Communications in Physics, Vol. 30, No. 3 (2020), pp. 221-230
DOI:10.15625/0868-3166/30/3/14963

ANOMALOUS MAGNETIC DIPOLE MOMENT $(g-2)_\mu$ IN A 3-3-1 MODEL WITH INVERSE SEESAW NEUTRINOS

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Received 7 April 2020
Accepted for publication 31 May 2020
Published 24 July 2020

Abstract. *We will show that the recent experimental value of the anomalous magnetic moment (AMM) of the charged lepton μ, denoted as $a_\mu \equiv (g-2)_\mu/2$, can be explained successfully in a 3-3-1 model with right handed neutrinos adding new heavy $SU(3)_L$ neutrino singlets. Allowed regions satisfying the recent AMM data are illustrated numerically.*

Keywords: 3-3-1 model, inverse seesaw, anomalous magnetic dipole moment.

Classification numbers: 12.60.-i, 14.60.Pq, 14.60.Ef.

I. INTRODUCTION

The well-known 3-3-1 model with right handed neutrinos (331RN) was introduced [1] not long after the appearance of the minimal 3-3-1 version [2]. Phenomenology of the 3-3-1 models is very interesting because it can explain the recent experimental data of neutrino oscillation [3], including the study of the AMM [4–12]. Theoretical and experimental aspects of the AMM were reviewed in detailed in Refs. [3, 9]. It was concerned recently [10, 12] that many 3-3-1models can not explain the recent experimental data of a_μ under the constraint of the symmetry breaking $SU(3)_L$ scale obtained by searching for the neutral heavy gauge boson Z' at LHC. Hence these 3-3-1 models should be extended. In this work, one-loop contributions to a_μ predicted by simple extended 331RN models, which contain heavy gauge singlet neutrinos needed for generating active neutrino masses through the inverse seesaw (ISS) mechanism [13–16]. In particular, the model (331ISS) introduced in Ref. [16] will be chosen for studying the effects of ISS neutrinos on one loop contributions to a_μ. Different from the simple Higgs potential chosen in Ref. [16], a more general one will be considered in this work [17, 18], where $\sqrt{2}\langle\rho^0\rangle \equiv v_1 \neq v_2 \equiv \sqrt{2}\langle\eta^0\rangle$. This leads to the constraint that $t_\beta \equiv \langle\eta^0\rangle/\langle\rho^0\rangle \geq 1/3$ because η^0 generates the top quark mass at tree level, $m_t \sim h_t v_2/\sqrt{2}$ and $v_1^2+v_2^2 = v^2 = (246)^2$ GeV^2 [19, 20]. We remind that Ref. [16] considered only the special case of $t_\beta = 1$. As we will show, the parameter t_β is very important for getting large a_μ satisfying the experimental data.

Our work is arranged as follows. Sec. II will summary the particle content as well as masses and mixing of all physical states appearing in the 331ISS model. Sec. III will show the analytical formulas for one-loop contributions to a_μ predicted by the 331ISS model. Sec. IV provides numerical results to illustrate the allowed regions of parameter space satisfying the experimental data of a_μ. Finally, our conclusions will be presented in Sec. V.

II. THE MODEL AND PARTICLE SPECTRUM

First, we will summary the particle content of the 331ISS model [16] where active neutrino masses and oscillations are originated from the ISS mechanism. The quark sector and $SU(3)_C$ representations are irrelevant in this work, and hence they are omitted here. The electric charge operator corresponding to the gauge group $SU(3)_L\times U(1)_X$ is $Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X$, where $T_{3,8}$ are diagonal $SU(3)_L$ generators. Each lepton family consists of a $SU(3)_L$ triplet $\psi_{aL} = (\nu_a,\ e_a, N_a)_L^T \sim (3,-\frac{1}{3})$ and a right-handed charged lepton $e_{aR} \sim (1,-1)$ with $a = 1,2,3$. Each left-handed neutrino $N_{aL} = (N_{aR})^c$ implies a new right-handed neutrino beyond the SM. The only difference from the usual 331RN model is that, the 331ISS model contains three right-handed neutrinos which are gauge singlets, $X_{aR} \sim (1,0)$, $a = 1,2,3$. The three Higgs triplets are $\rho = (\rho_1^+,\ \rho^0,\ \rho_2^+)^T \sim (3,\frac{2}{3})$, $\eta = (\eta_1^0,\ \eta^-, \eta_2^0)^T \sim (3,-\frac{1}{3})$, and $\chi = (\chi_1^0,\ \chi^-,\chi_2^0)^T \sim (3,-\frac{1}{3})$. The necessary vacuum expectation values for generating all tree-level quark masses and leptons are $\langle\rho\rangle = (0,\ \frac{v_1}{\sqrt{2}}, 0)^T$, $\langle\eta\rangle = (\frac{v_2}{\sqrt{2}}, 0, 0)^T$ and $\langle\chi\rangle = (0,\ 0,\ \frac{w}{\sqrt{2}})^T$.

