question: Which class of pattern does \( B \) belong to?

To answer this question, we practice the following steps:

Step 1. Compute the dissimilarity measures \( DM(A_i, B) \) of \( A_i (i = 1, 2, ..., m) \) and \( B \).
Step 2. We put $B$ to the class of pattern $A^*$, in which
\[ DM(A^*, B) = \min\{DM(A_i, B)\}_{i = 1, 2, ..., m}. \]

Example 3. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows
\[ A_1 = \{(u_1, 0.1, 0.1, 0.1), (u_2, 0.1, 0.4, 0.3), (u_3, 0.1, 0.9)\}, \]
\[ A_2 = \{(u_1, 0.7, 0.1, 0.2), (u_2, 0.1, 0.1, 0.8), (u_3, 0.1, 0.1, 0.7)\}. \]
Now, there is a sample $B = \{(u_1, 0.4, 0.4), (u_2, 0.6, 0.1, 0.2), (u_3, 0.1, 0.1, 0.8)\}$.

Question: Which class of pattern does $B$ belong to?
To answer this question, we consider the dissimilarity measures shown in eq.(6), eq.(8) with the weight vector $\omega = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

+ Applying the dissimilarity measure in eq.(6), we have
\[ DM_N(A_1, B) = 0.1417, \quad DM_N(A_2, B) = 0.1667. \]
It means that $B$ belongs to the class of pattern $A_1$.

+ Applying the dissimilarity measure in eq.(8) with $p = 2$, we have
\[ DM^p_N(A_1, B) = 0.0982, \quad DM^p_N(A_2, B) = 0.1741. \]
It means that $B$ belongs to the class of pattern $A_1$.

+ Applying the dissimilarity measure in eq.(8) with $p = 3$, we have
\[ DM^p_N(A_1, B) = 0.0935, \quad DM^p_N(A_2, B) = 0.161. \]
It means that $B$ belongs to the class of pattern $A_1$.

Example 4. Assume that there are three patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows
\[ A_1 = \{(u_1, 0.5, 0.4, 0.4), (u_2, 0.5, 0.2, 0.25), (u_3, 0.1, 0.9), (u_4, 0.1, 0.1, 0.65)\}, \]
\[ A_2 = \{(u_1, 0.7, 0.1, 0.2), (u_2, 0.1, 0.1, 0.8), (u_3, 0.1, 0.1, 0.7), (u_4, 0.4, 0.1, 0.5)\}, \]
\[ A_3 = \{(u_1, 0.6, 0.1, 0.2), (u_2, 0.6, 0.2, 0.15), (u_3, 0.1, 0.1, 0.9), (u_4, 0.15, 0.2, 0.6)\}. \]
Now, there is a sample
\[ B = \{(u_1, 0.5, 0.1, 0.4), (u_2, 0.6, 0.15, 0.2), (u_3, 0.1, 0.8), (u_4, 0.1, 0.2, 0.6)\}. \]

Question: Which class of pattern does $B$ belong to?
Using the weight vector $\omega = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and eq.(6), eq.(8), then:

+ Applying the dissimilarity measure in eq.(6), we have
\[ DM_N(A_1, B) = 0.0375, \quad DM_N(A_2, B) = 0.15, \quad DM_N(A_3, B) = 0.0594. \]
It means that $B$ belongs to the class of pattern $A_1$. 
Example 5. Assume that there are three patterns denoted by picture fuzzy sets on measures to solve a MCDM problem.

Question: Which class of pattern does \( \omega \) and eq.(8) with the weight vector \( \mathbf{w} \)?

Now, there is a sample

\[
 u = \{1, u_2, u_3, u_4\}
\]

Applying the dissimilarity measure in eq.(8) with \( p = 2 \), we have

\[
 DM_{P}^p(A_1, B) = 0.06, \\
 DM_{P}^p(A_2, B) = 0.303, \\
 DM_{P}^p(A_3, B) = 0.099.
\]

It means that \( B \) belongs to the class of pattern \( A_1 \).

Applying the dissimilarity measure in eq.(8) with \( p = 3 \), we have

\[
 DM_{P}^p(A_1, B) = 0.073, \\
 DM_{P}^p(A_2, B) = 0.3598, \\
 DM_{P}^p(A_3, B) = 0.1154.
\]

It means that \( B \) belongs to the class of pattern \( A_1 \).

