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THE PROCESS OF $e^+e^- \to \mu^+\mu^-$ IN THE RANDALL - SUNDRUM MODEL, SUPERSYMMETRIC MODEL AND UNPARTICLE PHYSICS

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Abstract. *The process of $e^+e^- \to \mu^+\mu^-$ in models, namely Randall - Sundrum, Supersymmetric and Unparticle physics are studied in details. We calculate the cross-section for γ, Z, U, h, ϕ, h_0, H_0, A_0 exchange when the e^+, e^-, μ^+, μ^- beams are polarized and unpolarized. The results show that the cross-section strongly depends on the exchange particles contributions and polarization of the initial and final beams.*

Keywords: standard model, Randall Sundrum model, Supersymmetric model, unparticle physics.

Classification numbers: 13.88.+e.

I. INTRODUCTION

The standard model (SM) of particle physics is a theory concerning the electromagnetic, weak, and strong nuclear interactions. From time to time, the discoveries of the top quark (1995), the neutrino tau (2000), and the Higgs boson (2012) [1,2], which agree with theoretical predictions have given further credence to the SM. However, there are still many existing problems to be solved in the SM. To be able to fully explain such problems, extensions of the SM, which predict many new interesting physical phenomena at the high-energy scale, are studied.

In this paper, we study extended models, namely Randall - Sundrum (RS), super-symmetric (SUSY) and unparticle physics (UP). The RS model [3] can solve the hierarchy problem by localizing all the SM particles on the IR brane. This model predicts two new particles beyond the SM. One is a spin-2 graviton and another is a scalar-field radion which is a metric fluctuation along the extra dimension. The mass of the radion is expected to be of the order of GeV. The radion, therefore, is expected to be the first signature of warped extra dimension models in direct search experiments such as the Large Hadron Collider (LHC) [4–10]. The SUSY model [11–18] is also

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quite an attractive extension model, which provides a natural solution to the hierarchy problem by canceling out the quadratically divergent quantum corrections to the Higgs boson mass with the introduction of the superpartner of the top quark and can explain the presence of Dark Matter in the Universe. Moreover, SUSY can unify the gauge couplings of the SM at high energy scales. According to some theoretical assumptions, some of the SUSY particles are less than one TeV, so their discovery can be found at the LHC.

UP [19] is also one hot topic for recent discussion. UP is possibly an interesting extension to the SM beyond the TeV scale. UP arises from the possibility of implementing scale invariance in the SM. This requires considering an additional set of fields, the Banks-Zaks (BZ) fields, with a non-trivial IR fixed point. The two fields interact through the exchange of particles if the energies of the particles are high enough. The collision energy of unparticle is high energy, but the unparticle is found in low energy domain. Scientists are trying to prove the correctness of the model from the experimental results [20–22]. In this paper, we study the process $e^+e^- \to \mu^+\mu^-$ in the RS model, SUSY model and UP in detail. In the process of searching for new particles, e^+e^- collision is one of the important processes. A lot of physical phenomena have been studied through the process e^+e^- collision [23–27]. The process of $e^+e^- \to \mu^+\mu^-$ is simple, clean products and no by-products attached. Besides that, it can occur in any extended patterns and the cross sections contain contribution only from s-channel. Therefore, the cross section for the contribution of exchange particles in the extended model can be compared within each other, easy to control and can be checked on the accelerator in the energy domain, which is not too large.

II. INTERACTION LAGRANGIAN AND UNPARTICLE PHYSICS

In this section, we list the relevant parts of Lagrangian and the Feynman rules for fermions interactions needed in the RS model, SUSY model and UP. First, in the RS model, the interactions of the radion with the SM particles on the brane are model-independent and are governed by 4-dimensional general covariance, and thus given by the following Lagrangian [27,28]:

$$L_{\text{int}} = \frac{\phi}{\Lambda_\phi} T_\mu^\mu(SM), \tag{1}$$

where $\Lambda_\phi \equiv \sqrt{6} M_{PL}\Omega_0$ is the VEV of the radion field, and T_μ^μ is the trace of SM energy-momentum tensor, which is given by

$$T_\mu^\mu(SM) = \sum_f m_f \bar{f} f - 2m_w^2 W_\mu^+ W^{-\mu} - m_z^2 Z_\mu Z^\mu + (2m_h^2 h^2 - \partial_\mu h \partial^\mu h) + ... \tag{2}$$

The Feynman rules for the $f\bar{f}$ couplings of the Higgs h and radion ϕ are shown in Fig. 1.