Gauge bosons in this model get masses through the covariant kinetic term of the Higgs bosons, $\mathscr{L}^H = \sum_{H=\chi,\eta,\rho}\left(D_\mu H\right)^\dagger\left(D^\mu H\right)$, where the covariant derivative for the electroweak symmetry is defined as $D_\mu = \partial_\mu - igW_\mu^a T^a - g_X T^9 X_\mu$, $a = 1,2,..,8$. Note that $T^9 \equiv \frac{I_3}{\sqrt{6}}$ and $\frac{1}{\sqrt{6}}$ for (anti)triplets and singlets [21]. It can be identified that $g = e/s_W$ and $\frac{g_X}{g} = \frac{3\sqrt{2}s_W}{\sqrt{3-4s_W^2}}$, where e and s_W are, respectively, the electric charge and sine of the Weinberg angle, $s_W^2 \simeq 0.231$.

As the usual 331RN model, the 331ISS model includes two pairs of singly charged gauge bosons, denoted as $W^\pm$ and $Y^\pm$, defined as

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}},\ Y_\mu^\pm = \frac{W_\mu^6 \pm iW_\mu^7}{\sqrt{2}},\ m_W^2 = \frac{g^2}{4}\left(v_1^2+v_2^2\right),\ m_Y^2 = \frac{g^2}{4}\left(w^2+v_1^2\right). \tag{1}$$

The bosons $W^\pm$ are identified with the SM ones, leading to the consequence obtained from experiments that

$$v_1^2+v_2^2 \equiv v^2 = (246\text{GeV})^2. \tag{2}$$

Different from Ref. [16], where $v_1 = v_2$ were assumed so that the Higgs potential given in Refs. [22, 23] was used to find the exact physical state of the SM-like Higgs boson, the general Higgs potential relating with the 331RN model will be applied in our work. The reason is that the physical states of the charged Higgs bosons are determined analytically from this Higgs potential, and only these Higgs bosons contribute significantly to one-loop corrections to the $(g-2)_\mu$. We will use the following parameters for this general case,

$$t_\beta \equiv \tan\beta = \frac{v_2}{v_1}, \quad v_1 = vc_\beta, \quad v_2 = vs_\beta, \tag{3}$$

The Higgs potential used here respects the new lepton number defined in Ref. [17], namely

$$\begin{aligned} V_h = &\sum_{S=\eta,\rho,\chi}\left[\mu_S^2 S^\dagger S + \lambda_S\left(S^\dagger S\right)^2\right] + \lambda_{12}(\eta^\dagger\eta)(\rho^\dagger\rho) + \lambda_{13}(\eta^\dagger\eta)(\chi^\dagger\chi) + \lambda_{23}(\rho^\dagger\rho)(\chi^\dagger\chi) \\ &+ \tilde{\lambda}_{12}(\eta^\dagger\rho)(\rho^\dagger\eta) + \tilde{\lambda}_{13}(\eta^\dagger\chi)(\chi^\dagger\eta) + \tilde{\lambda}_{23}(\rho^\dagger\chi)(\chi^\dagger\rho) + \sqrt{2}f\left(\epsilon_{ijk}\eta^i\rho^j\chi^k + \text{h.c.}\right), \end{aligned} \tag{4}$$

where f is a mass parameter. The model contains two pairs of singly charged Higgs bosons $H^\pm_{1,2}$ and Goldstone bosons of the gauge bosons $W^\pm$ and $Y^\pm$, which are denoted as $G^\pm_W$ and $G^\pm_Y$, respectively. The masses of all charged Higgs bosons are [21,24,25] $m^2_{H_1^\pm} = (v_1^2 + w^2)\left(\frac{\tilde{\lambda}_{23}}{2} - \frac{f}{w}t_\beta\right)$, $m^2_{H_2^\pm} = \left(\frac{\tilde{\lambda}_{12}v^2}{2} - \frac{fw}{s_\beta c_\beta}\right)$, and $m^2_{G^\pm_W} = m^2_{G^\pm_Y} = 0$. The relations between the original and mass eigenstates of the charged Higgs bosons are [25]

$$\begin{pmatrix} \rho_1^\pm \\ \eta^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} G_W^\pm \\ H_2^\pm \end{pmatrix}, \quad \begin{pmatrix} \rho_2^\pm \\ \chi^\pm \end{pmatrix} = \begin{pmatrix} -s_\theta & c_\theta \\ c_\theta & s_\theta \end{pmatrix}\begin{pmatrix} G_Y^\pm \\ H_1^\pm \end{pmatrix}, \tag{5}$$

where $t_\theta = v_1/w$.

