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LEPTOGENESIS IN A_4 FLAVOR SYMMETRY MODEL BY RENORMALIZATION GROUP EVOLUTION

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Abstract. *We study how leptogenesis can be implemented in the seesaw models with A_4 flavor symmetry, which lead to the tri-bimaximal neutrino mixing matrix. By considering renormalzation group evolution from a high energy scale of flavor symmetry breaking (the GUT scale is assumed) to the low energy scale of relevant phenomena, the off-diagonal terms in a combination of Dirac Yukawa-coupling matrix can be generated. As a result, the flavored leptogenesis is successfully realized. We also investigate how the effective light neutrino mass $|\langle m_{ee}\rangle|$ associated with neutrinoless double beta decay can be predicted by imposing the experimental data on the low energy observables. We find a link between the leptogenesis and the neutrinoless double beta decay characterized by $|\langle m_{ee}\rangle|$ through a high energy CP phase ϕ, which is correlated with the low energy Majorana CP phases. It is shown that the predictions of $|\langle m_{ee}\rangle|$ for some fixed parameters of the high energy physics can be constrained by the current observation of baryon asymmetry.*

Keywords: seesaw mechanism, leptogenesis, renormalization group.

I. INTRODUCTION

The evidence of neutrino oscillations absolutely confirmed that neutrinos have tiny mass and they are mixing. Based on neutrino experimental data, in 2002, P. F. Harrison et al. [1] proposed the structure of lepton mixing matrix which named Tri-bimaximal (TBM). According to this structure, the reactor mixing angle, θ_{13}, is zero and the Dirac CP violating phase is also absent. Subsequently, there were a lot of efforts to find a natural model that leads to TBM mixing pattern of leptons, and a fascinating way seems to be the use of some discrete non-Abelian flavor groups added to the gauge groups of the Standard Model (SM). There is a series of models based on the symmetry group A_4 [2,3], T' [4], and S_4 [5]. The common feature of these models is that they are realized at very high energy scale Λ and the groups are spontaneously broken due to a set of Higgs and scalar multiplets - the flavons. Based on the latest results of T2K [6], MINOS [7], RENO [8], Double CHOOZ [9] and Daya Bay [10] experiments, the newest values of lepton mixing angles are established where the reactor mixing angle is relatively large [11], $\theta_{13} \sim 8^\circ$. This leads to the necessary of re-evaluating the mentioned models in order to fit with the newest experimental results.

The tiny values of neutrino masses can be solved by seesaw mechanism [12]. In addition to the explanation of smallness of observed neutrino masses, the seesaw has another appearing feature so-called leptogenesis for generating the observed Baryon Asymmetry of the Universe (BAU) through the decay of heavy right-handed neutrinos (RHN) [13]. If the BAU was made via leptogenesis, then the CP violation in leptonic sector is required. For the Majorana neutrinos of three flavors, there are one Dirac-type phase and two Majorana-type phases, one (or a combination) of which in principle can be measured through neutrinoless double beta ($0\nu2\beta$) decays [14]. The exact TBM pattern forbids at low energy the CP violation in neutrino oscillations, due to $U_{e3} = 0$. Therefore, any observation of the leptonic CP violation, for instance in the $0\nu2\beta$ decay, can strengthen our belief in the leptogenesis by demonstrating that the CP is not a symmetry of leptons. It is interesting to explore this existence of the CP violation due to the Majorana CP-violating phases by measuring $|\langle m_{ee}\rangle|$ and examine a link between observable low-energy $0\nu2\beta$ decay and the BAU. The authors in Ref. [3] have shown that the TBM pattern can be generated naturally in the framework of the seesaw mechanism with $SU(2)_L \times U(1)_Y \times A_4$ symmetry. The textures of mass matrices as given in [3] also could not generate a lepton asymmetry which is essential for the leptogenesis. In this work, we investigate possibility of radiative leptogenesis when renormalization group (RG) effects are taken into account as well as the effects of RG on the reactor mixing angle θ_{13}. We will show that the leptogenesis can be linked to the $0\nu2\beta$ decay through the seesaw mechanism.

