SOME OPERATIONS ON TYPE-2 INTUITIONISTIC FUZZY SETS

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Abstract. In the last decades, there have been some extensions of fuzzy sets and their applications. Recently, type-2 fuzzy set and intuitionistic fuzzy set are two of them, drawing a great deal of scientist's attention because of their widespread range of applications. In this paper, we introduce a new concept - type-2 intuitionistic fuzzy set and propose some properties of their operations.

1. INTRODUCTION

Type-2 fuzzy sets - that is, fuzzy sets with fuzzy sets as truth values - seem destined to play an increasingly important role in applications. They were introduced by Zadeh [13], extending the notion of ordinary fuzzy sets. The Mendel’s book [5] on Uncertain Rule-based Fuzzy Logic Systems and other researches [6, 7, 8, 11] are discussions of both theoretical and practical aspects of type-2 fuzzy sets.

In [1] Atanassov K. introduced the concept of intuitionistic fuzzy set characterized by a membership function and a non-membership function, which is a generalization of fuzzy set. In [11] Atanassov K. also defined some operators of IFSs. Recently, intuitionistic fuzzy set (IFS) theory have been applied to many different fields, such as decision making, medical diagnosis, pattern recognition.

In this paper, we introduce concept of type-2 intuitionistic fuzzy sets. We define some basic operations and derive some their properties. The paper is organized as follow: Section 2 gives a briefly review some basic definitions of fuzzy sets, type-2 fuzzy sets and intuitionistic fuzzy sets. Section 3 is devoted to the new main definitions and some their properties. Section 4 is a first discussion of a subclass of the type-2 IFS.

2. BASIC DEFINITION

2.1. Definition of fuzzy sets

Let $T$, $S$ be nonempty sets. The $Map(S,T)$ be the set of all function from $S$ into $T$. A
fuzzy set $A$ of a set $S$ is a mapping $A : S \to [0, 1]$. The set $S$ has no operations on it. So
operations on the set $Map(S, [0, 1])$ of all fuzzy subsets of $S$ come from operations on $[0, 1]$.
Common operations on $[0, 1]$ of interest in fuzzy theory are $\lor, \land$, and $'$ given by
\[
x \land y = \min\{x, y\}, \quad x \lor y = \max\{x, y\}, \quad x' = 1 - x
\]
The constant 0 and 1 are generally considered as part of the algebraic structure. So the
algebra basic to fuzzy set theory is $[0, 1], \lor, \land, ', 0, 1$.
The corresponding operations on the set $Map(S, [0, 1])$ of all fuzzy subsets of $S$ are given
pointwise by the following formulas
\[
(A \land B)(s) = A(s) \land B(s), \quad (A \lor B)(s) = A(s) \lor B(s), \quad A'(s) = (A(s))'
\]
and the two nullary operations are given by $1(s) = 1$ and $0(s) = 0$ for all $s \in S$.
We use the same symbols for the pointwise operations on the elements of $Map(S, [0, 1])$. There
are many properties hold in the algebra $I = ([0, 1], \lor, \land, ', 0, 1)$ (see [9, 11]).
Thus, $I$ is a bounded distributive lattice with an involution $'$ that satisfies De Morgan’s
laws and the Kleene inequality. Thus is, $I$ is a Kleene algebra. Thus $Map(S, [0, 1])$ is also a
Kleene algebra. Basic knowledge of fuzzy sets has been presented in [3, 9].

2.2. Definition of type-2 fuzzy sets

Let $S$ be a universe of discourse, the a type-2 fuzzy set (T2 FS) is defined as following.