The Yukawa Lagrangian for generating lepton masses is:

$$\mathscr{L}_l^{\mathrm{Y}} = -h^e_{ab}\overline{\psi_{aL}}\rho e_{bR} + h^\nu_{ab}\epsilon^{ijk}\overline{(\psi_{aL})_i}(\psi_{bL})^c_j\rho^*_k - Y_{ab}\overline{\psi_{aL}}\chi X_{bR} - \frac{1}{2}(\mu_X)_{ab}\overline{(X_{aR})^c}X_{bR} + \text{H.c.}. \tag{6}$$

In the basis $\nu'_L = (\nu_L, N_L, (X_R)^c)^T$ and $(\nu'_L)^c = ((\nu_L)^c, (N_L)^c, X_R)^T$, Lagrangian (6) gives a neutrino mass term corresponding to a block form of the mass matrix [16], namely

$$-\mathscr{L}^\nu_{\text{mass}} = \frac{1}{2}\overline{\nu'_L}M^\nu(\nu'_L)^c + \text{H.c., where} \quad M^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix}, \tag{7}$$

where M_R is a 3×3 matrix $(M_R)_{ab} \equiv Y_{ab}\frac{w}{\sqrt{2}}$ with $a,b = 1,2,3$. Neutrino sub-bases are denoted as $\nu_R = ((\nu_{1L})^c, (\nu_{2L})^c, (\nu_{3L})^c)^T$, $N_R = ((N_{1L})^c, (N_{2L})^c, (N_{3L})^c)^T$, and $X_L = ((X_{1R})^c, (X_{2R})^c, (X_{3R})^c)^T$. In the model under consideration, the Dirac neutrino mass matrix m_D must be antisymmetric. The precise form of this matrix will be determined numerically.

The mass matrix M^ν is diagonalized by a 9×9 unitary matrix U^ν,

$$U^{\nu T}M^\nu U^\nu = \hat{M}^\nu = \text{diag}(m_{n_1}, m_{n_2}, ..., m_{n_9}) = \text{diag}(\hat{m}_\nu, \hat{M}_N), \tag{8}$$

where m_{n_i} $(i = 1,2,...,9)$ are masses of the nine physical neutrinos states n_{iL}, namely $\hat{m}_\nu = \text{diag}(m_{n_1},\ m_{n_2},\ m_{n_3})$ corresponding to the three active neutrinos n_{aL} $(a = 1,2,3)$, and $\hat{M}_N =$

diag$(m_{n_4},\, m_{n_5}, ...,\, m_{n_9})$ corresponding the six extra neutrinos n_{IL} $(I = 4, 5, .., 9)$. The ISS mechanism leads to the following approximation solution of U^ν,

$$U^\nu = \Omega \begin{pmatrix} U & \mathbf{O} \\ \mathbf{O} & V \end{pmatrix}, \quad \Omega = \exp \begin{pmatrix} \mathbf{O} & R \\ -R^\dagger & \mathbf{O} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}RR^\dagger & R \\ -R^\dagger & 1 - \frac{1}{2}R^\dagger R \end{pmatrix} + \mathscr{O}(R^3), \tag{9}$$

where U^ν and Ω are 9×9 matrices; $U \equiv U_{\text{PMNS}}$ is the well-known 3×3 matrix determined from the experiment of neutrino oscillation; V is a 6×6 matrix; and R is a 3×6 matrix satisfying the ISS condition that $\max|R_{ij}| \ll 1$. There are three zero matrices $\mathbf{O}$ have orders 3×6, 3×3, and 6×6. The ISS relations are

$$R^* \simeq \left(-m_D M^{-1}, \quad m_D (M_R^T)^{-1}\right), \quad M \equiv M_R \mu_X^{-1} M_R^T, \tag{10}$$

$$m_D M^{-1} m_D^T \simeq m_\nu \equiv U^*_{\text{PMNS}} \hat{m}_\nu U^\dagger_{\text{PMNS}}, \quad V^* \hat{M}_N V^\dagger \simeq M_N + \frac{1}{2} R^T R^* M_N + \frac{1}{2} M_N R^\dagger R, \tag{11}$$

where

$$M_N \equiv \begin{pmatrix} \mathbf{O} & M_R \\ M_R^T & \mu_X \end{pmatrix}.$$

The relations between the flavor and mass eigenstates are

$$\nu'_L = U^{\nu *} n_L, \quad \text{and } (\nu'_L)^c = U^\nu (n_L)^c, \tag{12}$$