The rest of this work is organized as follows. Section II is devoted to review the model as well as to analysis the low energy observables. We especially focus on the effective neutrino mass governing the $0\nu2\beta$ decay. In Sec. III, we study RG effects on the Yukawa couplings matrix of the Dirac neutrino so that the ingredients for leptogenesis become available. The numerical analysis is also given in this section. Finally, our conclusion is given in Sec. IV.

II. OVERVIEW OF THE MODEL

The non-Abelian A_4 is a group of even permutations of 4 objects and has $4!/2 = 12$ elements. The group is generated by two generators S and T satisfying the relations

$$S^2 = (ST)^3 = T^3 = 1. \tag{1}$$

There are three one-dimensional irreducible representations of the group denoted as

$$1: \qquad S = 1,\ T = 1 \tag{2}$$

$$1': \qquad S = 1,\ T = e^{i4\pi/3} \equiv \omega^2 \tag{3}$$

$$1'': \qquad S = 1,\ T = e^{i2\pi/3} \equiv \omega. \tag{4}$$

It is easy to check that there is no two-dimensional irreducible representation of this group. The three-dimensional unitary representations of T and S are given by

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix},\ S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \tag{5}$$

where T has been chosen to be diagonal. The multiplication rules for the singlet and triplet representations correspond to the above basis of two generators T, S are given as

$$1 \times 1 = 1,\ 1' \times 1'' = 1,\ 3 \times 3 = 3 + 3_A + 1 + 1' + 1''. \tag{6}$$

Table 1. Transformation properties of the lepton fields, Higss and flavons.

	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	φ_T	φ_S	ξ
A_4	3	1	$1''$	$1'$	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	1	ω^2	ω^2
$U(1)_{FN}$	0	4	2	0	0	0	-1	0	0	0

For triplets

$$a = (a_1, a_2, a_3),\ b = (b_1, b_2, b_3), \tag{7}$$

one can write

$$1 \equiv (ab) = (a_1b_1 + a_2b_3 + a_3b_2), \tag{8}$$
$$1' \equiv (ab)' = (a_3b_3 + a_1b_2 + a_2b_1), \tag{9}$$
$$1'' \equiv (ab)'' = (a_2b_2 + a_1b_3 + a_3b_1). \tag{10}$$

Note that while 1 remains invariant under the exchange of the second and the third elements of a and b, $1'$ is symmetric under the exchange of the first and the second elements while $1''$ is symmetric under the exchange of the first and the third elements.

$$3 \equiv (ab)_S = \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1), \tag{11}$$
$$3_A \equiv (ab)_A = \frac{1}{3}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1). \tag{12}$$

We will only focus only 3 since the 3_A terms are antisymmetric and hence cannot be used for neutrino mass matrix. In the triplet 3, we can see that the first element has 2-3 exchange symmetry, the second element has 1-2 exchange symmetry while the third element earns 1-3 interchange symmetry.

We consider the seesaw realization of the A_4 model proposed in [3]. In this model, the A_4 is accompanied with cyclic group Z_3 and Froggatt-Nielsen symmetry $U(1)_{FN}$ [15], i.e. $G_f = S_4 \times Z_3 \times U(1)_{FN}$. The matter fields and flavons are given in Table 1. The superpotential of the lepton sector is given as

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_A\xi)(\nu^c\nu^c) + x_B(\varphi_S\nu^c\nu^c) + h.c. + ... \tag{13}$$

where the dots denote high-order contribution, (33) transforms as 1, $(33)'$ transforms as $1'$ and $(33)''$ transforms as $1''$. To keep the superpotential to be commpacted, we omit to write the Higgs and flavon fields $h_{u,d}, \theta$ and the cut-off scale Λ. For instance $y_e e^c(\varphi_T l)$ stands for $y_e e^c(\varphi_T l)h_d\theta^4/\Lambda^5$.