Example 5. Assume that there are three patterns denoted by picture fuzzy sets on \( U = \{u_1, u_2, u_3, u_4\} \) as follows

\[
 A_1 = \{(u_1, 0.3, 0.4, 0.1), (u_2, 0.3, 0.4, 0.1), (u_3, 0.6, 0.1, 0.2), (u_4, 0.6, 0.1, 0.2)\}, \\
 A_2 = \{(u_1, 0.4, 0.4, 0.1), (u_2, 0.3, 0.2, 0.4), (u_3, 0.6, 0.1, 0.3), (u_4, 0.5, 0.2, 0.2)\}, \\
 A_3 = \{(u_1, 0.4, 0.4, 0.1), (u_2, 0.3, 0.1, 0.3), (u_3, 0.6, 0.1, 0.2), (u_4, 0.5, 0.2, 0.1)\}.
\]

Now, there is a sample

\[
 B = \{(u_1, 0.35, 0.65, 0), (u_2, 0.55, 0.35, 0.1), (u_3, 0.65, 0.1, 0.1), (u_4, 0.6, 0.15, 0.2)\}.
\]

Question: Which class of pattern does \( B \) belong to?

To answer this question, we consider the dissimilarity measures shown in eq.(6), eq.(7), and eq.(8) with the weight vector \( \omega = (0.4, 0.3, 0.2, 0.1) \)

Applying the dissimilarity measure in eq.(6), we have

\[
 DM_{N}(A_1, B) = 0.06875, \\
 DM_{N}(A_2, B) = 0.125, \\
 DM_{N}(A_3, B) = 0.10625.
\]

\[
 \Rightarrow DM_{N}(A_1, B) < DM_{N}(A_3, B) < DM_{N}(A_2, B).
\]

It means that \( B \) belongs to the class of pattern \( A_1 \).

Applying the dissimilarity measure in eq.(7), we have

\[
 DM_{P}^{\omega}(A_1, B) = 0.08625, \\
 DM_{P}^{\omega}(A_2, B) = 0.14125, \\
 DM_{P}^{\omega}(A_3, B) = 0.12375.
\]

\[
 \Rightarrow DM_{P}^{\omega}(A_1, B) < DM_{P}^{\omega}(A_3, B) < DM_{P}^{\omega}(A_2, B).
\]

It means that \( B \) belongs to the class of pattern \( A_1 \).

Applying the dissimilarity measure in eq.(8) with \( p = 2 \), we have

\[
 DM_{P}^{p}(A_1, B) = 0.06746, \\
 DM_{P}^{p}(A_2, B) = 0.10744, \\
 DM_{P}^{p}(A_3, B) = 0.09585.
\]

\[
 \Rightarrow DM_{P}^{p}(A_1, B) < DM_{P}^{p}(A_3, B) < DM_{P}^{p}(A_2, B).
\]

It means that \( B \) belongs to the class of pattern \( A_1 \).

5. APPLICATION IN MULTI-CRITERIA DECISION MAKING

In the MCDM problem, one has to find an optimal alternative from a set of alternatives \( A = \{A_1, A_2, \ldots, A_m\} \). In this section, we introduce a method based on the new dissimilarity measures to solve a MCDM problem.
**Step 1.** Determine the criteria set $C = \{C_1, C_2, \ldots, C_n\}$ for the MCDM.

**Step 2.** Express each alternative $A_i$ as a picture fuzzy set on the set $C = \{C_1, C_2, \ldots, C_n\}$,

$$A_i = \{(C_j, \mu_{ij}, \eta_{ij}, \gamma_{ij})|C_j \in C\}$$

for all $i = 1, 2, \ldots, m$.

**Step 3.** We choose the best alternative $A_b$ to be also a picture fuzzy set on the set $C = \{C_1, C_2, \ldots, C_n\}$.

**Step 4.** Determine the weight $\omega_j$ of criteria $C_j$ by considering $C_j = \{(A_i, \mu_{ij}, \eta_{ij}, \gamma_{ij})|A_i \in A\}$ as a picture fuzzy set on $A = \{A_1, A_2, \ldots, A_m\}$.