where $n_L \equiv (n_{1L}, n_{2L}, ..., n_{9L})^T$ and $(n_L)^c \equiv ((n_{1L})^c, (n_{2L})^c, ..., (n_{9L})^c)^T$. In this work, we will consider the normal order of the neutrino data given in [3]. The best-fit values are

$$\Delta m^2_{21} = 7.370 \times 10^{-5} \text{ eV}^2,\ \Delta m^2 = 2.50 \times 10^{-3} \text{ eV}^2,\ s^2_{12} = 0.297,\ s^2_{23} = 0.437,\ s^2_{13} = 0.0214,$$

where $\Delta m^2_{21} = m^2_{n_2} - m^2_{n_1}$ and $\Delta m^2 = m^2_{n_3} - \frac{\Delta m^2_{21}}{2}$. The detailed calculation shown in Ref. [16], using the ISS relations and the experimental data, gives

$$m_D \simeq z \times \begin{pmatrix} 0 & 0.545 & 0.395 \\ -0.545 & 0 & 1 \\ -0.395 & -1 & 0 \end{pmatrix},$$

where $z = \sqrt{2} v_1 h^\nu_{23}$. The perturbative limit requires that $h^\nu_{23} < \sqrt{4\pi}$, leading to the following upper bound of z,

$$z < \frac{1233 \text{ [GeV]}}{t_\beta}. \tag{13}$$

For simplicity in the numerical study, we will consider the diagonal matrix M_R in the degenerate case $M_R = M_{R_1} = M_{R_2} = M_{R_3} \equiv k \times z$. The parameter k will be fixed at small values that result in large effects on a_μ, namely $k \geq 5.5$ so that the exact numerical values of active neutrino masses are consistent with experimental data of the neutrino oscillation [16]. The choice of k and z as free parameters has an advantage mentioned in Ref. [16] that we can find numerically the eigenvalues $\hat{M}^\nu$ and the mixing matrix U^ν using the total neutrino mass matrix M^ν once z and k are fixed; and μ_X is assumed to be written as a function of M_R, m_D, U_{PMNS} and active neutrinos masses from the ISS relations given in Eq. (10) and (11). These results are generally different from the best-fit values used as inputs in this work, because the ISS relations are the approximate formulas to determine the U^ν at the order $\mathscr{O}(R^2)$. These formulas only work if $\max(R_{ij}) \sim m_D(M_R^T)^{-1} \sim 1/k \ll 1$. Our numerical investigation shows that when $k \geq 5.5$, the active neutrino masses in $\hat{M}^\nu$ lie in the 3σ allowed ranges of the experimental neutrino data. Regarding to μ_X, it can be seen from the ISS relations that $\mu_X \sim k^2 \hat{m}_\nu$, which is small enough so that the ISS mechanism works: $|\mu_X| \ll |m_D| \ll |M_R|$. The heavy neutrino masses are nearly degenerate and approximately equal to $M_R = k \times z$. The mixing matrix is estimated by diagonalizing the matrix M_N in the limit $\mu_X \simeq 0$.

All detailed steps for calculation to derive the couplings that give large one-loop contributions to a_μ are exactly the same as the couplings presented in Ref. [16], which relate to the lepton flavor violating decays $e_b \to e_a \gamma$, we therefore will not repeat here. We just give the final results with $t_\beta \neq 1$.

III. ANALYTIC FORMULAS OF a_μ

In general, Lagrangian of charged gauge bosons contributing to a_{e_a} with $e_a = e, \mu, \tau$ is

$$\mathscr{L}^{\ell n V} = \overline{\psi_{aL}} \gamma^\mu D_\mu \psi_{aL} \supset \frac{g}{\sqrt{2}} \sum_{i=1}^{9} \sum_{a=1}^{3} \left[U^\nu_{ai} \overline{n_i} \gamma^\mu P_L e_a W^+_\mu + U^\nu_{(a+3)i} \overline{n_i} \gamma^\mu P_L e_a Y^+_\mu \right], \tag{14}$$

leading to the following contributions to the a_μ [26]:

$$a^V_{e_a} \equiv -\frac{4 m_{e_a}}{e} \left(\Re[c^W_{aR}] + \Re[c^Y_{aR}] \right) = a^W_{e_a} + a^Y_{e_a},$$
$$c^W_{aR} = \frac{e g^2 m_{e_a}}{32 \pi^2 m_W^2} \sum_{i=1}^{9} U^\nu_{ai} U^{\nu *}_{ai} F_{LVV} \left(\frac{m^2_{n_i}}{m^2_W} \right),\ c^Y_{aR} = \frac{e g^2 m_{e_a}}{32 \pi^2 m_W^2} \sum_{i=1}^{9} U^\nu_{(a+3)i} U^{\nu *}_{(a+3)i} \frac{m^2_W}{m^2_Y} \times F_{LVV} \left(\frac{m^2_{n_i}}{m^2_Y} \right), \tag{15}$$

where $e = \sqrt{4 \pi \alpha_{\rm em}}$ being the electromagnetic coupling constant and

$$F_{LVV}(x) = -\frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \ln(x)}{24(x-1)^4}. \tag{16}$$