The VEV alignment of flavons is given as [3]

$$\langle\varphi_T\rangle = \begin{pmatrix} \upsilon_T & 0 & 0 \end{pmatrix}^T,\ \langle\varphi_S\rangle = \begin{pmatrix} \upsilon_S & \upsilon_S & \upsilon_S \end{pmatrix}^T,\ \langle\xi\rangle = u. \tag{14}$$

After spontaneous A_4 and electroweak symmetry breaking, the charged lepton mass matrix comes out diagonal

$$m_l = \upsilon_d \frac{\upsilon_T}{\Lambda} \mathrm{Diag.}\begin{pmatrix} y_e & y_\mu & y_\tau \end{pmatrix}, \tag{15}$$

and the neutrino sector gives rise to the following Dirac and Majorana neutrino mass matrices

$$m_\nu^d = xv_u \mathrm{Diag.}\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \quad (16)$$

$$M_R = \begin{pmatrix} A+\frac{2B}{3} & -\frac{B}{3} & -\frac{B}{3} \\ -\frac{B}{3} & \frac{2B}{3} & A-\frac{B}{3} \\ -\frac{B}{3} & A-\frac{B}{3} & \frac{2B}{3} \end{pmatrix} = A\begin{pmatrix} 1+\frac{2ke^{i\phi}}{3} & -\frac{ke^{i\phi}}{3} & -\frac{ke^{i\phi}}{3} \\ -\frac{ke^{i\phi}}{3} & \frac{2ke^{i\phi}}{3} & 1-\frac{ke^{i\phi}}{3} \\ -\frac{ke^{i\phi}}{3} & 1-\frac{ke^{i\phi}}{3} & \frac{2ke^{i\phi}}{3} \end{pmatrix}. \quad (17)$$

where $A = 2x_A u, B = 2x_B v_S, k = |B|/A$ and B is supposed to be the only complex parameter and ϕ is the only physical phase of the lepton sector. The Majorana neutrino mass matrix M_R is diagonalized by TBM mixing matrix:

$$M_R^D = V_R^T M_R V_R = \mathrm{Diag.}\big(M_1, M_2, M_3\big), \quad (18)$$
$$M_1 = A|ke^{i\phi}+1|,\ M_2 = A,\ M_3 = B|ke^{i\phi}-1|,$$

$$V_R = U_{TB}V_P,\ V_P = \mathrm{Diag.}\big(e^{i\gamma_1/2}, 1, e^{i\gamma_2/2}\big), \quad (19)$$

$$\gamma_{1,2} = -\arg(ke^{i\phi} \pm 1), \quad (20)$$

$$U_{TB} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (21)$$

Integrating out the heavy degrees of freedom, we get the effective light neutrino mass matrix, which is given by the seesaw relation, $m_{\rm eff} = -(m_\nu^d)^T M_R^{-1} m_\nu^d$, and diagonalized by the TBM matrix:

$$U_\nu^T m_{\rm eff} U_\nu = \mathrm{Diag.}(m_1, m_2, m_3) = -\mathrm{Diag.}\Big(\frac{x^2v_u^2}{M_1}, \frac{x^2v_u^2}{M_2}, \frac{x^2v_u^2}{M_3}\Big), \quad (22)$$

$$U_\nu = U_{TB}\mathrm{Diag.}(e^{-i\gamma_1/2}, 1, e^{-i\gamma_2/2}). \quad (23)$$

Since the charged lepton mass matrix is diagonal leading to the lepton mixing matrix is the diagonalizing matrix of neutrino mass matrix

$$U_{\rm PMNS} = U_\nu = e^{-i\gamma_1/2}\, U_{\rm TB}\, \mathrm{Diag.}(1, e^{i\beta_1}, e^{i\beta_2}), \quad (24)$$

where $\beta_1 = \gamma_1/2$ and $\beta_2 = (\gamma_1 - \gamma_2)/2$ are Majorana CP violating phases. The phase factored out to the left has no physical meaning, since it can be eliminated by a redefinition of the charged lepton fields. The light neutrino mass eigenvalues are simply the inversions of that of the heavy neutrino ones, a part from a minus sign and the global factor from m_ν^d, as can be seen in Eq. (22). There are nine physical parameters consisting of the three light neutrino masses, three mixing angles and three CP-violating phases, in general. The mixing angles are entirely fixed by the A_4 symmetry group, predicting TBM and in turn no Dirac CP-violating phase. The remaining five physical parameters $\beta_1,\ \beta_2,\ m_1,\ m_2$ and m_3, are determined by the five real parameters $A,\ k,\ v_u,\ x$ and ϕ.