The second Lagrangian is SUSY Lagrangian describing the interaction among scalar field A_i, fermion field ψ_i and auxiliary field F_i in Eq. (3) [17,18,29]

$$L = i\partial_\mu \bar{\psi}_i \sigma^\mu \psi_i + A_i^* W A_i - \frac{1}{2} m^{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \bar{\psi}^i \bar{\psi}^j - g^{ijk} \psi_i \psi_j A_k - g_{ijk}^* \bar{\psi}^i \bar{\psi}^j A^{*k} - V(F_i^* F_i). \tag{3}$$

The Feynman rules for the $f\bar{f}$ couplings of the h_0, H_0 and A_0 are shown in Fig. 2.

As we know that the UP is built from the very high energy theory, which contains the fields of the SM and BZ. The two sets interact through the exchange of particles with a large mass scale

Fig. 1. Feynman rules for the $f\bar{f}$ couplings of the Higss h and radion ϕ.

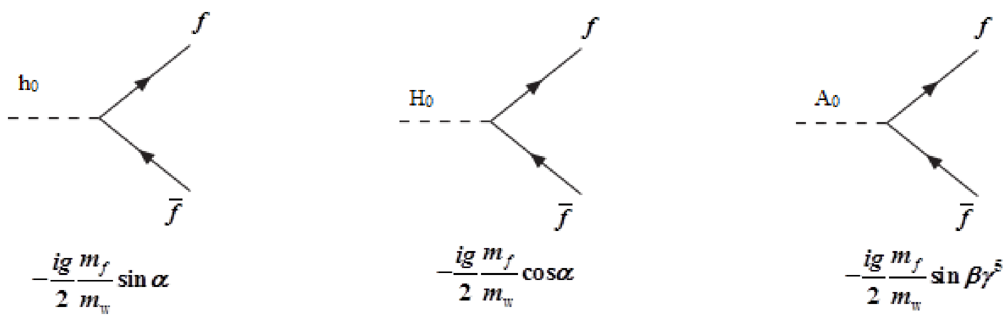


Fig. 2. Feynman rules for the $f\bar{f}$ couplings of the h_0, H_0 and A_0.

M_u. The effective Lagrangian can be written as [19, 30]:

$$L_u = C_{O_u}\frac{\Lambda_u^{d_{BZ}-d_u}}{M_u^{d_{SM}+d_{BZ}-4}}O_{SM}O_u, \tag{4}$$

where d_u is the scale dimension of the unparticle operator O_u, O_{SM} is an operator with mass dimension d_{SM} built out of SM fields and C_{O_u} is a coefficient function fixed by the matching. Interactions of unparticles with SM particles are introduced in Ref. [31]. The common effective interactions that satisfy the SM gauge symmetry for the vector and tensor unparticle operators with SM fields are given by [30]

$$\lambda_1\frac{1}{\Lambda_u^{d_u-1}}\bar{f}\gamma_\mu f O_u^\mu,\ \lambda_1\frac{1}{\Lambda_u^{d_u-1}}\bar{f}\gamma_\mu\gamma_5 f O_u^\mu, \tag{5}$$

$$-\frac{1}{4}\lambda_2\frac{1}{\Lambda_u^{d_u}}\bar{\psi}i\left(\gamma_\mu\overleftrightarrow{D}_\nu+\gamma_\nu\overleftrightarrow{D}_\mu\right)\psi O_u^{\mu\nu},\ \lambda_2\frac{1}{\Lambda_u^{d_u}}G_{\mu\alpha}G_\nu^\alpha O_u^{\mu\nu}. \tag{6}$$

The Feynman rules for the operators in Eqs. (5), (6) are shown in Fig. 3.

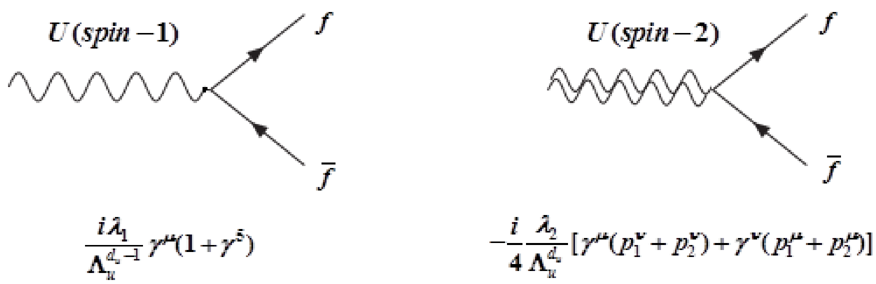


Fig. 3. Feynman rules for the $f\bar{f}$ couplings of the vector unparticle and tensor unparticle.