**Definition 2.2.1.** [9] A type-2 fuzzy set, denoted by $A$, is characterized by a type-2 mem-
bership function $\mu_A(x, u)$, where $x \in S$ and $u \in J_x \subseteq [0, 1]$, i.e. $A = \{(x, u), \mu_A(x, u)\}\forall x \in
X, \forall u \in J_x \subseteq [0, 1]\}$, in which $0 \leq \mu_A(x, u) \leq 1$. $A$ can be express as $A = \int \int \mu_A(x, u)/(x, u),
J_x \subseteq [0, 1]$

2.3. Basic definition and some properties of IFS

Let $Y$ be a universe of discourse, then a fuzzy set $A = \{< y, \mu_A(y) > | y \in Y \}$ defined by
Zadeh [13] is characterized by a membership function $\mu_A : Y \to [0, 1]$, where $\mu_A(y)$ denotes
the degree of membership of element $y$ to the set $A$.

**Definition 2.3.1.** [7] An intuitionistic fuzzy set (IFS) $A = \{< y, \mu_A(y), \nu_A(y) > | y \in Y \}$
is characterized by a membership function $\mu_A : Y \to [0, 1]$, and a non-membership function
$\nu_A : Y \to [0, 1]$ with the condition $0 \leq \mu_A(y) + \nu_A(y) \leq 1$ for all $y \in Y$, where the numbers
$\mu_A(y)$ and $\nu_A(y)$ represent the degree of membership and the degree of non-membership
of the element $y$ to the set $A$, respectively.

**Definition 2.3.2.** [7, 9] If $A$ and $B$ are two IFS of the set $Y$, then
$A \subseteq B$ iff $\forall y \in Y, \mu_A(y) \leq \mu_B(y)$ and $\nu_A \geq \nu_B(y), \quad A \supseteq B$ iff $B \subseteq A$, $A = B$ iff $\forall y \in Y, \mu_A(y) = \mu_B(y)$ and $\nu_A = \nu_B(y)$,
$A \cap B = \{< y, \min(\mu_A(y), \mu_B(y)), \max(\nu_A(y), \nu_B(y)) > | y \in Y \}$,
$A \cup B = \{< y, \max(\mu_A(y), \mu_B(y)), \min(\nu_A(y), \nu_B(y)) > | y \in Y \}$,
Definition 3.1.1. Let \( S \) be a nonempty set. \( A \) is a type-2 intuitionistic fuzzy set (T2IFS) of \( S \). \( A \) is defined by:
\[
A : S \to Map(D, [0, 1]) \times Map(D, [0, 1]), \text{ where } D = \{(u, v) \in [0, 1] \times [0, 1] : u + v \leq 1\}.
\]

For convenience in description, the binary operations between \( f(u, v) \) and \( g(u, v) \) in \( Map(D, [0, 1]) \) are written in the forms \( (f \land g)(u, v) = f(u, v) \land g(u, v) \), \( (f \lor g)(u, v) = f(u, v) \lor g(u, v) \), and \( (f, g) = \langle f(u, v), g(u, v) \rangle \).

Now we give the definitions of main operations in T2IFS theory.

3. DEFINITION OF OPERATIONS ON TYPE-2 IFS

3.1. Definition

Definition 3.1.1. Let \( S \) be a nonempty set. \( A \) is a type-2 intuitionistic fuzzy set (T2IFS) of \( S \). \( A \) is defined by:
\[
A : S \to Map(D, [0, 1]) \times Map(D, [0, 1]), \text{ where } D = \{(u, v) \in [0, 1] \times [0, 1] : u + v \leq 1\}.
\]

For convenience in description, the binary operations between \( f(u, v) \) and \( g(u, v) \) in \( Map(D, [0, 1]) \) are written in the forms \( (f \land g)(u, v) = f(u, v) \land g(u, v) \), \( (f \lor g)(u, v) = f(u, v) \lor g(u, v) \), and \( (f, g) = \langle f(u, v), g(u, v) \rangle \).

Now we give the definitions of main operations in T2IFS theory.