The light neutrino mass spectrum can be both normal hierarchy (NH) or inverted hierarchy (IH) depending on the sign of $\cos\phi$ and the ratio k where $\cos\phi > -k/2$ is required to have $|m_2| > |m_1|$. If $-k/2 < \cos\phi < 0$ one has IH, whereas $k/2 < \cos\phi < 1$ for that of NH.

Because there is no Dirac CP-violating phase as mentioned, the only contribution from the Majorana phases to the $0\nu2\beta$ decay comes from β_1. The effective neutrino mass governing the $0\nu2\beta$ decay is given by

$$|\langle m_{ee}\rangle| \;=\; \frac{1}{3}|2m_1 + m_2 e^{2i\beta_1}|, \tag{25}$$

where using Eq. (20) we can obtain the explicit relation between ϕ and β_1:

$$\sin 2\beta_1 = \frac{-k\sin\phi}{1+2k\cos\phi+k^2}\,. \tag{26}$$

In a basis where the charged current is flavor diagonal and the RHN mass matrix M_R is diagonal and real, the Dirac mass matrix m^d_ν gets modified to

$$m^d_\nu \to Y_\nu \upsilon_u = V_R^T m^d_\nu, \tag{27}$$

where $\upsilon_u = \upsilon\sin\beta$, $\upsilon = 176$ GeV, and then the coupling of N_i with leptons and scalar, Y_ν, is given by

$$Y_\nu \;=\; x e^{i\gamma_1/2}\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \\ \frac{e^{-i\beta_1}}{\sqrt{3}} & \frac{e^{-i\beta_1}}{\sqrt{3}} & \frac{e^{-i\beta_1}}{\sqrt{3}} \\ 0 & \frac{e^{-i\beta_2}}{\sqrt{2}} & \frac{-e^{-i\beta_2}}{\sqrt{2}} \end{pmatrix}. \tag{28}$$

Concerned with the CP violation, we notice that the CP phase ϕ originating from M_R obviously takes part at the low-energy CP violation as the Majorana phases β_1 and β_2. On the other hand, the leptogenesis is associated with both the Yukawa coupling Y_ν and its combination,

$$H \equiv Y_\nu Y_\nu^\dagger = x^2\cdot \mathrm{Diag.}(1,1,1). \tag{29}$$

This directly indicates that all off-diagonal H_{ij} vanish, so the CP asymmetry could not be generated and neither leptogenesis. For the leptogenesis to be viable, the off-diagonal H_{ij} have to be generated.

III. RELEVANT RG EQUATIONS

In the exact A_4 model, the CP asymmetries due to the decay of RHN at leading order is vanished due to the diagonal of Hermitian matrix H, Eq.(29), consequently the leptogenesis could not take place. The radiative effects due to RG running from a high to low energy scale can naturally lead to an enhancement in vanished off-diagonal terms of H, which are necessary ingredients for a successful leptogenesis mechanism.

The radiative behavior of heavy RH-Majorana mass matrix $\mathbf{M}_R$ is dictated by the following RG equation [16]:

$$\frac{d\mathbf{M}_R}{dt} \;=\; 2[(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)\mathbf{M}_R + \mathbf{M}_R(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)^T], \tag{30}$$

where $t = \frac{1}{16\pi^2}\ln(M/\Lambda')$, and M is an arbitrary renormalization scale. The cutoff scale Λ' can be regarded as the G_f breaking scale $\Lambda' = \Lambda$ and assumed to be in order of the GUT scale, $\Lambda' \sim 10^{16}$ GeV. The RG equation for the Dirac neutrino Yukawa coupling can be written as

$$\frac{d\mathbf{Y}_\nu}{dt} \;=\; \mathbf{Y}_\nu[(T - 3g_2^2 - \frac{3}{5}g_1^2) + \mathbf{Y}_l^\dagger\mathbf{Y}_l + 3\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\,, \tag{31}$$

where $T = Tr(3Y_u^\dagger Y_u + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)$, Y_u and $\mathbf{Y}_l$ are the Yukawa couplings of up-type quarks and charged leptons and $g_{2,1}$ are the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge coupling constants, respectively.

