# $e^{+} e^{-} \rightarrow h Z$ COLLISION IN RANDALL-SUNDRUM MODEL 

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#### Abstract

We study the production of Higgs and $Z$ bosons which has been proposed as an option of $e^{+} e^{-}$collision at the ILC, CLIC with the polarization of the electron and positron beams in Randall-Sundrum model (RSM). The differential cross-sections are presented and numerical evaluation is given. Based on the results, we show that the advantageous direction to collect Higgs is perpendicular to the direction of the initial $e^{-}$beam. With the high integrated luminosity and at the high degree of polarization, the reaction can give observable cross-sections in future accelerators (ILC, CLIC).


Keywords: ZZ coupling, Randall-Sundrum, electron beams.
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## I. INTRODUCTION

In 1999, Randall and Sundrum proposed a 5-dimensional model for solving the gauge hierarchy problem [1]. The RSM allows for a natural generation of the Planck-weak and fermion mass hierarchies [2]. Goldberger and Wise have proposed an attractive mechanism to stabilize the distance between two branes introducting a bulk scalar field which has scalar potentials on both branes [1]. In RSM, the extra dimension is assumed to be located on a $S^{1} / Z_{2}$ orbifold, which has two fixed points, $\phi=0$ and $\phi=\pi$. They correspond to high energy brane and the brane we live on, respectively. Graviton is the only particle propagating through the bulk between these two branes [3]. The space-time metric is given by

$$
\begin{equation*}
d s^{2}=e^{-2 k y} \eta_{\mu \nu} d x^{\mu} d x^{\nu}-d y^{2}, \tag{1}
\end{equation*}
$$

where $x^{\mu}(\mu=0,1,2,3), y$ and $k$ denote the coordinate of 4D space-time, that of a fifth dimension, and the $A d S_{5}$ curvature, respectively. The Minkowski metric is $\eta_{\mu \nu}=\operatorname{diag}(1,-1,-1,-1)$ and $e^{-2 k y}$ is called a warp factor [1]. In four-dimensional effective theory of RSM, there are two new particles beyond the Standard model. One is a spin-2 graviton and the other is a scalar-field radion $\phi$ which is a metric fluctuation along the extra dimension. Having determined the vacuum
structure of the model, we discuss the possibility of mixing between gravity and the electroweak sector. The gravity-scalar mixing is described by the following action [4-6]

$$
\begin{equation*}
S_{\xi}=-\xi \int d^{4} x \sqrt{-g_{v i s}} R\left(g_{v i s}\right) \hat{H}^{+} \hat{H} \tag{2}
\end{equation*}
$$

where $R\left(g_{v i s}\right)$ is the Ricci scalar for the metric induced on the visible brane, $g_{v i s}^{\mu \nu}=\Omega_{b}^{2}(x)\left(\eta^{\mu \nu}+\right.$ $\left.\varepsilon h^{\mu \nu}\right)$. $\hat{H}$ is the Higgs field in the 5D context before rescaling to canonical normalization on the brane. The parameter $\xi$ denotes the size of the mixing term $[1-8]$. With $\xi \neq 0$, there is neither a pure Higgs boson nor pure radion mass eigenstate.
We define the mixing angle $\theta$ by

$$
\begin{equation*}
\tan 2 \theta=12 \gamma \xi Z \frac{m_{h_{0}}^{2}}{m_{\phi_{0}}^{2}-m_{h_{0}}^{2}\left(Z^{2}-36 \xi^{2} \gamma^{2}\right)}, \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
Z^{2} \equiv 1+6 \xi \gamma^{2}(1-6 \xi) \equiv \beta-36 \xi^{2} \gamma^{2}, \gamma=v_{0} / \Lambda_{\phi} \tag{4}
\end{equation*}
$$