3.1. The operation AND

Definition 3.1.2. Let \( (f_1, g_1) \) and \( (f_2, g_2) \) be in \( Map(D, [0, 1]) \times Map(D, [0, 1]) \). We define the intersection operation denoted \( \land \) and it is defined by:
\[
(f_1, g_1) \land (f_2, g_2) = (f, g)
\]
where for any \((u, v) \in D\)
\[
\begin{align*}
    f(u, v) & = \bigvee_{u_1 \land u_2 = u, v_1 \lor v_2 = v} f_1(u_1, v_1) \land f_2(u_2, v_2), \\
    g(u, v) & = \bigvee_{u_1 \land u_2 = u, v_1 \lor v_2 = v} g_1(u_1, v_1) \land g_2(u_2, v_2)
\end{align*}
\]

3.1.2. The operation OR

Definition 3.1.3. Let \( (f_1, g_1) \) and \( (f_2, g_2) \) be in \( Map(D, [0, 1]) \times Map(D, [0, 1]) \). We define the union operation denoted \( \lor \) and it is defined by:
\[
(f_1, g_1) \lor (f_2, g_2) = (f, g)
\]
where for any \((u, v) \in D\)
\[
\begin{align*}
    f(u, v) & = \bigvee_{u_1 \lor u_2 = u, v_1 \land v_2 = v} f_1(u_1, v_1) \lor f_2(u_2, v_2), \\
    g(u, v) & = \bigvee_{u_1 \lor u_2 = u, v_1 \land v_2 = v} g_1(u_1, v_1) \lor g_2(u_2, v_2)
\end{align*}
\]

3.1.3. The operation NEGATION

\[
(f(u, v), g(u, v))^* = (f(v, u), g(v, u))
\]
The followings are definitions of identities. They are \( 1 = (1_1, 1_0) \) and \( 0 = (1_0, 1_1) \).
Thus, we defined the algebra $M = (Map(D, [0, 1]) \times Map(D, [0, 1]), \cap, \cup, *, 1, 0)$ for T2IFS with operations $\cap, \cup, *$. Our aim in this paper is to examine some properties on the algebra of T2IFS such as idempotent, involution, commutative laws, associative laws or distributive laws.

### 3.2. Some properties of these operations.

In this section, we are going to demonstrate some properties of the operations on T2IFS. We start with the below theorem which clarifies the first properties such as idempotent, commutative, absorption laws with identities, involution, and De Morgan’s laws.

**Theorem 3.2.1.** For every $(f, g), (f_1, g_1), (f_2, g_2) \in M$, we have

1. $(f, g) \cap (f, g) = (f, g)$ and $(f, g) \cup (f, g) = (f, g)$
2. $(f_1, g_1) \cap (f_2, g_2) = (f_2, g_2) \cap (f_1, g_1)$ and $(f_1, g_1) \cup (f_2, g_2) = (f_2, g_2) \cup (f_1, g_1)$
3. $(1_1, 1_0) \cap (f, g) = (f, g)$ and $(1_0, 1_1) \cup (f, g) = (f, g)$
4. $(f, g)^* = (f, g)$
5. $((f_1, g_1) \cap (f_2, g_2))^* = (f_1, g_1)^* \cup (f_2, g_2)^*$ and
   $((f_1, g_1) \cup (f_2, g_2))^* = (f_1, g_1)^* \cap (f_2, g_2)^*$

**Proof** We omit properties 1, 2 and 4.

The proof of property 3. First, we handle the absorption law of identity $(1_1, 1_0)$. Let $(f, g)$ be in $M$. Suppose that $(1_1, 1_0) \cap (f, g) = (f', g')$, we have

$$f'(u, v) = \bigvee_{u_1 \land u_2 = u, v_1 \lor v_2 = v} 1_1(u_1, v_1) \land f(u_2, v_2)$$

To find $f'(u, v)$ we look for all values of $1_1(u_1, v_1) \land f(u_2, v_2)$, where $u_1 \land u_2 = u, v_1 \lor v_2 = v$ and then find their $\sup$. In the first case, if $(u_1, v_1) = (1, 0)$ then $1_1(u_1, v_1) = 1$ and $(u_2, v_2)$ must be $(u, v)$. We have $1_1(u_1, v_1) \land f(u_2, v_2) = 1 \land f(u, v) = f(u, v)$.