Based on the union of picture fuzzy sets we propose a method to determine the weight $\omega_j$ of criteria $C_j (j = 1, 2, \ldots, n)$ as follows:

- We calculate

$$d_j = d_{1j} + d_{2j} + d_{3j} \quad (10)$$

where $d_{1j} = \max_{1 \leq i \leq m} \mu_{ij}$, $d_{2j} = \min_{1 \leq i \leq m} \eta_{ij}$, and $d_{3j} = \min_{1 \leq i \leq m} \gamma_{ij}$ for all $j = 1, 2, \ldots, n$.

Then, $A^* = \{(C_j, d_{1j}, d_{2j}, d_{3j})|C_j \in C| = \bigcup_{i=1}^{m} A_i$ and $d_j$ in the eq.(10) is referred to frequency of $C_j (j = 1, 2, \ldots, n)$ in $A^*$.

So that, we can determine the weight $\omega_j$ of criteria $C_j (j = 1, 2, \ldots, n)$ based on frequency $d_j (j = 1, 2, \ldots, n)$.

- Put

$$\omega_j^{(k)} = \frac{d_j^{(k)}}{\sum_{j=1}^{n} d_j^{(k)}} \quad (11)$$

for all $j = 1, 2, \ldots, n$; $k = 0, 1, 2, \ldots$

Note that, when $k = 0$ then we have the weight $\omega_j = \frac{1}{n}$ for all $j = 1, 2, \ldots, n$.

**Step 5.** Compute the dissimilarity measures $DM(A_i, A_b)$ between $A_i (i = 1, 2, \ldots, m)$ and $A_b$.

**Step 6.** Rank the alternatives based on the dissimilarity measures as follows

$$A_i \prec A_k \text{ if } DM(A_i, A_b) < DM(A_k, A_b) (i, k = 1, 2, \ldots, m).$$

**Example 6.** Consider a supplier section problem. Suppose a construction company wants to procure the material for their upcoming project. The company invites the tenders for procuring the required material. Given five suppliers are $\{A_1, A_2, A_3, A_4, A_5\}$. To find an optimal supply, we apply the six steps for solving this MCDM problem as follows:

**Step 1.** The company has fixed criteria for supplier selection: $C_1$: quality of material; $C_2$: price; $C_3$: services; $C_4$: delivery; $C_5$: technical support if required; $C_6$: behavior.
Step 2. Alternatives $A_i$ is expressed as a picture fuzzy set on a criteria set $\{C_1, C_2, \ldots, C_6\}$ in Table 1 and Table 2.

Step 3. The best alternative $A_b$ is

$$A_b = \{(C_j, 1, 0, 0) | j = 1, 2, 3, 4, 5, 6\}.$$

Step 4. Using the eq.(1), we get $d_1 = 0.85$, $d_2 = 1$, $d_3 = 0.9$, $d_4 = 1$, $d_5 = 0.95$, $d_6 = 0.9$.

To calculate the weight $\omega_j$ of criteria $C_j(j = 1, 2, \ldots, 6)$ we use the eq.(11):

- $k = 0$ we have the weight vector $\omega_0 = (1/6, 1/6, 1/6, 1/6, 1/6, 1/6)$.
- $k = 1$ we have the weight vector $\omega_1 = (0.145, 0.171, 0.171, 0.171, 0.171, 0.171)$.
- $k = 2$ we have the weight vector $\omega_2 = (0.125, 0.175, 0.175, 0.175, 0.175, 0.175)$.

Table 1. The picture fuzzy decision matrix for the supplier selection

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.4, 0.05, 0.5)</td>
<td>(0.1, 0.1, 0.8)</td>
<td>(0.7, 0.0, 0.3)</td>
<td>(0.6, 0.1, 0.2)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.7, 0.05, 0.2)</td>
<td>(0.5, 0.1, 0.3)</td>
<td>(0.3, 0.3, 0.4)</td>
<td>(0.8, 0.05, 0.1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.6, 0.2, 0.1)</td>
<td>(0.7, 0.0, 0.3)</td>
<td>(0.6, 0.1, 0.2)</td>
<td>(0.4, 0.3, 0.1)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.5, 0.05, 0.4)</td>
<td>(0.4, 0.2, 0.3)</td>
<td>(0.8, 0.1, 0.1)</td>
<td>(0.7, 0.05, 0.2)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.4, 0.3, 0.3)</td>
<td>(0.1, 0.15, 0.7)</td>
<td>(0.5, 0.25, 0.2)</td>
<td>(0.9, 0.0, 1)</td>
</tr>
</tbody>
</table>

Table 2. The picture fuzzy decision matrix for the supplier selection (cont.)