Lagrangian of charged Higgs bosons giving one loop contributions to a_{e_a} is

$$\mathscr{L}^{\ell n H} = -\frac{g}{\sqrt{2} m_W} \sum_{k=1}^{2} \sum_{a=1}^{3} \sum_{i=1}^{9} H^+_k \overline{n_i} \left(\lambda^{L,1}_{ai} P_L + \lambda^{R,1}_{ai} P_R \right) e_a + H.c., \tag{17}$$

where

$$\lambda^{R,1}_{ai} = \frac{m_a c_\theta U^\nu_{(a+3)i}}{c_\beta}, \quad \lambda^{L,1}_{ai} = \sum_{c=1}^{3} \frac{c_\theta}{c_\beta} \times \left[(m^*_D)_{ac} U^{\nu *}_{ci} + t^2_\theta (M^*_R)_{ac} U^{\nu *}_{(c+6)i} \right],$$
$$\lambda^{R,2}_{ai} = m_a U^\nu_{ai} t_\beta, \quad \lambda^{L,2}_{ai} = -t_\beta \sum_{c=1}^{3} (m^*_D)_{ac} U^{\nu *}_{(c+3)i}. \tag{18}$$

We note that the formulas given in Eq. (18) are more general than those in Ref. [16] because of the appearance of t_β or c_β. The two results are the same when $t_\beta = 1$. We emphasize that the couplings $\lambda^{L,R,k} \sim t_\beta$, hence they are large with large t_β. In contrast, it does not affect the couplings of W and Y with charged leptons. This is one of the reasons to explain that contributions of W and Y to AMM are much smaller than those of charged Higgs bosons, as we will point it out numerically in Section IV.

The corresponding one-loop contribution to a_{e_a} caused by charged Higgs bosons is [26]:

$$a^H_{e_a} \equiv -\frac{4m_{e_a}}{e}\sum_{k=1}^{2}\Re[c^{H,k}_{aR}] = \sum_{k=1}^{2} a^{H,k}_{e_a},$$

$$c^{H,k}_{aR} = \frac{eg^2}{32\pi^2 m_W^2 m_{H_k}^2} \times \sum_{i=1}^{9}\left[\lambda^{L,k*}_{ai}\lambda^{R,k}_{ai} m_{n_i} F_{LHH}\left(\frac{m^2_{n_i}}{m^2_{H_k}}\right) + m_{e_a}\left(\lambda^{L,k*}_{ai}\lambda^{L,k}_{ai} + \lambda^{R,k*}_{ai}\lambda^{R,k}_{ai}\right)\tilde{F}_{LHH}\left(\frac{m^2_{n_i}}{m^2_{H_k}}\right)\right], \quad (19)$$

where

$$F_{LHH}(x) = -\frac{1-x^2+2x\ln(x)}{4(x-1)^3}, \quad \widetilde{F}_{LHH}(x) = -\frac{-1+6x-3x^2-2x^3+6x^2\ln(x)}{24(x-1)^4}. \quad (20)$$

We remind that the one loop contributions from neutral Higgs bosons are very suppressed hence they are ignored here. The deviation of a_μ between predictions by the two models 331ISS and SM is

$$\Delta a^{331\text{ISS}}_\mu \equiv \Delta a^W_\mu + a^Y_\mu + a^{H,1}_\mu + a^{H,2}_\mu, \quad \Delta a^W_\mu = a^W_\mu - a^{\text{SM},W}_\mu, \quad (21)$$

where $a^{\text{SM},W}_\mu = 3.887\times 10^{-9}$ [27] is the one-loop contribution from W-boson in the SM framework. In this work, $\Delta a^{331\text{ISS}}_\mu$ will be considered as new physics (NP) predicted by the 331ISS and will be used to compare with experimental data in the following numerical investigation.