III. THE CROSS SECTION OF THE $e^+e^- \to \mu^+\mu^-$

The process $e^+e^- \to \mu^+\mu^-$ in extended models is described by the Feynman diagram presented in Fig. 4. The process $e^+e^- \to \mu^+\mu^-$ proceeds via γ, Z, $U-spin\,1$, $U-spin\,2$, h, ϕ, h_0, H_0, A_0 exchange in the UP, RS model and SUSY model, (see Fig. 4) in which, γ and Z exchange contributions in all three models. In the UP model, there are exchange contributions of $U-spin\,1$ and $U-spin\,2$. In the RS model, there are exchange contributions of h and ϕ. In the SUSY model, there are exchange contributions of h_0, H_0 and A_0.

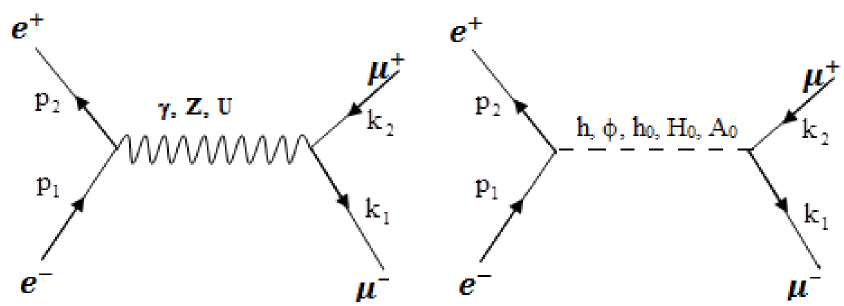


Fig. 4. Feynman diagram for $e^+e^- \to \mu^+\mu^-$ in expansion models.

Using Feynman rules, we calculated the matrix element of this process for each case as following:

+ Via γ, Z exchange:

$$M_\gamma = i\frac{e^2}{q^2}\bar{v}(p_2)\gamma_\mu u(p_1)\bar{u}(k_1)\gamma^\mu v(k_2), \tag{7}$$

$$\begin{aligned} M_Z = & \frac{ig^2}{16c_W^2(q^2-m_z^2)}\bar{v}(p_2)\gamma_\mu\left[\left(-1+4s_W^2\right)+\gamma_5\right]u(p_1)\left[g^{\mu\nu}-\frac{q^\mu q^\nu}{m_z^2}\right]c \\ & \times\bar{u}(k_1)\gamma_\nu\left[\left(-1+4s_W^2\right)+\gamma_5\right]v(k_2). \end{aligned} \tag{8}$$

+ Via spin-1 unparticle and spin-2 unparticle exchange:

$$M_{U-spin\,1} = \frac{-i\lambda_1^2 A_{d_U}\left(-q^2\right)^{d_U-2}}{2\sin(d_U\pi)\Lambda_U^{d_U-1}}\bar{v}(p_2)\gamma_\mu u(p_1)\left[-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right]\bar{u}(k_1)\gamma_\nu v(k_2), \tag{9}$$

$$\begin{aligned} M_{U-spin\,2} = &-\frac{1}{8}\frac{\lambda_2^2 A_{d_U}}{2\sin(d_U\pi)\Lambda_U^4}\left(-\frac{q^2}{\Lambda_U^2}\right)^{d_U-2}\bar{v}(p_2)\gamma^\mu u(p_1)\bar{u}(k_1)\gamma^\nu v(k_2) \\ &\times\left[(p_1+p_2)(k_1+k_2)g_{\mu\nu}+(p_1+p_2)_\nu(k_1+k_2)_\mu\right]. \end{aligned} \tag{10}$$

+ Via Higgs h and radion ϕ exchange in the RS model:

$$M_h = \frac{g^2 m_e m_\mu (d+\gamma b)^2}{4m_w(q^2-m_h^2)}\bar{v}(p_2)u(p_1)\bar{u}(k_1)v(k_2), \tag{11}$$

$$M_\phi = \frac{g^2 m_e m_\mu (c+\gamma a)^2}{4m_w(q^2-m_\phi^2)}\bar{v}(p_2)u(p_1)\bar{u}(k_1)v(k_2). \tag{12}$$

+ Via Higgs h_0, H_0 and A_0 exchange in the SUSY model:

$$M_{h_0} = -\frac{ih_e h_\mu \sin^2\alpha}{2(q^2-m_{h_0}^2)}\bar{v}(p_2)u(p_1)\bar{u}(k_1)v(k_2), \tag{13}$$

$$M_{H_0} = -\frac{ih_e h_\mu \cos^2\alpha}{2(q^2-m_{H_0}^2)}\bar{v}(p_2)u(p_1)\bar{u}(k_1)v(k_2), \tag{14}$$

$$M_{A_0} = -\frac{ih_e h_\mu \sin^2\beta}{2(q^2-m_{A_0}^2)}\bar{v}(p_2)\gamma^5 u(p_1)\bar{u}(k_1)\gamma^5 v(k_2), \tag{15}$$

where $q = p_1 + p_2 = k_1 + k_2$.