Let us first reformulate (30) in the basis where $\mathbf{M}_R$ is diagonal. Since $\mathbf{M}_R$ is symmetric, it can be diagonalized by a unitary matrix V_R as mentioned,

$$V_R^T \mathbf{M}_R V_R = \mathrm{Diag.}(M_1, M_2, M_3). \tag{32}$$

As the structure of $\mathbf{M}_R$ changes with the evolution of the scale, the V_R depends on the scale too. The RG evolution of $V_R(t)$ can be written as

$$\frac{d}{dt} V_R = V_R A, \tag{33}$$

where A is an anti-Hermitian matrix $A^\dagger = -A$ due to the unitary of V_R. Differentiating (32) we obtain

$$\begin{aligned} \frac{dM_i \delta_{ij}}{dt} &= A_{ij}^T M_j + M_i A_{ij} + 2\{V_R^T[(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)\mathbf{M}_R \\ &\quad + \mathbf{M}_R(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)^T] V_R\}_{ij} \, . \end{aligned} \tag{34}$$

Absorbing the unitary factor into the Dirac Yukawa coupling $Y_\nu \equiv V_R^T \mathbf{Y}_\nu$, the real diagonal part of (34) becomes

$$\frac{dM_i}{dt} = 4 M_i (Y_\nu Y_\nu^\dagger)_{ii}. \tag{35}$$

The RG equation for Y_ν in the basis of diagonal $\mathbf{M}_R$ is given by

$$\begin{aligned} \frac{dY_\nu}{dt} &= Y_\nu[(T - 3g_2^2 - \frac{3}{5}g_1^2) + \mathbf{Y}_l^\dagger \mathbf{Y}_l + 3Y_\nu^\dagger Y_\nu)] \\ &\quad + A^T Y_\nu. \end{aligned} \tag{36}$$

Finally, we obtain the RG equation for H responsible for the leptogenesis:

$$\begin{aligned} \frac{dH}{dt} &= 2\big(T - 3g_2^2 - \frac{3}{5}g_1^2\big)H + 2Y_\nu(\mathbf{Y}_l^\dagger \mathbf{Y}_l)Y_\nu^\dagger \\ &\quad + 6H^2 + A^T H + HA^*, \end{aligned} \tag{37}$$

where H is defined in (29).

Notice that a nonvanishing CP asymmetry requires $\mathrm{Im}[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)_{j\alpha}^*] \neq 0$ with Y_ν defined in (28). Therefore, to have a viable radiative leptogenesis we need to induce a nonvanishing $H_{ij}(i \neq j)$ at the leptogenesis scale. Indeed, this is possible since the RG effects due to the τ-Yukawa coupling contribution imply at the leading order yields [17]

$$H_{ij}(t) \simeq 2y_\tau^2 (Y_\nu)_{i3}(Y_\nu)_{j3}^* \times t. \tag{38}$$

The flavored CP asymmetries ε_i^α can then be obtained from (28), (29), (38) and (39).

IV. Radiatively-induced Flavored Leptogenesis

As already noticed, the leptogenesis cannot be realized in the A_4 models at the leading order, so this section is devoted to study the flavored leptogenesis with the effects of RG evolution. The lepton asymmetries, which are produced by out-of-equilibrium decays of heavy RHNs in early Universe at temperatures above $T \sim (1+\tan^2\beta)\times 10^{12}$ GeV, do not distinguish among lepton flavors, called conventional or unflavored leptogenesis. However, if the scale of RHN masses are about $M \leq (1+\tan^2\beta)\times 10^{12}$ GeV, we need to take into account lepton flavor effects, called flavored leptogenesis. In this case, the CP asymmetry as generated by the decay of *i*-th heavy RHN far from almost degenerate is given by [18, 19]

$$\varepsilon_i^\alpha \quad = \quad \frac{1}{8\pi H_{ii}}\sum_{j\neq i}\mathrm{Im}\Big[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)^*_{j\alpha}\Big]g\Big(\frac{M_j^2}{M_i^2}\Big), \tag{39}$$

where Y_ν and $H = Y_\nu Y_\nu^\dagger$ are in the basis where M_R is real and diagonal. Here the loop function $g(M_j^2/M_i^2)$ is

$$g\Big(\frac{M_j^2}{M_i^2}\Big) \quad \equiv \quad g_{ij}(x) = \sqrt{x}\Big[\frac{2}{1-x} - \ln\frac{1+x}{x}\Big]. \tag{40}$$