The one-loop contributions from Z and Z' bosons were ignored in our calculation because they relate to couplings with only charged lepton μ but not new heavy neutral leptons. Hence the contribution from the Z boson is nearly the same as that in the SM. The contribution from the Z' boson is estimated based on the Z contribution, namely with $m_{Z'} \sim 2$ TeV, we have $\Delta a^{Z'}_\mu \sim a^Z_\mu \times m^2_Z/m^2_{Z'} \sim 10^{-2} a^Z_\mu = \mathscr{O}(10^{-11}) \ll \Delta a^{NP}_\mu = \mathscr{O}(10^{-9})$ [3]. This is also consistent with the result shown in Ref. [12], where $m_{Z'} = 160$ GeV is needed to explain $\Delta a^{Z'}_\mu \sim \Delta a^{NP}_\mu$, leading to $\Delta a^{Z'}_\mu \sim \Delta a^{NP}_\mu \times (160\,\text{GeV})^2/m^2_{Z'} = O(10^{-11})$ with $m_{Z'} \geq 2$ TeV.

IV. NUMERICAL RESULTS

Apart from the experimental neutrino data used as the input we mentioned above, the relevant experimental data is taken from Ref. [3], namely $m_W = 80.385$ GeV, $g = 0.652$, $\alpha_{em} = 1/137$, $m_\mu = 0.105$ GeV, $s^2_W = 0.231$, $e^2 = 4\pi\alpha_{em}$.

We adopt the contribution from new physics to a_μ given in Ref. [3],

$$\Delta a^{\text{NP}}_\mu \equiv a^{\text{exp}}_\mu - a^{\text{SM}}_\mu = (255\pm 77)\times 10^{-11} \Leftrightarrow 178\times 10^{-11} \leq \Delta a^{\text{NP}}_\mu \leq 332\times 10^{-11}, \quad (22)$$

which is also the same order with the choice adopted in [12], namely $\Delta a^{\text{NP}}_\mu \equiv a^{\text{exp}}_\mu - a^{\text{SM}}_\mu = (261\pm 78)\times 10^{-11}$. This is the discrepancy of the experimental value and the SM's prediction. If the 331ISS model explains successfully the experimental data, we will have $\Delta a^{331\text{ISS}}_\mu = \Delta a^{\text{NP}}_\mu$ that must belong to the experimental range given in Eq. (22).

The free input parameters in the 331ISS model are z, k, $m_{H^\pm_1}$, $m_{H^\pm_2}$, t_β and m_Y. As concerned in Ref. [12] that heavier m_Y will give smaller gauge contribution to a_μ hence we will fix $m_Y = 1.7$ TeV corresponding the allowed lower bound of $w = 5.06$ TeV. This leads to the upper bound of $M_R = kz < \sqrt{4\pi}w/\sqrt{2} = 12.7$ TeV. As we discussed above, the coupling of top quark with η generates the top quark mass, leading to the consequence that $t_\beta > 0.3$ in order to satisfy the perturbative limit. Hence, the range of t_β is taken as $0.3 \le t_\beta \le 20$ in our numerical investigation. t_β must have a upper bound in this model because the ρ couples with quark to generate quarks masses at tree level. Similarly to the well-known models such as THDM and the minimal supersymmetric standard model, this upper bound may be $t_\beta < 60$. In addition, because of the perturbative limit given in Eq. (13), large t_β gives small z, which will result in small Δa_μ^{NP}. Hence very large t_β is not interesting to explain the AMM.

The singly charged Higgs masses $m_{H^\pm_{1,2}}$ contain different free parameters and they can be light if f is small enough, which is still acceptable in recent discussions [20, 28]. We note that although f is a coupling beyond the SM, it can be small when the model under consideration respects a discrete symmetry, for example the Z_2 one mentioned in Ref. [29], where ρ and η are even, while χ is odd. Then f is a soft breaking parameter, hence it can be small. In addition, $H_2^\pm$ can be considered nearly as the ones predicted by the Two Higgs Doublet model [29], where the lower bound is $m^2_{H_1^\pm} \ge m^2_{H_2^\pm} \ge 300$ GeV [30]. In this work, we will use the lower bound concerned in Ref. [31], $m^2_{H_1^\pm} \ge m^2_{H_2^\pm} > 600$ GeV . We have checked numerically that $\Delta a_\mu^{331\mathrm{ISS}}$ will be large with small charged Higgs masses. Hence we will fix $m^2_{H_1^\pm} = m^2_{H_2^\pm} = 650$ GeV in the numerical investigation.

To start the numerical investigation, our scan shows that the sign of $\Delta a_\mu^{331\mathrm{ISS}}$ depends strongly on both t_β and z, see the illustration in Table 1, where we fix $k = 10$, $t_\beta = 15$, and $m_{H_1^\pm} = m_{H_2^\pm} = 650$ GeV.