Similarly, we obtained the matrix element of the process $e^+e^- \to \mu^+\mu^-$ when the e^+, e^- or μ^+, μ^- beams are polarized by substituting $u(k)$ into $P_L u(k)$ or $P_R u(k)$ and $v(k)$ into $P_L v(k)$ or $P_R v(k)$. Here $k = p_1$, p_2, k_1, k_2 and

$$P_L = \frac{1-\gamma_5}{2}, P_R = \frac{1+\gamma_5}{2}. \tag{16}$$

By using these matrix elements, we evaluated the differential cross-section (DCS) of $e^+e^- \to \mu^+\mu^-$ by the expression: $\frac{d\sigma}{d\cos\theta} = \frac{1}{64\pi s}\frac{|\vec{k}_1|}{|\vec{p}_1|}|M|^2$, (17) where M is the matrix element, $s = (p_1+p_2)^2$ and θ is the angle between $\vec{p}_1$ and $\vec{k}_1$.

In Fig. 5, Fig. 6, we showed the dependence of the DCS on $\cos\theta$ at the collision energy $\sqrt{s} = 3000$ GeV. For , Z, spin-1 unparticle and spin-2 unparticle - exchange, the DCS has a maximum value at $\cos\theta = \pm 1$ and a minimum value at $\cos\theta = 0$. When the e^+, e^- beams are polarized, the DCS is always greater than that when the e^+, e^- beams are unpolazired. Moreover, the calculated DCS for the case of spin 1 unparticle exchange contribution is the largest and Z exchange contribution is the smallest. For the h, ϕ, h_0, H_0 and A_0 exchange contributions, the DCS does not depend on the scattering angle. We obtained numerical values 8.04543×10^{-20} pbar, 2.9431×10^{-23} pbar, 9.44461×10^{-21} pbar, 3.68855×10^{-20} pbar and 6.779×10^{-20} pbar for h,

ϕ, h_0, H_0 and A_0, respectively. When the e^+, e^- or μ^+, μ^- beams are polarized, the DCS value is doubled when the e^+, e^- or μ^+, μ^- beams are unpolazired.

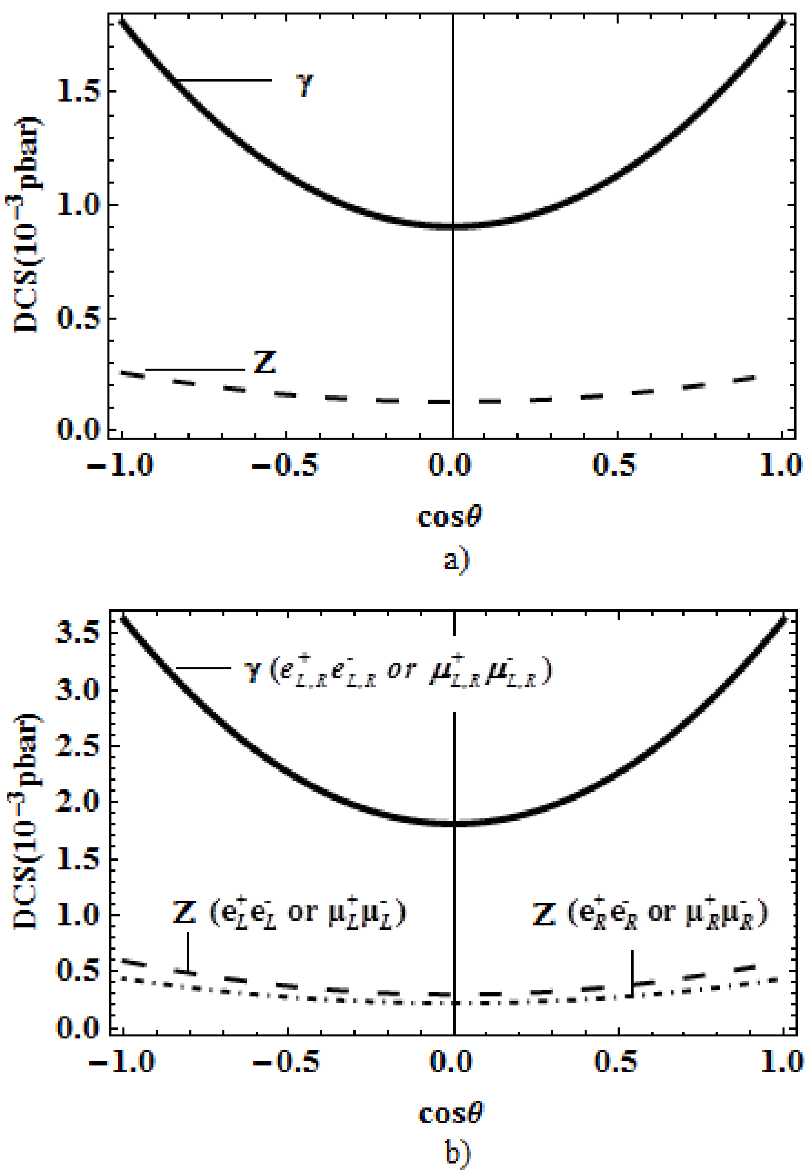


Fig. 5. The differential cross-section of $e^+e^- \to \mu^+\mu^-$ via γ, Z exchange as a function of $\cos\theta$ when the e^+, e^-, μ^+, μ^- beams are unpolazired (a) and polarized (b).