In terms of these quantities, the new fields h and $\phi$ are the states that diagonalize the kinetic energy and have canonical normalization with:

$$
\begin{align*}
h_{0} & =\left(\cos \theta-\frac{6 \xi \gamma}{Z} \sin \theta\right) h+\left(\sin \theta+\frac{6 \xi \gamma}{Z} \cos \theta\right) \phi \equiv d h+c \phi,  \tag{5}\\
\phi_{0} & =-\frac{1}{Z} \cos \theta \phi+\frac{1}{Z} \sin \theta h \equiv a \phi+b h . \tag{6}
\end{align*}
$$

The corresponding mass-squared eigenvalues are [9]

$$
\begin{equation*}
m_{h, \phi}^{2}=\frac{1}{2 Z^{2}}\left[m_{\phi_{0}}^{2}+\beta m_{h_{0}}^{2} \pm \sqrt{\left(m_{\phi_{0}}^{2}+\beta m_{h_{0}}^{2}\right)^{2}-4 Z^{2} m_{\phi_{0}}^{2} m_{h_{0}}^{2}}\right] . \tag{7}
\end{equation*}
$$

When $\xi \neq 0$, there are four independent parameters that must be specified to fix the state mixing parameters $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ of Eqs.(5) and (6) defining the mass eigenstates:

$$
\begin{equation*}
\Lambda_{\phi}, m_{h}, m_{\phi}, \xi \tag{8}
\end{equation*}
$$

We consider the case of $\Lambda_{\phi}=5 \mathrm{TeV}$ and $\frac{m_{0}}{M_{P}}=0.1$, which makes the radion stabilization model most natural [10].

The search experiments of the Higgs boson at the LHC give stringent constraints on the parameters of the radion (a radion mass $m_{\phi}$ and a scale parameter $\Lambda_{\phi}$ ). The recently discovered 125 GeV scalar at the LHC Run-I [11,12], behaves like the SM Higgs boson and this fixes the last free parameter of the SM Lagrangian [13]. Additionally, the Higgs bosons were also studied in the $\gamma \gamma, Z Z, W^{+} W^{-}, \ldots$ processes at $\operatorname{LHC}[14,15]$.

In this paper, we study the production of Higgs and $Z$ bosons which has been proposed as an option of $e^{+} e^{-}$collision at the ILC, CLIC. This paper is organised as follows. In Sec.II, we briefly review the interactions of Higgs to SM fields. In Sec.III, the cross-section and our numerical results at $e^{+} e^{-} \rightarrow h Z$ collision are shown. Sec. IV is devoted to summary and discussion.

## II. INTERACTIONS

We turn to the important interactions of the $h, \phi$ and $h_{\mu \nu}^{n}$. We begin with the ZZ couplings of the $h$ and $\phi$. The $h_{0}$ has standard ZZ or fermionic couplings and the $\phi_{0}$ has ZZ or fermionic couplings from the interaction $-\frac{\phi_{0}}{\Lambda_{\phi}} T_{\mu}^{\mu}$ using the Yukawa interaction contributions of $T_{\mu}^{\mu}$. The results are obtained as:

$$
\begin{align*}
& \bar{g}_{Z Z h}=\frac{g m_{Z}}{c_{W}}(d+\gamma b),  \tag{9}\\
& \bar{g}_{Z Z \phi}=\frac{g m_{Z}}{c_{W}}(c+\gamma a),  \tag{10}\\
& \bar{g}_{f \bar{f} h}=-\frac{g m_{f}}{2 m_{W}}(d+\gamma b),  \tag{11}\\
& \bar{g}_{f \bar{f} \phi}=-\frac{g m_{f}}{2 m_{W}}(c+\gamma a) . \tag{12}
\end{align*}
$$

where $g$ and $c_{W}$ denote the $\mathrm{SU}(2)$ gauge coupling and cosine of the Weinberg angle, respectively.