As reminded in the previous section, by properly taking into account the RG effects, the non-zero flavored CP asymmetries ε_i^α as given above can be obtained. Once the initial values of ε_i^α are fixed, the final result of BAU, η_B, can be given by solving a set of flavor dependent Boltzmann equations including the decay, inverse decay, and scattering processes as well as the nonperturbative sphaleron interaction. In order to estimate the wash-out effects, we introduce parameters K_i^α which are the wash-out factors due to the inverse decay of Majorana neutrino N_i into the lepton flavor α. The explicit form of K_i^α is given by

$$K_i^\alpha = \frac{\Gamma_i^\alpha}{H(M_i)} = (Y_\nu^\dagger)_{\alpha i}(Y_\nu)_{i\alpha}\frac{\upsilon_u^2}{m_* M_i}, \tag{41}$$

where Γ_i^α is the partial decay width of N_i into the lepton flavors and Higgs scalars, $H(M_i) \simeq (4\pi^3 g_*/45)^{\frac{1}{2}} M_i^2/M_{Pl}$, with the Planck mass $M_{Pl} = 1.22\times 10^{19}$ GeV and the effective number of degrees of freedom $g_* \simeq 228.75$, is the Hubble parameter at temperature $T = M_i$, and the equilibrium neutrino mass $m_* \simeq 10^{-3}$. From (28), and (41) we can obtain the washout parameters corresponding to the model.

Each lepton asymmetry for a single flavor ε_i^α is weighted differently by the corresponding washout parameter K_i^α, appearing with a different weight in the final formula for the baryon asymmetry [20],

$$\eta_B \simeq -10^{-2}\sum_{N_i}\left[\varepsilon_i^e\kappa\Big(\frac{93}{110}K_i^e\Big) + \varepsilon_i^\mu\kappa\Big(\frac{19}{30}K_i^\mu\Big) + \varepsilon_i^\tau\kappa\Big(\frac{19}{30}K_i^\tau\Big)\right], \tag{42}$$

provided that the scale of heavy RH neutrino masses is about $M \leq (1+\tan^2\beta)\times 10^9$ GeV where the μ and τ Yukawa couplings are in equilibrium and all the flavors are to be treated separately.

And

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[\varepsilon_i^2 \kappa\Big(\frac{541}{761} K_i^2\Big) + \varepsilon_i^\tau \kappa\Big(\frac{494}{761} K_i^\tau\Big)\right] \quad (43)$$

is given if $(1+\tan^2\beta)\cdot 10^9$ GeV $\leq M_i \leq (1+\tan^2\beta)\cdot 10^{12}$ GeV where only the τ Yukawa coupling is in equilibrium and treated separately while the e and μ flavors are indistinguishable; here $\varepsilon_i^2 = \varepsilon_i^e + \varepsilon_i^\mu$, $K_i^2 = K_i^e + K_i^\mu$.

In the above expressions, the wash-out factors, $\kappa \equiv \kappa_i^\alpha$, are given by

$$\kappa_i^\alpha \simeq \left(\frac{8.25}{K_i^\alpha} + \Big(\frac{K_i^\alpha}{0.2}\Big)^{1.16}\right)^{-1}. \quad (44)$$