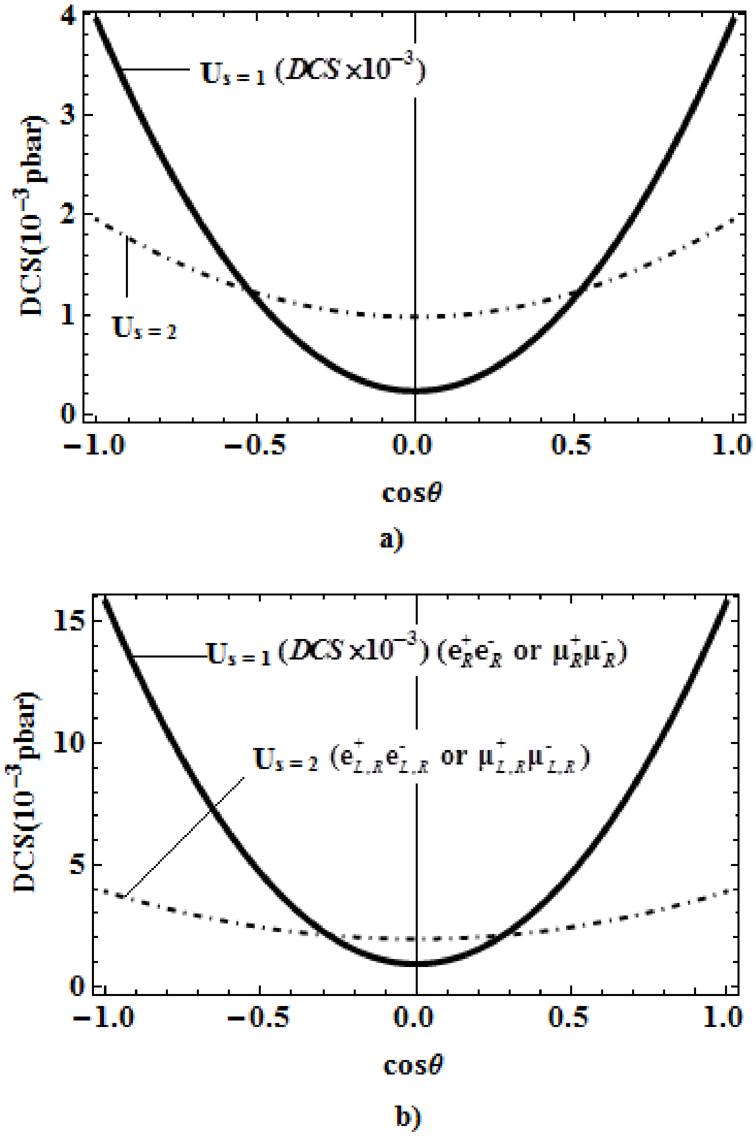
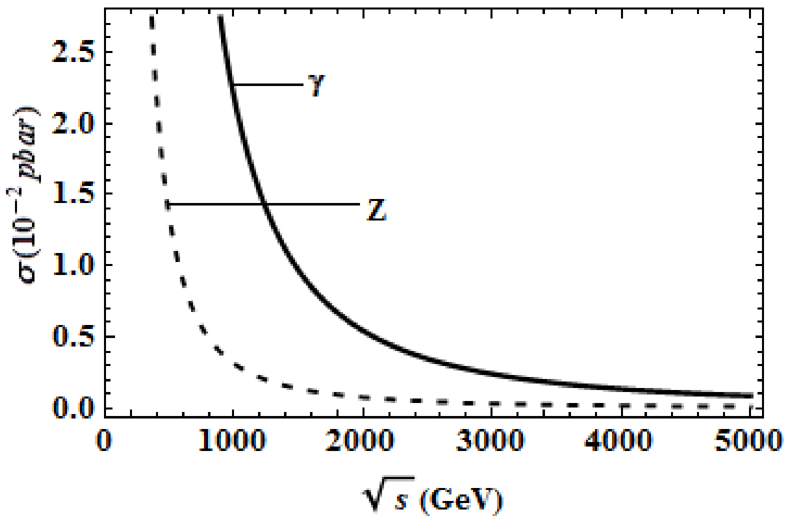
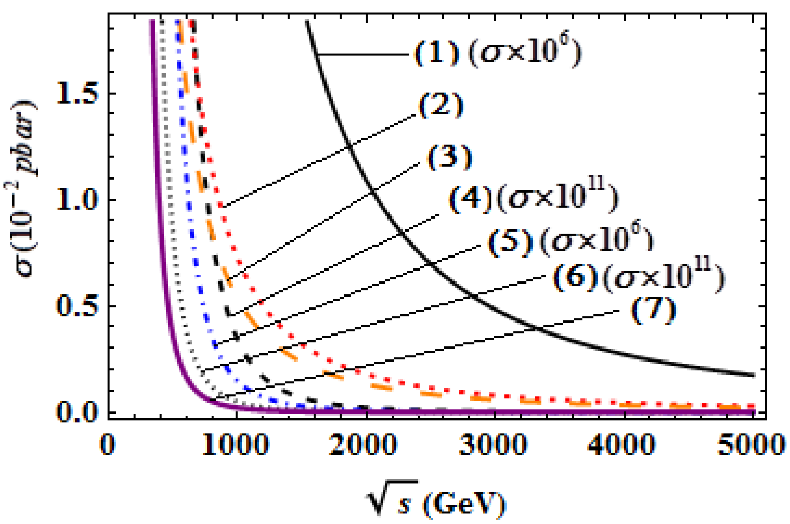