## III. $e^{+} e^{-} \rightarrow h Z$ COLLISION IN RANDALL-SUNDRUM MODEL

In this section, we consider the collision process in which the initial state contains an electron and a positron, the final state contains a pair of Higgs and $Z$ bosons,

$$
\begin{equation*}
e^{+}\left(p_{1}\right)+e^{-}\left(p_{2}\right) \rightarrow h\left(k_{1}\right)+Z\left(k_{2}\right) . \tag{13}
\end{equation*}
$$

Here $p_{i}, k_{i}(i=1,2)$ stand for the momentum. There are three Feynman diagrams contributing to reaction (13), representing the $s, u, t$ channels exchange depicted in Fig. 1.


Fig. 1. The Feynman diagrams $e^{+} e^{-} \rightarrow h Z$ collision.
The amplitude squared of this collision process can be written as

$$
\begin{equation*}
|M|^{2}=\left|M_{s}\right|^{2}+\left|M_{u}\right|^{2}+\left|M_{t}\right|^{2}+2 \operatorname{Re}\left(M_{s}^{+} M_{u}+M_{s}^{+} M_{t}+M_{u}^{+} M_{t}\right), \tag{14}
\end{equation*}
$$

where

$$
\begin{align*}
M_{s} & =-\frac{i g^{2} m_{Z}}{4 c_{w}^{2}\left(q_{s}^{2}-m_{Z}^{2}\right)}(d+\gamma b)\left(\eta_{v \mu}-\frac{q_{s \mu} q_{s v}}{m_{Z}^{2}}\right) \eta^{v \alpha} \varepsilon_{\alpha}^{*}\left(k_{2}\right) \bar{v}\left(p_{2}\right) \gamma^{\mu}\left(v_{e}-a_{e} \gamma^{5}\right) u\left(p_{1}\right),  \tag{15}\\
M_{u} & =\frac{i g^{2} m_{e}}{8 c_{w}^{2}\left(q_{u}^{2}-m_{e}^{2}\right)}(d+\gamma b) \varepsilon_{\mu}^{*}\left(k_{2}\right) \bar{v}\left(p_{2}\right) \gamma^{\mu}\left(v_{e}-a_{e} \gamma^{5}\right)\left(\widehat{q}_{u}+m_{e}\right) u\left(p_{1}\right),  \tag{16}\\
M_{t} & =\frac{i g^{2} m_{e}}{8 c_{w}^{2}\left(q_{t}^{2}-m_{e}^{2}\right)}(d+\gamma b) \varepsilon_{\mu}^{*}\left(k_{2}\right) \bar{v}\left(p_{2}\right) \gamma^{\mu}\left(v_{e}-a_{e} \gamma^{5}\right)\left(\widehat{q}_{t}+m_{e}\right) u\left(p_{1}\right) . \tag{17}
\end{align*}
$$

When the $e^{+}, e^{-}$beams are polarized, we change $v\left(p_{2}\right)$ into $v_{L}\left(p_{2}\right)$ or $v_{R}\left(p_{2}\right)$ and $u\left(p_{1}\right)$ into $u_{L}\left(p_{1}\right)$ or $u_{R}\left(p_{1}\right)$. Here,

$$
\begin{align*}
& \bar{v}_{R}\left(p_{2}\right)=\bar{v}\left(p_{2}\right) \frac{1-\gamma_{5}}{2}  \tag{18}\\
& \bar{v}_{L}\left(p_{2}\right)=\bar{v}\left(p_{2}\right) \frac{1+\gamma_{5}}{2}  \tag{19}\\
& u_{R}\left(p_{1}\right)=\frac{1+\gamma_{5}}{2} u\left(p_{1}\right)  \tag{20}\\
& u_{L}\left(p_{1}\right)=\frac{1-\gamma_{5}}{2} u\left(p_{1}\right) \tag{21}
\end{align*}
$$

From the expressions of the differential cross-section and the total cross-section:

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s} \frac{|\vec{k}|}{|\vec{p}|}\left|M_{f i}\right|^{2} \tag{22}
\end{equation*}
$$

where $M_{f i}$ is the the scattering amplitude, we assess the number and make the identification, evaluation of the results obtained from the dependence of the differential cross-section by $\cos \theta$, the total cross-section fully follows $\sqrt{s}$. We study the cross-section in case of the polarization factors of $e^{+}, e^{-}$beams $P_{1}=P_{2}=-1 ; P_{1}=P_{2}=0 ; P_{1}=-1, P_{2}=0$.