In the other case, if $(u_1, v_1) \neq (1, 0)$, then $1_1(u_1, v_1) = 0$, we have $1_1(u_1, v_1) \land f(u_2, v_2) = 0 \lor f(u_2, v_2) = 0$.

Hence, $f'(u, v)$ is the $\sup$ of two results for the two cases or $f'(u, v) = 0 \lor f(u, v) = f(u, v)$.

In similar way, we prove that $g'(u, v) = g(u, v)$.

With the same arguments, the absorption law of the other identity can be proven.

For property 5, we prove only the first formula. The second formula automatically has the analogous proof.

Let $(f_1, g_1), (f_2, g_2) \in M$. Suppose that $(f', g') = (f_1, g_1) \cap (f_2, g_2)$, we have

$$f'(u, v) = \bigvee_{u_1 \land u_2 = u, v_1 \lor v_2 = v} f_1(u_1, v_1) \land f_2(u_2, v_2),$$

$$g'(u, v) = \bigvee_{u_1 \land u_2 = u, v_1 \lor v_2 = v} g_1(u_1, v_1) \land g_2(u_2, v_2).$$

then $(f'(u, v), g'(u, v))^* = (f'(v, u), g'(v, u))$, where

$$f'(v, u) = \bigvee_{u_1 \land u_2 = u, v_1 \lor v_2 = v} f_1(u_1, v_1) \land f_2(v_2, u_2),$$

$$g'(v, u) = \bigvee_{u_1 \land u_2 = u, v_1 \lor v_2 = v} g_1(v_1, u_1) \land g_2(v_2, u_2).$$
Otherwise,
\[(f_1, g_1)^* \sqcup (f_2, g_2)^* = (f_1(u, v), g_1(u, v))^* \sqcup (f_2(u, v), g_2(u, v))^*\]
\[= (f_1(v, u), g_1(v, u)) \sqcup (f_2(v, u), g_2(v, u) = (f''(v, u), g''(v, u)),\]
where
\[f''(v, u) = \bigvee_{u_1 \vee u_2 = u_1 \wedge u_2 = u} f_1(v_1, u_1) \wedge f_2(v_2, u_2),\]
\[g''(v, u) = \bigvee_{u_1 \vee u_2 = u_1 \wedge u_2 = u} g_1(v_1, u_1) \wedge g_2(v_2, u_2).\]

Comparing \((f', g')\) and \((f'', g'')\), we can imply \((f', g') = (f'', g'')\). Thus, we have \((f_1, g_1)^* \sqcup (f_2, g_2)^* = (f_1, g_1)^* \sqcup (f_2, g_2)^*\) is proved.

**Definition 3.2.2.** For \(f \in \text{Map}(D, [0, 1])\). Let \(f^L, f^R, f_L, f_R\) be elements of \(\text{Map}(D, [0, 1])\) defined by \(f^L(u, v) = \bigvee_{u' \leq u} f(u', v)\), \(f^R(u, v) = \bigvee_{u' \geq u} f(u', v)\), \(f_L(u, v) = \bigvee_{v' \leq v} f(u, v')\), \(f_R(u, v) = \bigvee_{v' \geq v} f(u, v')\).

The below figures visualize our definitions.

![Figure 3.1: Geometrical interpretation of \(f, f^L, f^R, f_L, f_R\)](image)

**Theorem 3.2.3.** The following properties hold for all \((f_1, g_1)(f_2, g_2) \in M:\)

1. \((f_1, g_1) \sqcap (f_2, g_2) = (f, g)\) provided

\[f = (f_1 \sqcap f_2^R) \vee (f_1^R \sqcap f_2) \vee (f_1^R \sqcap f_2^R),\]
\[g = (g_1 \sqcap g_2^R) \vee (g_1^R \sqcap g_2) \vee (g_1^R \sqcap g_2^R).\]