Fig. 6. The differential cross-section of $e^+e^- \to \mu^+\mu^-$ via spin-1 unparticle, spin-2 unparticle exchange as a function of $\cos\theta$ when the e^+, e^-, μ^+, μ^- beams are unpolazired (a) and polarized (b). We chose $\Lambda_U = 1TeV$, $d_U = 1.1$, $\lambda_1 = 1$ for the spin-1 unparticle and $\Lambda_U = 4$ TeV, $d_U = 1.8$, $\lambda_2 = 1$ for the spin-2 unparticle [30].

In Fig. 7, Fig. 8, Fig. 9 and Fig. 10, we plotted the total cross-section as a function of the collision energy$\sqrt{s}$. The figures showed that the total cross-section decreased while $\sqrt{s}$ increased for the photon, Z, spin-1 unparticle, Higgs h_0, H_0, A_0, Higgs h and radion ϕ exchange contributions. For the spin-2 unparticle exchange contribution, the total cross-section increased while $\sqrt{s}$ increased. This can be explained by the spin-2 structure of the operator.

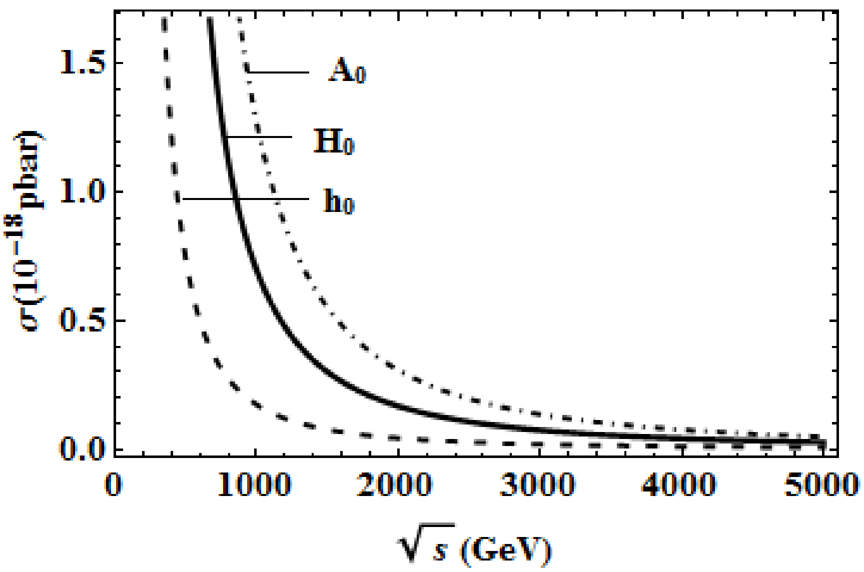


a)