In the considering model, the RHN masses are strongly hierarchy. For the IH case of light neutrino masses, the lightest RHN mass is M_2, then the leptogenesis is governed by the decay of the second generation of RHN neutrino, N_2. The flavored CP asymmetries ε_2^α are obtained as

$$\begin{aligned}
\varepsilon_2^e &\simeq \frac{y_\tau^2 x^2}{36\pi} \sin 2\beta_1 \cdot g_{21} \cdot t, \\
\varepsilon_2^\mu &\simeq \frac{y_\tau^2 x^2}{24\pi} \Big(-\frac{1}{3}\sin 2\beta_1 \cdot g_{21} + \sin 2(\beta_1 - \beta_2)\cdot g_{23}\Big)\cdot t, \\
\varepsilon_2^\tau &\simeq -\frac{y_\tau^2 x^2}{24\pi} \Big(\frac{1}{3}\sin 2\beta_1 \cdot g_{21} + \sin 2(\beta_1 - \beta_2)\cdot g_{23}\Big)\cdot t,
\end{aligned} \quad (45)$$

with corresponding washout parameters

$$K_2^e = K_2^\mu = K_2^\tau = \frac{m_0}{3m_*}, \quad (46)$$

where $m_0 = x^2 \upsilon_u^2/A$.

Now we want to see numerically how the parameters of the model are constrained by experimental data as well as the effects of RG on leptogenesis and on θ_{13}. To do so, we use the latest data given in the reference [11] at 3σ C.L. for numerical analysis (with an exception that

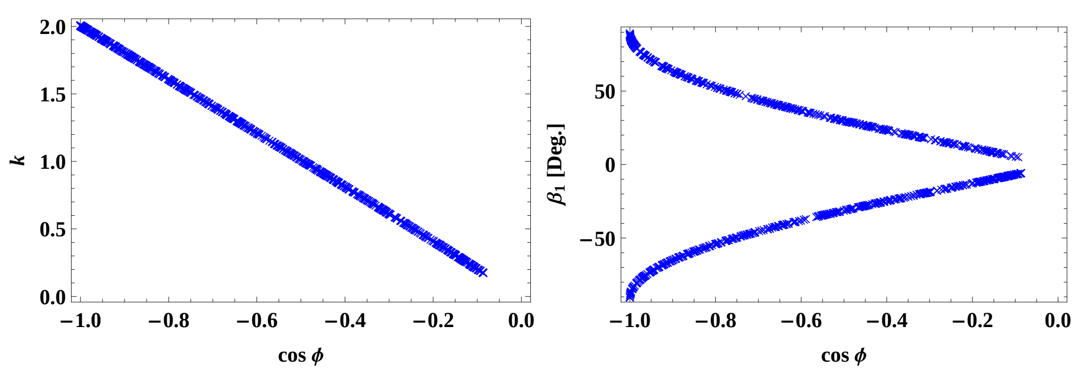

Fig. 1. The allowed parameters $k, \cos\phi$ (left panel) and the relation between the CP phase β_1 and $\cos\phi$ (right panel) by the 3σ experimental constrain (47) (right panel).

$0 \leq \theta_{13} \leq 12^\circ$ is used):

$$31.3^\circ \leq \theta_{12} \leq 37.5^\circ\ ,\ 37^\circ \leq \theta_{23} \leq 55.5^\circ\ ,\ 7.5^\circ \leq \theta_{13} \leq 10.5^\circ\ ,$$
$$7.12 \leq \Delta m^2_{21}[10^{-5}\text{eV}^2] \leq 8.20\ ,\ 2.31(2.21) \leq |\Delta m^2_{31}|[10^{-3}\text{eV}^2] \leq 2.74(2.64), \quad (47)$$

where numbers inside the parentheses correspond to the IH of neutrino masses. (And the Dirac CP violating phase, $\delta_{\rm CP}$, is unrestricted at 3σ C.L). The scale of RHN mass $A = M_2 = 10^{12}$ GeV, the supersymmetric parameter $\tan\beta = 30$ are used as inputs. The overall factor x of Dirac neutrino coupling Y_ν can be estimated by $x^2 \simeq \frac{A\sqrt{|\Delta m^2_{31}|}}{v_u^2}$

The correlation between k and $\cos\phi$ is presented in the left panel of Fig. 1. Whereas in the right panel, we show the relation between $\cos\phi$ and the Majorana CP violating phase β_1. The behavior of $|\langle m_{ee}\rangle|$ as a function of $\cos\phi$ is plotted in the left panel of Fig. 2. The horizontal line (0.2 eV) is the current lower bound sensitivity [21] while the dashed-line (10^{-2} eV) is a future sensitivity [22].