2. \((f_1, g_1) \sqcup (f_2, g_2) = (f, g)\) provided

\[f = (f_1 \sqcup f_2^R) \vee (f_1^R \sqcup f_2^L) \vee (f_1^R \sqcup f_2),\]
\[g = (g_1 \sqcup g_2^R) \vee (g_1^R \sqcup g_2^L) \vee (g_1^R \sqcup g_2).\]

**Proof**

Let \((f_1, g_1), (f_2, g_2) \in M\). We have \((f_1, g_1) \sqcap (f_2, g_2) = (f, g)\) such that

\[f(u, v) = \bigvee_{u_1 \vee u_2 = u, v_1 \vee v_2 = v} (f_1(u_1, v_1) \wedge f_2(u_2, v_2)),\]
\[g(u, v) = \bigvee_{u_1 \vee u_2 = u, v_1 \vee v_2 = v} (g_1(u_1, v_1) \wedge g_2(u_2, v_2)).\]
Corollary 3.2.4. Let $f, g \in Map(D, [0, 1])$. We have

1. $(f \cup g)L = (f_L \wedge g_L) \vee (f^L \wedge g_R)\wedge (f^L \wedge g_R)\wedge (f^L \wedge g_R)$
2. $(f \cap g)_R = (f_R \vee g_R) \vee (f_R \vee g_R)\wedge (f_R \vee g_R)$
3. $(f \cup g)^R_L = f^R_L \wedge g^R_R$

Proof

1. We have
(f \uplus g)^L(u, v) = \bigvee_{u' \leq u} (f \uplus g)(u', v) = \bigvee_{u' \leq u} \big( \bigwedge_{u_1 \vee u_2 = u', v_1 \wedge v_2 = v} (f(u_1, v_1) \land g(u_2, v_2)) \big) \\
= \bigvee_{u_1 \vee u_2 = u, v_1 \wedge v_2 = v} (f(u_1, v_1) \land g(u_2, v_2)) \\
= \bigvee_{u_1 \vee u_2 \leq u} (f(u_1, v_1) \land g(u_2, v_2)) \vee \bigvee_{u_1 \leq u} (f(u_1, v_1) \land g_R(u_2, v_2)) \\
= \bigvee_{u_1 \vee u_2 \leq u} (f(u_1, v_1) \land g_R(u_2, v_2)) \vee \bigvee_{u_1 \leq u} (f(u_1, v_1) \land g_R(u_2, v_2)) \\
= (f_R(u, v) \land g_R(u, v)) \vee (f_L(u, v) \land g_R(u, v)). \square \\

There is no difficulty to prove the other formula by using the same arguments used in the above proof.

2. We have 
\[
(f \sqcap g)_R(u, v) = \bigvee_{v' \geq v} (f \sqcap g)(u, v') = \bigvee_{v' \geq v} \big( \bigwedge_{u \vee u_2 = u, v_1 \wedge v_2 = v'} f(u_1, v_1) \land g(u_2, v_2) \big) \\
= \bigvee_{u_1 \wedge u_2 = u, v_1 \wedge v_2 = v'} f(u_1, v_1) \land g(u_2, v_2) \\
= \bigvee_{u_1 \wedge u_2 = u, v_1 \wedge v_2 = v'} \big( \bigvee_{u_2 \geq u} f(u_1, v_1) \land g(u_2, v_2) \big) \\
= \bigvee_{u_1 \wedge u_2 = u} \big( \bigvee_{u_2 \geq u} f(u_1, v_1) \land g(u_2, v_2) \big) \\
= \bigvee_{u_1 \wedge u_2 = u} \big( \bigvee_{u_2 \geq u} f(u_1, v_1) \land g(u_2, v_2) \big) \\
= \bigvee_{u_1 \wedge u_2 = u} \big( \bigvee_{u_2 \geq u} f(u_1, v_1) \land g_R(u_2, v_2) \big) \\
= \bigvee_{u_1 \wedge u_2 = u} \big( \bigvee_{u_2 \geq u} f(u_1, v_1) \land g_R(u_2, v_2) \big) \\
= \bigvee_{u_1 \wedge u_2 = u} \big( \bigvee_{u_2 \geq u} f_R(u_1, v_1) \land g_R(u_2, v_2) \big) \\
= f_R(u, v) \land g_R(u, v).
\]