Table 1. One loop contributions to a_μ in the 331ISS model as functions of z, where free parameters are fixed as $k = 10$, $t_\beta = 15$, and $m_{H_1^\pm} = m_{H_2^\pm} = 650$ GeV.

z [GeV]	$\Delta a_\mu^W \times 10^{11}$	$a_\mu^Y \times 10^{11}$	$a_\mu^{H,1} \times 10^{11}$	$a_\mu^{H,2} \times 10^{11}$	$\Delta a_\mu^{331\mathrm{ISS}} \times 10^{11}$
60	-8.503	0.8284	110.2	154.1	256.6
70	-8.573	0.8202	110.2	177.9	280.3
80	-8.624	0.8114	101.7	199.1	293.0
90	-8.663	0.8023	85.08	218.0	295.2
100	-8.693	0.7929	60.51	234.9	287.5
110	-8.718	0.7833	28.26	249.9	270.3
120	-8.737	0.7737	-11.40	263.4	244.0
130	-8.753	0.7640	-58.24	275.5	209.3
140	-8.767	0.7545	-112.0	286.4	166.3
150	-8.778	0.7450	-172.6	296.2	115.6
160	-8.788	0.7356	-239.7	305.1	57.27
170	-8.796	0.7264	-313.3	313.1	-8.264
180	-8.803	0.7174	-393.2	320.5	-80.83

There is an interesting result that $\Delta\, a_\mu^{331\mathrm{ISS}}$ can reach the order of $\mathcal{O}(10^{-9})$, which is the same order with a_μ^{NP} given in Eq. (22). In addition the values of $z \in [60\mathrm{GeV},\ 130\mathrm{GeV}]$ can explain a_μ^{NP}. For $z \ge 170$ GeV, $a_\mu^{H,1}$ becomes negative, resulting in that $\Delta\, a_\mu^{331\mathrm{ISS}}$ decreases. In deed, the perturbative limit gives a constraint that $z < 1233/t_\beta = 82$ GeV, hence all values relating with $z > 80$ GeV in Table 1 are excluded. Fortunately, the values of z giving $\Delta a_\mu^{331\mathrm{ISS}}$ consistent with experimental data are still allowed.

Table 2. One loop contributions to a_μ in the 331ISS model as functions of k and z, where free parameters are fixed as $t_\beta = 10$, and $m_{H_1^\pm} = m_{H_2^\pm} = 650$ GeV.

$\{k,\ z\,[\mathrm{GeV}]\}$	$\Delta a_\mu^W \times 10^{11}$	$a_\mu^Y \times 10^{11}$	$a_\mu^{H,1} \times 10^{11}$	$a_\mu^{H,2} \times 10^{11}$	$\Delta a_\mu^{331\mathrm{ISS}} \times 10^{11}$
{6,50}	-11.62	0.8480	77.37	83.11	149.7
{6,60}	-12.03	0.8447	95.07	105.8	189.7
{6,70}	-12.33	0.8411	110.1	127.9	226.5
{6,80}	-12.56	0.8370	122.0	149.0	259.3
{6,90}	-12.75	0.8326	130.4	169.0	287.5
{6,100}	-12.89	0.8279	135.2	187.8	310.9
{6,110}	-13.01	0.8230	136.3	205.4	329.6
{6,120}	-13.11	0.8178	133.8	221.9	343.3
{7,50}	-10.41	0.8454	68.03	75.25	133.7
{7,60}	-10.68	0.8412	81.20	94.34	165.7
{7,70}	-10.88	0.8365	91.22	112.5	193.7
{7,80}	-11.03	0.8313	97.72	129.5	217.0
{7,90}	-11.14	0.8257	100.6	145.3	235.6
{7,100}	-11.24	0.8198	99.68	160.0	249.3
{7,110}	-11.31	0.8137	95.10	173.5	258.1
{7,120}	-11.37	0.8073	86.88	186.0	262.3
{8,50}	-9.536	0.8425	59.65	68.30	119.3
{8,60}	-9.723	0.8373	69.08	84.45	144.6
{8,70}	-9.859	0.8314	75.02	99.49	165.5
{8,80}	-9.961	0.8250	77.29	113.3	181.5
{8,90}	-10.04	0.8183	75.83	126.0	192.6
{8,100}	-10.10	0.8112	70.66	137.6	199.0
{8,110}	-10.15	0.8038	61.85	148.1	200.7
{8,120}	-10.19	0.7963	49.50	157.8	197.9
{9,50}	-8.892	0.8394	52.17	62.16	106.3
{9,60}	-9.026	0.8330	58.47	75.90	126.2
{9,70}	-9.122	0.8260	61.12	88.46	141.3
{9,80}	-9.193	0.8184	60.02	99.84	151.5
{9,90}	-9.248	0.8104	55.21	110.1	156.9
{9,100}	-9.290	0.8022	46.76	119.4	157.6
{9,110}	-9.324	0.7937	34.77	127.7	154.0
{9,120}	-9.352	0.7851	19.35	135.2	146.0

For small $t_\beta = 10$, corresponding to $z \leq 123$ GeV obtained from Eq. (13), we can see the dependence of different one-loop contributions to $a_\mu^{331\mathrm{ISS}}$ as functions of z and k in Table 2.