We choose $\sqrt{s}=3 \mathrm{TeV}$ (CLIC), $m_{h}=125 \mathrm{GeV}$ (CMS), $\Lambda_{\phi}=5 \mathrm{TeV}, \xi=1 / 6, v_{0}=246$ $\mathrm{GeV}[12,14,15]$. We give some estimates for the cross-section as follows:
i) In Fig. 2, we plot the total cross-section as the function of the polarization coefficients $P_{1}, P_{2}$, which are polarized coefficients of $e^{+}$and $e^{-}$beams, respectively. The figure indicates that the total cross-section achieves the maximum value $\left(\sigma_{\max }\right)$ when $P_{1}=P_{2}=-1$ and the minimum value when $P_{1}=P_{2}=1$.


Fig. 2. The cross-section of $e^{+} e^{-} \rightarrow h Z$ collision as a function of the polarization coefficients $P_{1}, P_{2}$.
ii) In Fig.3, we plot the differential cross-section as a function of $\cos \theta$. The collision energy is chosen as $\sqrt{s}=3 \mathrm{TeV}$. The figure shows that when the final Higgs direction is perpendicular to the direction of the initial $e^{-}$direction $(\cos \theta \simeq 0)$, the differential cross-sections reach the maximum values. The differential cross-sections reach the minimum values when the $\cos \theta \simeq \pm 1$. The result shows that the advantageous direction to collect Higgs is perpendicular direction to the initial $e^{-}$beam.


Fig. 3. The differential cross-section of $e^{+} e^{-} \rightarrow h Z$ collision as a function of $\cos \theta$.
iii) In Fig. 4, we plot the total cross-section as a function of the collision energy $\sqrt{s}$. We can see that the total cross-sections decreases fast in the region $\sqrt{s}<1 \mathrm{TeV}$. When $1 \mathrm{TeV}<\sqrt{s}<3 \mathrm{TeV}$, the total cross-section decreases gradually. With the high integrated luminosity $L=2 \times 10^{34}$ $\mathrm{cm}^{-2} s^{-1}$ [16], the number of events in a year with some different values of the collision energy are given in Table 1. From these results, we can see that with the high integrated luminosity and at the high degree of polarization, the production cross-section of the Higgs may give observable values at CLIC.


Fig. 4. The differential cross-section of $e^{+} e^{-} \rightarrow h Z$ collision as a function of the collision energy $\sqrt{s}$.

Table 1. The total cross-section of $e^{+} e^{-} \rightarrow h Z$ collision as a function of the collision energy $\sqrt{s}$.

| $\sqrt{s}(\mathrm{TeV})$ | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $N\left(P_{1}=P_{2}=-1\right)$ | 0.30113 | 0.06748 | 0.02925 | 0.01628 | 0.01035 | 0.00715 |
| $N\left(P_{1}=-1, P_{2}=0\right)$ | 0.15056 | 0.03374 | 0.01462 | 0.008134 | 0.00517 | 0.00357 |
| $N\left(P_{1}=P_{2}=0\right)$ | 0.07634 | 0.01709 | 0.007395 | 0.004105 | 0.0026 | 0.00179 |

## IV. CONCLUSION

In this paper, we have evaluated the Higgs production in $e^{+} e^{-} \rightarrow h Z$ collision. The result shows that the cross-section of Higgs depends on the polarization of $e^{+}, e^{-}$beams $P_{1}, P_{2}$ and the collision energy $\sqrt{s}$. We also show that the advantageous direction to collect Higgs is perpendicular to the direction of the initial $e^{-}$beam. With the high integrated luminosity, the number of events in a year with some different values of the collision energy may give observable values at the future accelerators.

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