Thus,
\[
(f \sqcap g)_R = (f_R \sqcap g_R) \sqcap (f_R \sqcap g_R).
\]

With the same arguments, it becomes easy to prove the following formula:
\[
(f \uplus g)_L = (f_R \sqcup g_R) \sqcup (f_L \sqcup g_R).
\]

3. We have 
\[
(f \uplus g)^R(u, v) = \bigvee_{u', v' \geq u, v} (f \uplus g)(u', v') = \bigvee_{u', v' \geq u, v} \big( \bigwedge_{u_1 \vee u_2 = u', v_1 \wedge v_2 = v'} (f(u_1, v_1) \land g(u_2, v_2)) \big) \\
= \bigvee_{u_1 \vee u_2 = u, v_1 \wedge v_2 = v} (f(u_1, v_1) \land g(u_2, v_2)) \\
= \bigvee_{u_1 \vee u_2 = u, v_1 \wedge v_2 = v} (f(u_1, v_1) \land g_R(u_2, v_2)) \\
= \bigvee_{u_1 \vee u_2 = u, v_1 \wedge v_2 = v} (f(u_1, v_1) \land g_R(u_2, v_2)) \\
= \bigvee_{u_1 \leq u, v_2 \geq v} (f_R(u_1, v_1) \land g_R(u_2, v_2)) \\
= f_R(u, v) \land g_R(u, v).
\]

Theorem 3.2.5. The following associative laws hold for \( \sqcup \) and \( \sqcap \).
\[
(f_1, f_2) \sqcup [(g_1, g_2) \sqcup (h_1, h_2)] = [(f_1, f_2) \sqcup (g_1, g_2)] \sqcup (h_1, h_2)
\]
We are going to prove the following formula. For \(f, g, h \in \text{Map}(D, [0, 1])\)

\[
f \uplus (g \uplus h) = (f \uplus g) \uplus h
\]

(3.1)

Proof

We are going to prove the following formula. For \(f, g, h \in \text{Map}(D, [0, 1])\)

\[
f \uplus (g \uplus h) = (f \uplus g) \uplus h
\]

From the right side of (1), we have

\[
(f \uplus g) \uplus h = [(f \uplus g)^L \land h_R] \lor [(f \uplus g)^R \land h_L] \lor \left(\left(\left((f \uplus g)^L \land h_R\right) \lor \left((f \uplus g)^R \land h_L\right)\right) \lor \left(f \uplus (g \uplus h)^L \right)\right) \lor \left(f \uplus \left((g \uplus h)^R \land h_R\right)\right)
\]

From Corollary 3.2.6.

the left side of (1), we have

\[
f \uplus (g \uplus h) = \left[\left(f^L \land \left(\left(g \uplus h\right)^L\right)\right) \lor \left(f^R \land \left(\left(g \uplus h\right)^R\right)\right)\right] \lor \left(\left(\left(f^L \land \left(\left(g \uplus h\right)^L\right)\right) \lor \left(f^R \land \left(\left(g \uplus h\right)^R\right)\right)\right) \lor \left(f \land (g \uplus h)^L\right)\right)
\]

Hence, \(f \uplus (g \uplus h) = (f \uplus g) \uplus h\) is proved. Because of this property, we imply

\[
(f_1, f_2) \sqcup [(g_1, g_2) \sqcup (h_1, h_2)] = (f_1 \uplus (g_1 \uplus h_1), f_2 \uplus (g_2 \uplus h_2))
\]

\[
= ((f_1 \uplus g_1) \uplus h_1, (f_2 \uplus g_2) \uplus h_2)
\]

\[
= [(f_1, f_2) \sqcup (g_1, g_2)] \sqcup (h_1, h_2)
\]

To prove the absorption law of \(\sqcup\), we have the similar proof.