We can see that the allowed smallest $k = 6$ allows large $a_\mu^{331\mathrm{ISS}}$ which enhances with increasing z. The maximal values correspond to the allowed largest z which is constrained by the perturbative limit. This value of k can explain successfully the experimental data. On the other hand, the maximal max($a_\mu^{331\text{ISS}}$) decreases with larger k. When $k \geq 9$, it can be seen that max($a_\mu^{331\text{ISS}}$) $< 178 \times 10^{-11}$ which is lower bound allowed by the experimental data. Hence, in order to get large max($a_\mu^{331\text{ISS}}$) we will choose $k = 6$ for studying the case of small t_β.

There are some interesting properties in the Table 1 as the following: i) Δa_μ^W and a_μ^Y are much smaller than those from the Higgs contributions; ii) $a_\mu^{H,1}$ may be negative with large z ; iii) $a_\mu^{331\text{ISS}}$ has a maximal value for a value of z, namely $z \in$ [80GeV, 100GeV]. The first property is consistent with the previous studies mentioned in this work. Two remaining properties imply that the dependence of $a_\mu^{331\text{ISS}}$ on z, k and t_β is rather complicated. Our numerical scan suggests that large values of $a_\mu^{331\text{ISS}}$ require small k and large t_β, see the numerical illustration shown in Table 2.

For illustration the case of small t_β and k, we choose $t_\beta = 5$ corresponding to $z < 245$ GeV, and $k = 6$ to get large max($a_\mu^{331\text{ISS}}$). The numerical results are shown in Table 3. We get

Table 3. One loop contributions to a_μ in the 331ISS model as functions of z, where free parameters are fixed as $k = 6$, $t_\beta = 5$, and $m_{H_1^\pm} = m_{H_2^\pm} = 650$ GeV.

z [GeV]	$\Delta a_\mu^W \times 10^{11}$	$a_\mu^Y \times 10^{11}$	$a_\mu^{H,1} \times 10^{11}$	$a_\mu^{H,2} \times 10^{11}$	$\Delta a_\mu^{331\text{ISS}} \times 10^{11}$
90	-12.75	0.8326	33.46	42.25	63.80
100	-12.89	0.8279	34.69	46.95	69.57
110	-13.01	0.8230	34.96	51.35	74.13
120	-13.11	0.8178	34.29	55.47	77.46
130	-13.19	0.8124	32.67	59.31	79.60
140	-13.26	0.8069	30.13	62.89	80.56
150	-13.32	0.8013	26.67	66.23	80.38
160	-13.37	0.7956	22.33	69.34	79.09
170	-13.42	0.7899	17.10	72.24	76.72
180	-13.45	0.7841	11.02	74.95	73.31
190	-13.49	0.7782	4.108	77.49	68.88
200	-13.52	0.7724	-3.629	79.86	63.48
210	-13.55	0.7665	-12.17	82.08	57.13
220	-13.57	0.7607	-21.50	84.16	49.85
230	-13.59	0.7549	-31.60	86.11	41.67
240	-13.61	0.7491	-42.47	87.94	32.61

max($a_\mu^{331\text{ISS}}$) $\simeq 81 \times 10^{-11}$ which is still smaller than that obtained from $t_\beta = 10$ and 15 indicated in the two Tables 1 and 2. The reason is that negative $a_\mu^{H,1}$ does not allow large and positive $a_\mu^{331\text{ISS}}$. In general, after checking numerically, we conclude that small $t_\beta < 5$ does not give large $a_\mu^{331\text{ISS}}$ enough to explain successfully the experimental data (22).

V. CONCLUSIONS

In this work, we have shown that the 331ISS [16] can explain successfully the recent experimental data of a_μ if the relation $v_1 = v_2$ is released. In addition, more necessary conditions for the free parameters resulting in consistent a_μ with experiment are: i) $t_\beta \equiv v_2/v_1$ should be large, ii)

$k = M_R/z$ should be small. These conditions should be paid attention to in further studies relating with the 331ISS model discussed in this work. The conclusion may be still true for other 3-3-1 models with the ISS mechanism in order to explain successfully the experimental data of a_μ. This will be our future topic.

ACKNOWLEDGMENT

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2018.331.

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