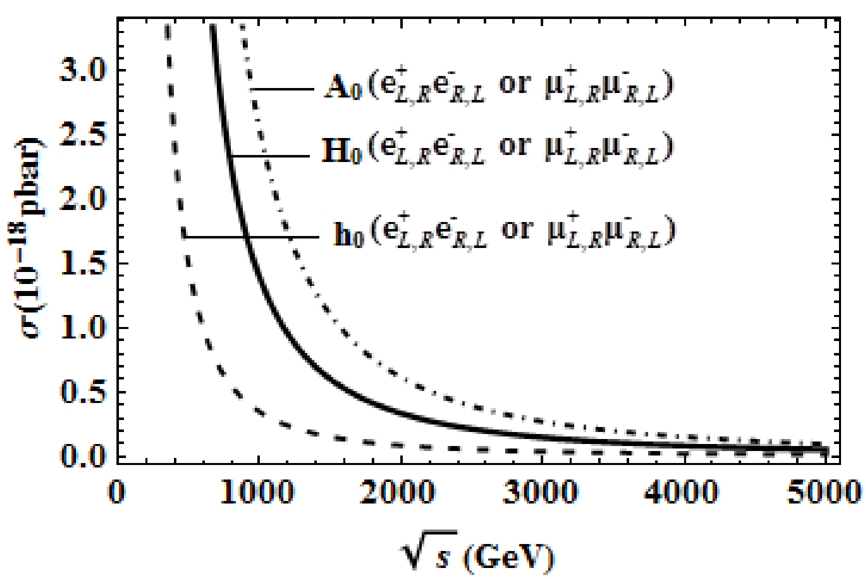


b)

Fig. 7. The total cross-section of $e^+e^- \to \mu^+\mu^-$ via γ, Z exchange as a function of the collision energy $\sqrt{s}$ when the e^+, e^-, μ^+, μ^- beams are unpolazired (a) and polarized (b). In fig.7b, (1), (2), (3), (6) are Z exchange in the case of $\mu^+_{L,R}\mu^-_{R,L}$, $e^+_L e^-_L$ or $\mu^+_L \mu^-_L$, $e^+_R e^-_R$ or $\mu^+_R \mu^-_R$, $e^+_{L,R}e^-_{R,L}$, respectively; (4), (5), (7) are γ exchange in the case of $e^+_{L,R}e^-_{R,L}$, $\mu^+_{L,R}\mu^-_{R,L}$, $e^+_{L,R}e^-_{L,R}$ or $\mu^+_{L,R}\mu^-_{L,R}$, respectively.



a)



b)

Fig. 8. The total cross-section of $e^+e^- \to \mu^+\mu^-$ via Higgs h_0, H_0, A_0 exchange as a function of the collision energy $\sqrt{s}$ when the e^+, e^-, μ^+, μ^- beams are unpolazired (a) and polarized (b).