Corollary 3.2.6. The following distributive laws hold for operations \(\sqcap, \triangle\) and \(\lor\):

\[
f \sqcap (g \lor h) = (f \sqcap g) \lor (f \sqcap h)
\]

\[
f \triangle (g \lor h) = (f \triangle g) \lor (f \triangle h)
\]

This is one of the results which has been proved for type-2 fuzzy sets in [11].

The following results will show that the operations of type-2 intuitionistic fuzzy sets do not have distributive laws.

Theorem 3.2.7. Let \(f, g, h\) be in \(M\) We have

1. \(f \uplus (g \sqcap h) = [f_R \triangle (g_L \sqcap h)] \lor [f_R \triangle (g \sqcap h_R)] \lor \left(f \triangle (g_R \lor h_R)\right)\)

2. \((f \uplus g) \sqcap (f \uplus h) = \left[\left(f_R \triangle g_L\right) \lor \left(f \triangle h\right)\right] \lor \left((f_R \triangle g) \lor \left(f \triangle (g_R \lor h_R)\right)\right)\)

\[
\lor \left((f \triangle g) \lor \left(f_R \triangle h\right)\right] \lor \left((f \triangle g) \lor \left(f \triangle (g_R \lor h_R)\right)\right)\)
\]

\[
\lor \left((f \triangle g) \lor \left(f_R \triangle h\right)\right] \lor \left((f \triangle g) \lor \left(f \triangle (g_R \lor h_R)\right)\right)\)
\]
Proof
1. From the left side, we have

\[ f \uplus (g \uplus h) = [fR \triangledown (g \uplus h)] \lor [f \triangledown (g \uplus h)_R] \]
\[ = \{fR \triangledown [(gL \triangle h) \lor (g \triangle h_L)] \} \lor \{f \triangledown [(gR \triangle h_{RL}) \lor (g_{RL} \triangle h_R)]\} \]
\[ = \{fR \triangledown (gL \triangle h)] \lor [fR \triangledown (g \triangle h_L)] \lor [f \triangledown (gR \triangle h_{RL})] \lor [f \triangledown (g_{RL} \triangle h_R)] \]

2. We have \((f \uplus g) \ominus (f \uplus h) = [(f \uplus g)_L \triangledown (f \uplus h)] \lor [(f \uplus g) \triangledown (f \uplus h)_L] \)

Deal with the second expression, we have

\[ (f \uplus g) \triangledown (f \uplus h)_L = \]
\[ = [(fR \triangle g) \triangledown (fR \triangle h_L)] \lor [(fR \triangle g) \triangledown (fL \triangle h_{RL})] \]
\[ \lor [(f \triangle gR) \triangledown (f_{RL} \triangle h_L)] \lor [(f \triangle gR) \triangledown (fL \triangle h_{RL})] \]

Hence,

\[ (f \uplus g) \ominus (f \uplus h) = [(fR \triangle gL) \triangledown (fR \triangle h)] \lor [(fR \triangle gL) \triangledown (f \triangle h_R)] \]
\[ \lor [(fL \triangle g_{RL}) \triangledown (fL \triangle h_L)] \lor [(fL \triangle g_{RL}) \triangledown (f \triangle h_{RL})] \]
\[ \lor [(f \triangle g) \triangledown (f_{RL} \triangle h_L)] \lor [(f \triangle g) \triangledown (fL \triangle h_{RL})] \]
\[ \lor [(f \triangle g) \triangledown (f_{RL} \triangle h_L)] \lor [(f \triangle g) \triangledown (fL \triangle h_{RL})] \]

In short, this theorem shows that the distributive laws do not hold.

4. CONCLUSION

In this paper, we have introduced a new concept of type-2 intuitionistic fuzzy set and their operations. It is hopefully more general and applicable than the ordinary intuitionistic fuzzy set. There would be an overwhelmingly large amount of applications on many different fields that derive from it.

REFERENCES


Received on March 5, 2012