For the flavored leptogenesis, the prediction of η_B is shown in the right panel of Fig. 2 as a function of $|\langle m_{ee}\rangle|$. The solid horizontal line and the dotted horizontal lines respectively corresponds to the experimental value of baryon asymmetry [23], $\eta_B^{\rm CMB} = 6.1 \times 10^{-10}$, and phenomenologically allowed regions $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$.

As seen in Fig. 2, the current observation of $\eta_B^{\rm CMB}$ can narrowly constrain the value of $|\langle m_{ee}\rangle|$. Then, combining the result in the left panel of Fig. 2, we can narrowly constrain the value of $\cos\phi$. Finally, we can pin down the Majorana CP phase β_1 via cosine of high energy phase ϕ via their relation in Fig. 1 (right panel).

V. CONCLUSIONS

We have studied the A_4 model in the context of a supersymmetric seesaw model which naturally lead to the TBM form for the lepton mixing matrix in which the reactor mixing angle, θ_{13}, is zero. In this model, the combination $Y_\nu Y_\nu^\dagger$ is proportional to unity this would forbid the desirable leptogenesis to occur. Therefore, for a viable leptogenesis the off-diagonal terms of

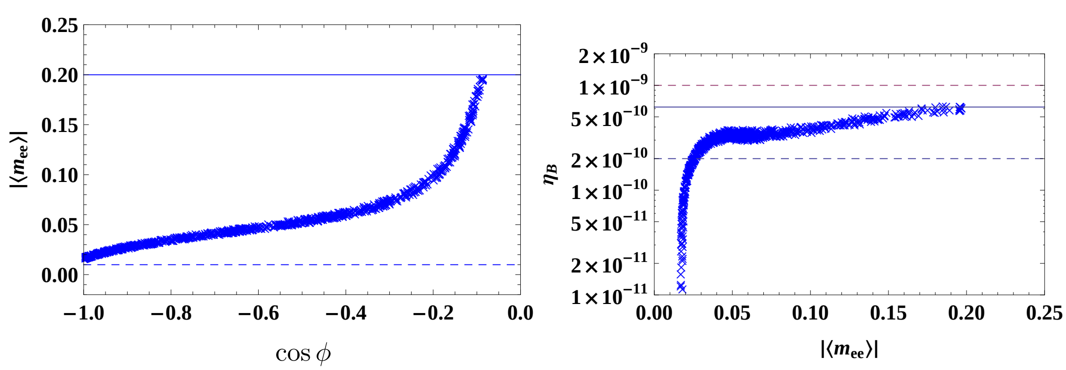

Fig. 2. The predictions of effective mass $|\langle m_{ee}\rangle|$ responsible for $0\nu2\beta$ decay as a function of $\cos\phi$ by the 3σ experimental constrain (47) (left panel) and the predictions η_B as a function of $|\langle m_{ee}\rangle|$.

$Y_\nu Y_\nu^\dagger$ have to be generated. We have shown that these terms can be easily achieved by the RG effects from a high energy scale to the low energy scale which result in the successful leptogenesis.

We have also studied implications to the low-energy observables such as the $0\nu\beta\beta$ decay. It gives the definite predictions for $0\nu 2\beta$ decay parameter $|\langle m_{ee}\rangle|$. Interestingly we have found a link between the leptogenesis and amplitude of $0\nu 2\beta$ decay $|\langle m_{ee}\rangle|$ through a high energy CP phase ϕ. We have shown how the high energy CP phase ϕ is correlated to the low energy Majorana CP phase β_1, and examined how the leptogenesis can be related with the $0\nu 2\beta$ decay. It is pointed out that the predictions of $|\langle m_{ee}\rangle|$ for the IH spectrum can be constrained by the current observation of the baryon asymmetry of the universe as 6.1×10^{-10}.

We have also studied the effects of RG on the mixing angle θ_{13}. The result comes out that the radiatively generated of this angle (and other mixing angles) is negligible. It is also to remark that this work study numerically for the case of IH spectrum of light neutrino mass, it could be similarly studied for the case of NH spectrum.

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