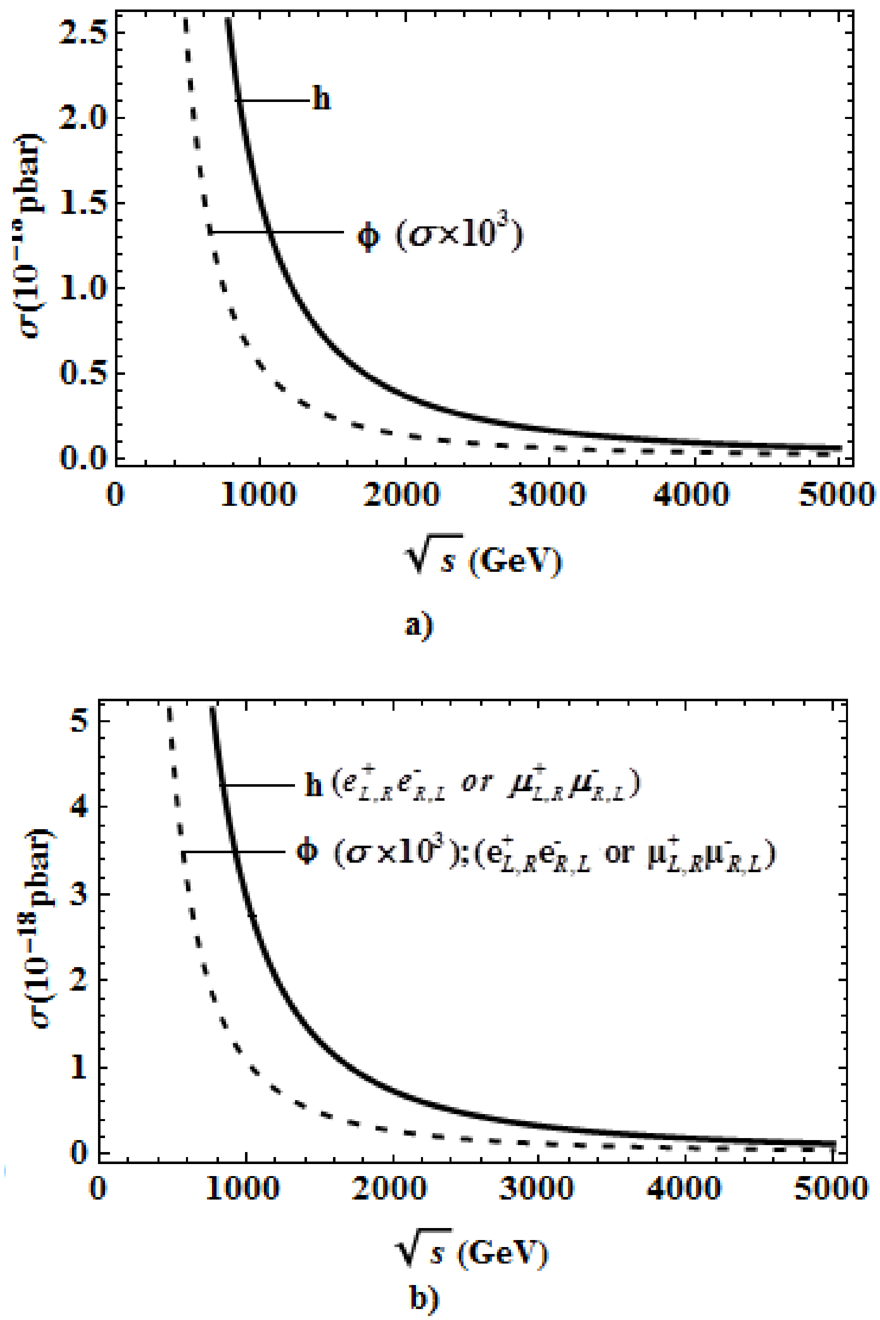


Fig. 9. The total cross-section of $e^+e^- \to \mu^+\mu^-$ via Higgs h, radion ϕ exchange as a function of the collision energy $\sqrt{s}$ when the e^+, e^-, μ^+, μ^- beams are unpolazired (a) and polarized (b).

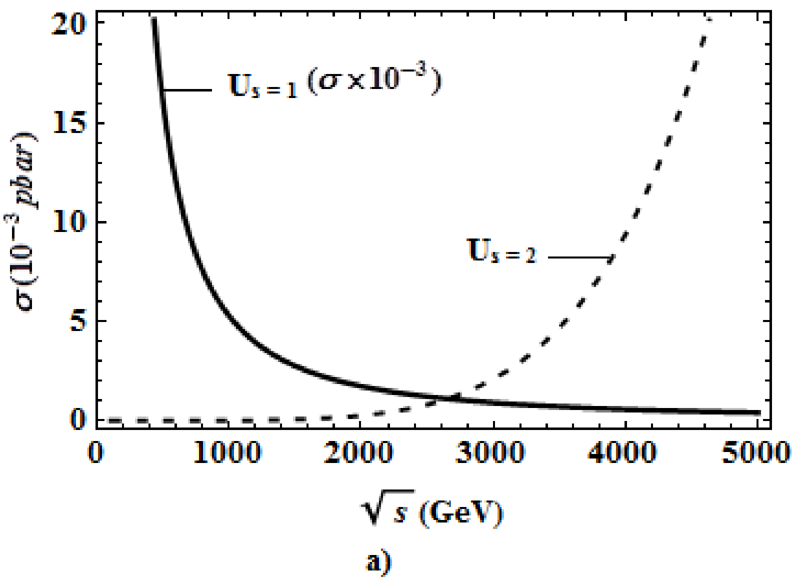


Fig. 10. The total cross-section of $e^+e^- \to \mu^+\mu^-$ via spin-1 unparticle $(Us = 1)$, spin-2 unparticle $(Us = 2)$ exchange as a function of the collision energy $\sqrt{s}$ when the e^+, e^-, μ^+, μ^- beams are unpolazired (a) and polarized (b). In Fig. 10b, (1) is $Us = 1$ in the case of $e^+_{L,R}e^-_{L,R}$ or $\mu^+_{L,R}\mu^-_{L,R}$; (2), (3), (4) are $Us = 2$ in the case of $\mu^+_{L,R}\mu^-_{R,L}$, $e^+_{L,R}e^-_{L,R}$ or $\mu^+_{L,R}\mu^-_{L,R}$, $e^+_{L,R}e^-_{R,L}$, respectively. We chose $\Lambda_U = 1$ TeV, $d_U = 1.1$, $\lambda_1 = 1$ for the spin-1 unparticle and $\Lambda_U = 4$ TeV, $d_U = 1.8$, $\lambda_2 = 1$ for the spin-2 unparticle.

IV. CONCLUSION

In this paper, we calculate the differential cross-sections and the total cross-sections in the process $e^+e^- \to \mu^+\mu^-$ in RS model, SUSY model and UP when the e^+, e^- and μ^+, μ^- beams are polarized and unpolazired. The results show that the cross-section depends on the polarization of e^+, e^- or μ^+, μ^- beams. The cross-section for the spin-1 unparticle exchange is the largest. For the h, ϕ, h_0, H_0 and A_0 exchange, the cross-section is so much smaller than the photon, Z boson, spin-1 unparticle exchange.

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