<table>
<thead>
<tr>
<th></th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.5, 0.1, 0.4)</td>
<td>(0.3, 0.2, 0.4)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.2, 0.1, 0.6)</td>
<td>(0.4, 0.0, 0.5)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.3, 0.2, 0.4)</td>
<td>(0.8, 0.0, 0.2)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.6, 0.25, 0.1)</td>
<td>(0.7, 0.2, 0.1)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.8, 0.05, 0.1)</td>
<td>(0.6, 0.0, 0.4)</td>
</tr>
</tbody>
</table>

Step 5. Compute the dissimilarity measures $DM(A_i, A_b)$ between $A_i (i = 1, 2, \ldots, m)$ and $A_b$ using the eq.(8) with $p = 1$ and $p = 2$.

Step 6. Rank the alternatives based on the dissimilarity measure.

The results of Step 5 and Step 6 with the various weight vectors are shown in Table 3, 4, 5.

- With the weight vector $\omega_0 = (1/6, 1/6, 1/6, 1/6, 1/6, 1/6)$, we have the dissimilarity measure and ranking of alternatives as in Table 3.

- With the weight vector $\omega_1 = (0.145, 0.171, 0.171, 0.171, 0.171, 0.171)$, we have the dissimilarity measure and ranking of alternatives as in Table 4.
Table 3. The dissimilarity measure and ranking of alternatives with the weight vector \( \omega_0 \)

<table>
<thead>
<tr>
<th></th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( DM(A_i, A_b) )</td>
<td>0.2688</td>
<td>0.2229</td>
<td>0.1833</td>
<td>0.1813</td>
</tr>
<tr>
<td>Rank</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( DM(A_i, A_b) )</td>
<td>0.2651</td>
<td>0.2281</td>
<td>0.1776</td>
<td>0.1693</td>
</tr>
<tr>
<td>Rank</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4. The dissimilarity measure and ranking of alternatives with the weight vector \( \omega_1 \)

<table>
<thead>
<tr>
<th></th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( DM(A_i, A_b) )</td>
<td>0.2657</td>
<td>0.2245</td>
<td>0.1842</td>
<td>0.1778</td>
</tr>
<tr>
<td>Rank</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( DM(A_i, A_b) )</td>
<td>0.2919</td>
<td>0.2548</td>
<td>0.1957</td>
<td>0.1829</td>
</tr>
<tr>
<td>Rank</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

- With the weight vector \( \omega_2 = (0.125, 0.175, 0.175, 0.175, 0.175, 0.175) \), we have the dissimilarity measure and ranking of alternatives as in Table 5.

Table 5. The dissimilarity measure and ranking of alternatives with the weight vector \( \omega_2 \)

<table>
<thead>
<tr>
<th></th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( DM(A_i, A_b) )</td>
<td>0.2632</td>
<td>0.2263</td>
<td>0.1852</td>
<td>0.175</td>
</tr>
<tr>
<td>Rank</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( DM(A_i, A_b) )</td>
<td>0.2902</td>
<td>0.257</td>
<td>0.197</td>
<td>0.1802</td>
</tr>
<tr>
<td>Rank</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

6. CONCLUSION

In this paper, we introduce some new dissimilarity measures on picture fuzzy sets. These new measures overcome the limitations of the previous dissimilarity measures on picture fuzzy sets in [7, 34]. After that, we apply the proposed dissimilarity measures in the pattern recognition. We also use these new dissimilarity measures for a MCDM problem to select an optimal supplier. In the future, we also continue to study about the dissimilarity measures on picture fuzzy sets and the relationship of them and other measures on picture fuzzy sets. Beside, we also find new applications of them to deal with the real problems.

REFERENCES

NEW DISSIMILARITY MEASURES ON PICTURE FUZZY SETS


Received on October 24, 2018
Revised on November 01, 2018